Plan for Lecture 12 (Chapters 30):

Sources of Magnetic fields

1. “Permanent” magnets
2. Biot-Savart Law; magnetic fields from a current-carrying wire
3. Ampere’ Law
4. Magnetic fields in a solenoid

Remember to send in your chapter reading questions...

Review of magnetic forces:

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

$$d\mathbf{F}_B = I
ds \times \mathbf{B}$$
Vector cross product

\[ \hat{k} \times \hat{j} = \hat{i} \]
\[ \hat{j} \times \hat{k} = \hat{i} \]
\[ \hat{k} \times \hat{i} = \hat{j} \]
\[ \hat{j} \times \hat{i} = -\hat{k} \]

Sources of magnetic fields \( B \)

Permanent magnet materials – Fe, Fe₂O₃, Co, Ni, alloys

Internal atomic level magnet dipole moments \( \mu \)

\[ \Rightarrow \] Energy incentive for neighboring magnetic dipoles to align at temperatures below Curie temperature.

Visualization of intrinsic spin magnetic moment of electron.


\[ B = 0 \]

In an unmagnetized substance, the atomic magnetic dipoles are randomly oriented.

\[ B > 0 \]

When an external field \( B \) is applied, the magnetic moments, components of magnetic moment to line up in the same direction as \( B \) and increase the sample's net magnetization.

\[ B \gg 0 \]

In the field's intense core, atoms align with the external field; become very magnetic.

“Permanent” magnets controlled by temperature and external magnetic fields.
Magnetic fields generated by moving charge:

Electrical field generated by charge distribution
\[
\mathbf{E}(r) = \sum \frac{Q}{4\pi \|r-r'\|} \mathbf{r}'
\]

Magnetic field generated by current distribution
\[
\mathbf{J}(r) = \sum \frac{Q v}{4\pi \|r-r'\|} \mathbf{r}'
\]

Biot-Savart Law:
Units: \( B \rightarrow T \) (Tesla) \( J \rightarrow I/\text{area} \rightarrow A/m^2 \)

Constant: \( \mu_0 \) “permeability” of free space \( 4\pi \times 10^{-7} \ Tm/A \)
\[
\mathbf{B}(r) = \frac{\mu_0 I}{4\pi} \int \frac{\mathbf{J}(r') \times \mathbf{r} - \mathbf{r}' \, d^3r}{\|r-r'\|}
\]

For thin wire of constant current \( I \), where \( s \) denotes direction along wire:
\[
\mathbf{B}(r) = \frac{\mu_0 I}{4\pi} \int ds \times \frac{r-s}{\|r-s\|}
\]

Magnetic field produced by straight wire:
\[
B(r_p) = \frac{\mu_0 I}{2 \pi a} \int_{x_{\min}}^{x_{\max}} \frac{dx}{\left(a^2 + x^2\right)^{3/2}}
\]
\[
= \frac{\mu_0 I}{4 \pi a} \cos \theta \, d\theta \quad B(r_p) = \frac{\mu_0 I}{4 \pi a} \left(\sin \theta_2 - \sin \theta_1\right)
\]
Consider 2 (infinitely long) wires. Vote for the least incorrect answer:
A. The two wires cancel each other completely
B. The two wires generate exactly twice the magnet field compared to a single wire
C. The two wires attract each other.
D. The two wires repel each other.
\[ F = \ell I \times B \]

\[ \text{wires repel each other} \]

Magnetic field produced by wire loop:

\[ B(r_{p}) = \hat{x} \frac{\mu_{0} I^{2} a}{4\pi} \frac{a^{2} d\theta}{(a^{2} + x^{2})^{3/2}} \]

\[ = \hat{x} \frac{\mu_{0} I^{2} a}{4\pi} \frac{2\pi a^{2}}{2(a^{2} + x^{2})^{3/2}} = \hat{x} \frac{\mu_{0} I^{2} a}{2(a^{2} + x^{2})^{3/2}} \]

Easier way for situations with high symmetry

– Ampere’s Law

\[ \oint B \cdot ds = \mu_{0} I_{in} \]

In this case

\[ \oint B \cdot ds = 2\pi a B = \mu_{0} I \]

\[ \Rightarrow B = \frac{\mu_{0} I}{2\pi a} \]
Example of Ampere’s law:
The figure below shows a wire coming out of the screen. Which of the paths for \( \int B \cdot ds \) has the smallest magnitude?

\[ A. \ a \quad B. \ b \quad C. \ c \quad D. \ d \]

Magnetic field in a solenoid

Ideal solenoid

Magnetic field inside ideal solenoid:

Ampere’s Law:
\[ \int B \cdot ds = \mu_0 I_n \]
\[ \int B \cdot ds = B \ell = \mu_0 N I \]

\[ B = \frac{\mu_0 N I}{\ell} \]
$B = \frac{\mu_0 NI}{\ell}$

Experimental MRI at FSU

http://www.magnet.fsu.edu/education/tutorials/magnetacademy/mri/images/mri-scanner.jpg

$B = 21 \ T$

MRI signal – detects H nuclei using the magnetic moment of $H - \mu_H$.

$$E = -\mu_H \cdot B$$

Summary:

Ampere's law:

- Integral form: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{ext}}$
- Differential form: $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$

Gauss's law:

- Integral form: $\oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = \frac{\rho}{\varepsilon_0}$
- Differential form: $\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$
- Note:
  - $\oint \mathbf{B}(\mathbf{r}) \cdot d\mathbf{A} = 0$
  - $\nabla \cdot \mathbf{B} = 0$