PHY 114 A General Physics II
11 AM-12:15 PM TR Olin 101

Plan for Lecture 14 (Chapter 32):

Inductance

1. Inductors as a circuit element
2. RL, LC, and RLC circuits
3. Energy stored in an inductor

Remember to send in your chapter reading questions…

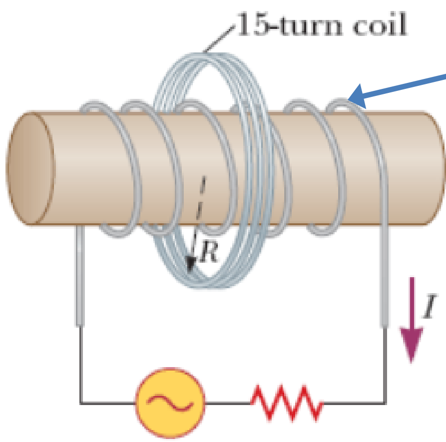
7	02/14/2012	Current and resistance	27.1-27.6	27.3,27.12,27.29	02/16/2012
8	02/16/2012	Direct current circuits	28.1-28.2	28.3,28.7,28.19	02/21/2012
9	02/21/2012	Direct current circuits	28.3-28.5	28.23,28.25,28.34	02/23/2012
10	02/23/2012	Review	26.1-28.5	(Review for exam)	
	02/28/2012	Exam			
11	03/01/2012	Magnetic fields	29.1-29.6	29.5,29.12,29.47	03/06/2012
12	03/06/2012	Magnetic field sources	30.1-30.6	30.5,30.21,30.29	03/08/2012
13	03/08/2012	Faraday's law	31.1-31.5	31.12,31.23,31.40	03/20/2012
	03/13/2012	*No class (Spring Break)*			
	03/15/2012	*No class (Spring Break)*			
14	03/20/2012	Induction and AC circuits	32.1-32.6	32.4,32.20,32.43	03/22/2012
15	03/22/2012	AC circuits	33.1-33.9	33.8,33.24,33.71	03/27/2012
16	03/27/2012	Electromagnetic waves	34.1-34.3	34.3,34.10,34.13	03/29/2012
17	03/29/2012	Electromagnetic waves	34.4-34.7	34.22,34.46,34.57	04/03/2012
18	04/03/2012	Ray optics	35.1-35.8		

Web assign question -- hint

2. **–/0.334 points** SerPSE8 31.P.012.WI.

A coil of 15 turns and radius 10.0 cm surrounds a long solenoid of radius 2.10 cm and 1.00×10^3 turns/meter (see figure below). The current in the solenoid changes as $I = 8.00 \sin 120\, t$, where I is in amperes and t is in seconds. Find the induced emf (in volts) in the 15-turn coil as a function of time.

$\mathcal{E}$ = [ ]



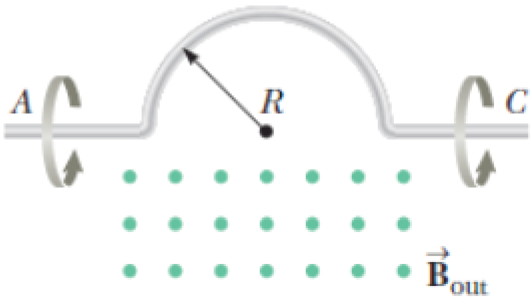
1000 turns/m

$$\mathcal{E} = -N_c \frac{d\Phi_B}{dt}$$

Web assign question -- hint

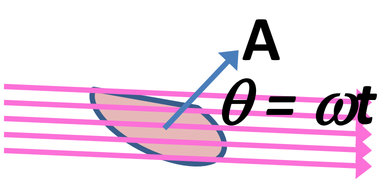
0/0.333 points SerPSE8 31.P.040. [1470785]

In the figure below, a semicircular conductor of radius R = 0.260 m is rotated about the axis AC at a constant rate of 160 rev/min. A uniform magnetic field of magnitude 1.28 T fills the entire region below the axis and is directed out of the page.



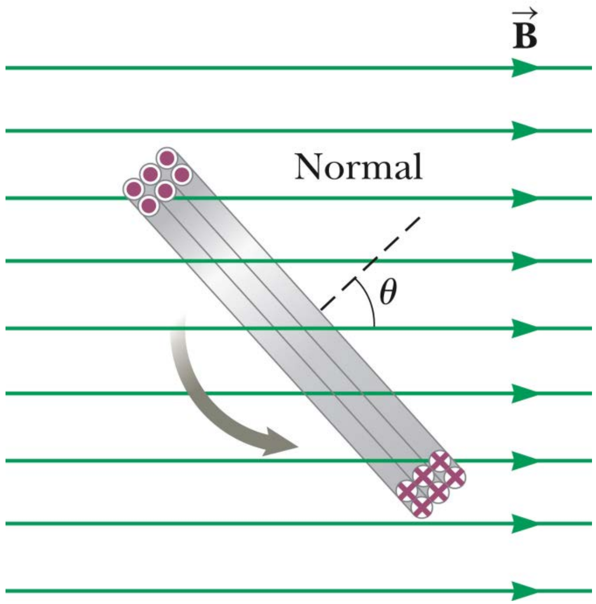
(a) Calculate the maximum value of the emf induced between the ends of the conductor.

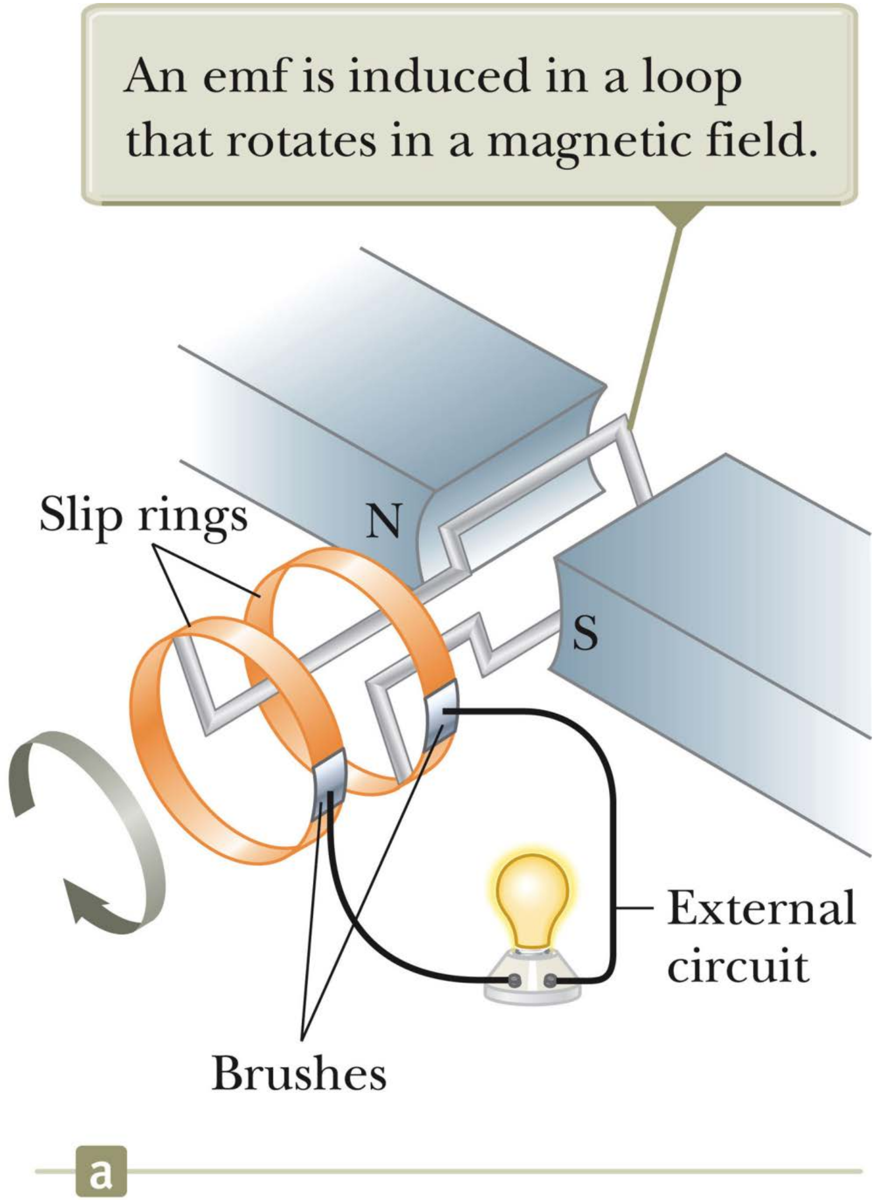
(b) What is the value of the average induced emf for each complete rotation?



$$\Phi_B = \mathbf{B} \cdot \mathbf{A} = \frac{B\pi R^2}{2}\cos(\theta) = \frac{B\pi R^2}{2}\cos(\omega t)$$

Electric generator:





$$\theta = \omega t$$

$$\mathcal{E} = -N\frac{d\left(BA\cos(\omega t)\right)}{dt}$$

$$= NBA\omega\sin(\omega t)$$

Electric generator:

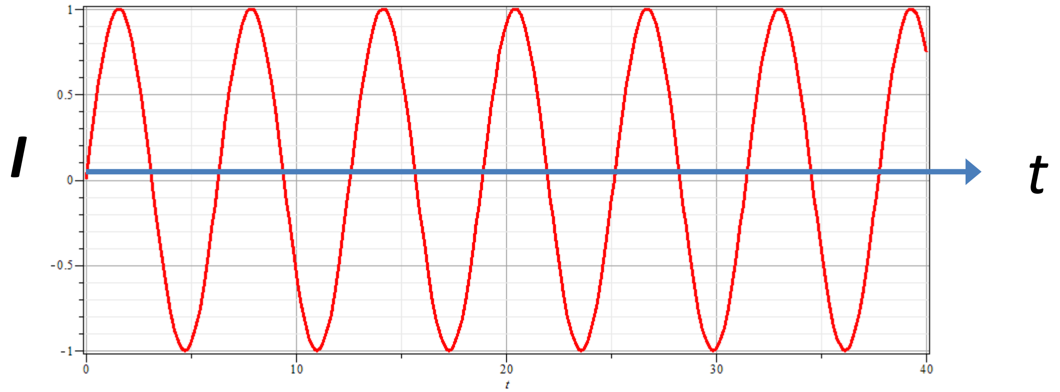
$$\mathcal{E} = -N\frac{d\left(BA\cos(\omega t)\right)}{dt}$$

$$= NBA\omega\sin(\omega t)$$

If the resistance in the generator coil is R,

current in the coil is given by:

$$I = \frac{\mathcal{E}}{R} = \frac{NBA\omega}{R}\sin(\omega t) \equiv I_{\max}\sin(\omega t)$$



Summary:

Ampere's law:

Integral form: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{in}$

Differential form: $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$

Faraday's law:

$$\int E(\mathbf{r}) \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B}(r) \cdot dA$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Gauss's law:

Integral form: $\oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = \frac{Q_{in}}{\varepsilon_0}$

Differential form: $\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$

$$\oint \mathbf{B}(\mathbf{r}) \cdot d\mathbf{A} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

Faraday's law for EMF induced in a current loop:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

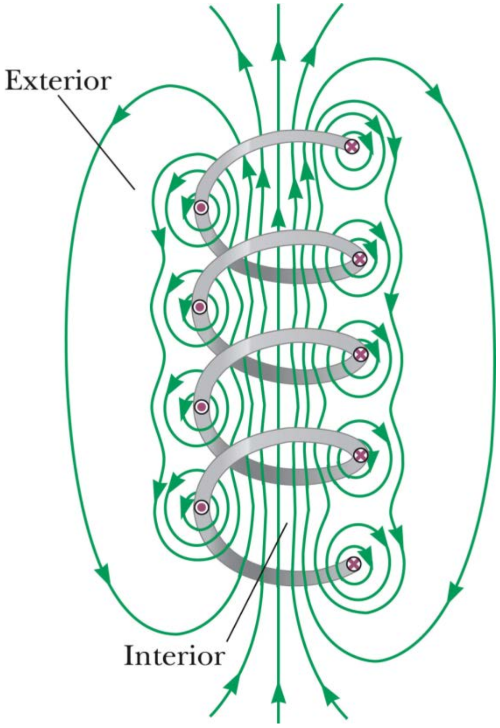
Examples of $\frac{d\Phi_B}{dt}$:

Field inside solenoid:

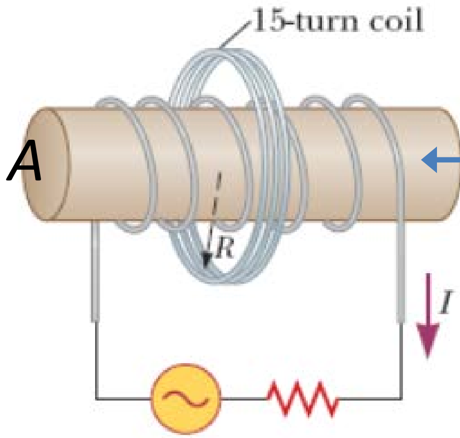
$$B = \frac{\mu_0 N I}{\ell}$$

If I is changing with time:

$$\frac{dB}{dt} = \frac{\mu_0 N}{\ell}\frac{dI}{dt}$$



"Mutual" inductance

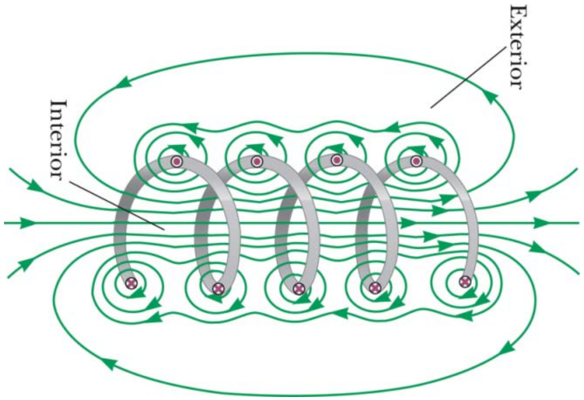


$=N_c$

$$\frac{N_s}{\ell} = 1000 \text{ turns/m}$$

$$\mathcal{E}_c = -N_c \frac{d\Phi_B}{dt} = -N_c \frac{\mu_0 N_s A}{\ell} \frac{dI}{dt}$$

"Self" inductance



$$\mathcal{E}_s = -N_s \frac{d\Phi_B}{dt} = -N_s \frac{\mu_0 N_s A}{\ell} \frac{dI}{dt} = -\underbrace{\frac{\mu_0 N_s^2 A}{\ell}}_{L} \frac{dI}{dt}$$

L inductance

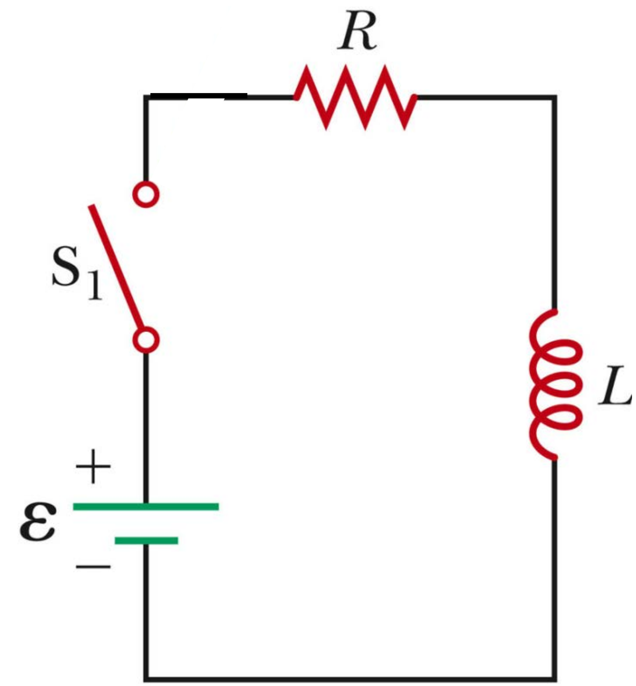
Units of inductance:

$$1\,\text{Henry} = \frac{1\,\text{Volt}}{1\,\text{Amp/s}}$$

Inductors in a circuit

$$\mathcal{E} = -L\frac{dI}{dt}$$

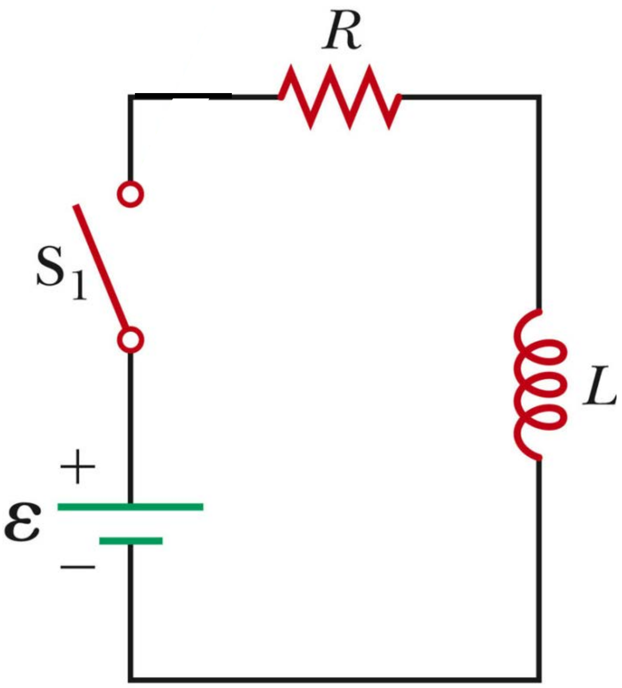
Example: LR circuit



With switch closed:

$$\mathcal{E}_{EMF} - RI - L\frac{dI}{dt} = 0$$

Example: LR circuit



With switch closed:

$$\mathcal{E}_{EMF} - RI - L\frac{dI}{dt} = 0$$

Solve differential equation:

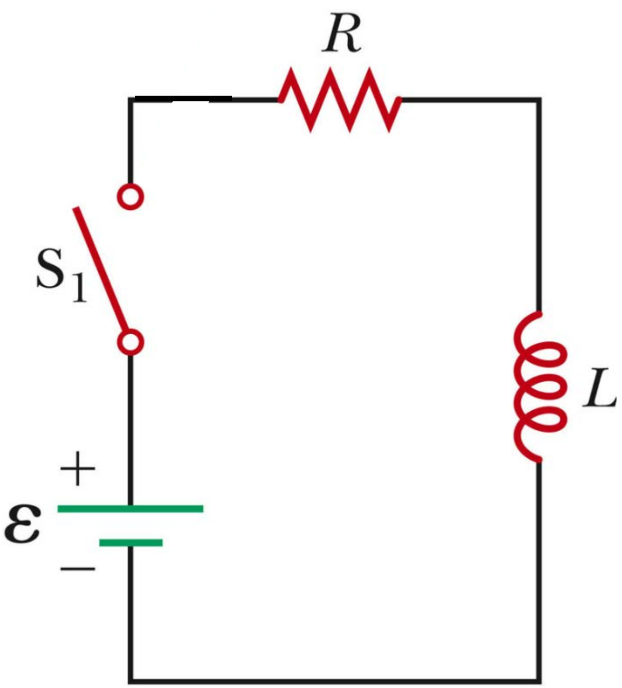
$$\frac{dI}{dt} = \frac{\mathcal{E}_{EMF}}{L} - \frac{R}{L}I = \frac{R}{L}\left(\frac{\mathcal{E}_{EMF}}{R} - I\right)$$

$$\frac{dI}{\frac{\mathcal{E}_{EMF}}{R} - I} = \frac{R}{L}dt$$

$$-\ln\left(\frac{\mathcal{E}_{EMF}}{R} - I\right) = \frac{R}{L}(t - t_0)$$

$$I(t) = \frac{\mathcal{E}_{EMF}}{R}\left(1 - e^{-\frac{R}{L}(t-t_0)}\right)$$

Example: LR circuit

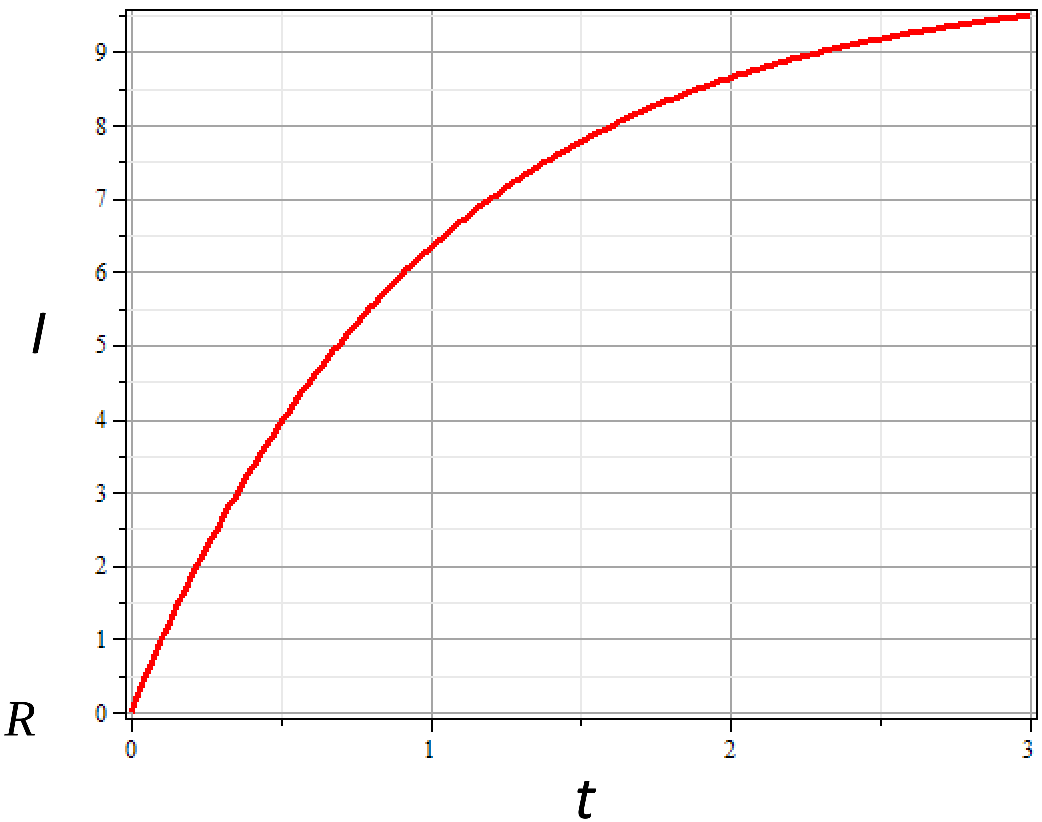


With switch closed:

$$I(t) = \frac{\mathcal{E}_{EMF}}{R}\left(1 - e^{-\frac{R}{L}(t-t_0)}\right)$$

$$= \frac{\mathcal{E}_{EMF}}{R}\left(1 - e^{-(t-t_0)/\tau}\right) \qquad \tau = L/R$$

$$\tau = \frac{\text{Volts/(Amp/s)}}{\text{Volts/Amp}}$$



The LR circuit reminds us of:

A. Why physics class is so beautiful.
B. Why physics class is so terrible.
C. RC circuit.
D. Money in the bank.

Energy storage in inductor:

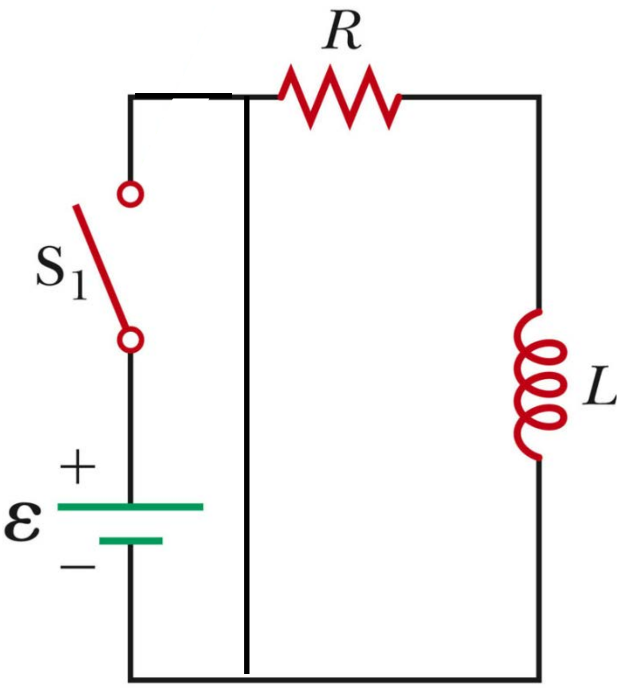
$$\mathcal{E} = -L\frac{dI}{dt}$$

$$dU = VdQ = V\frac{dQ}{dt}dt = L\frac{dI}{dt}I\ dt = \frac{d}{dt}\left(\frac{1}{2}LI^2\right)dt$$

$$\Rightarrow dU = d\left(\frac{1}{2}LI^2\right)$$

$$\Rightarrow U = \frac{1}{2}LI^2$$

Example: LR circuit continued



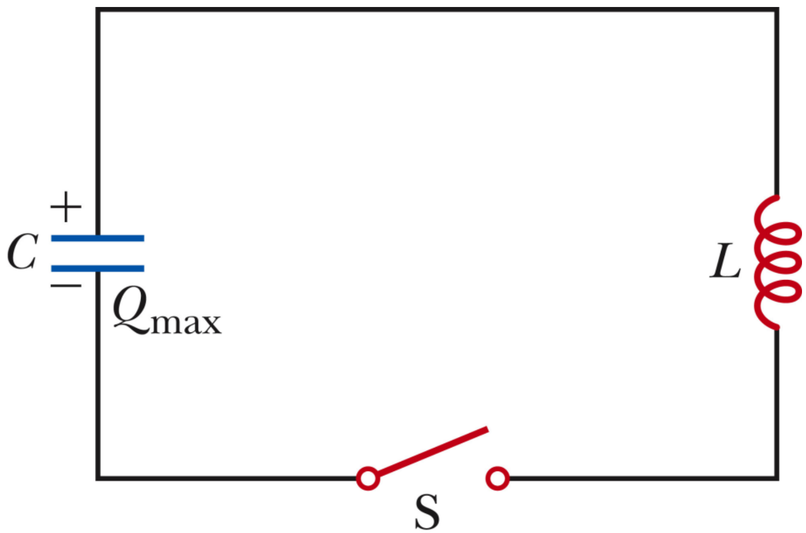
With switch open and wire placed:

$$-RI - L\frac{dI}{dt} = 0$$

$$\frac{dI}{I} = -\frac{R}{L}dt$$

$$I(t) = I_1 e^{-\frac{R}{L}(t-t_1)} = \frac{\mathcal{E}_{EMF}}{R} e^{-\frac{R}{L}(t-t_1)}$$

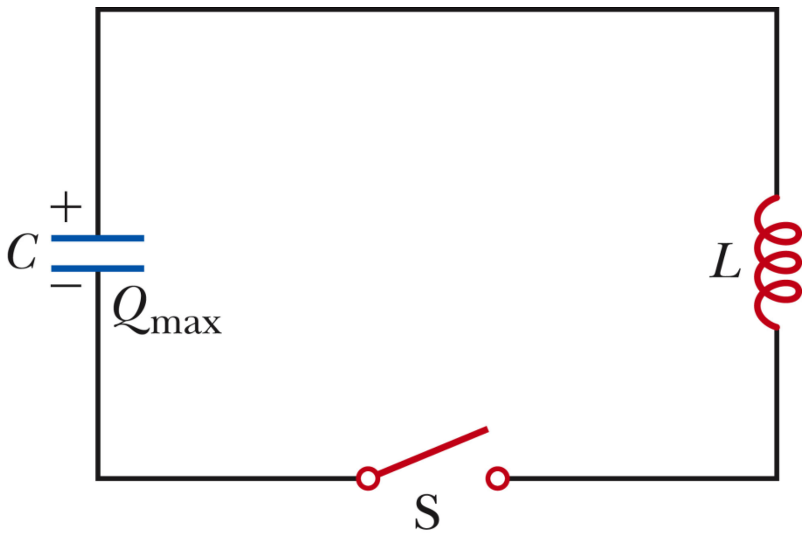
LC circuit



This circuit has the following time dependence after the switch is closed:

A. Charge decays with time
B. Charge increases with time
C. Charge remains constant with time
D. Charge increases and decreases with time

LC circuit



With switch closed:

$$-\frac{Q}{C} - L\frac{dI}{dt} = 0$$

$$-\frac{Q}{C} - L\frac{d^2Q}{dt^2} = 0$$

$$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

Check units:

$$\omega \equiv \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\text{Henry}\cdot\text{Farad}}}$$

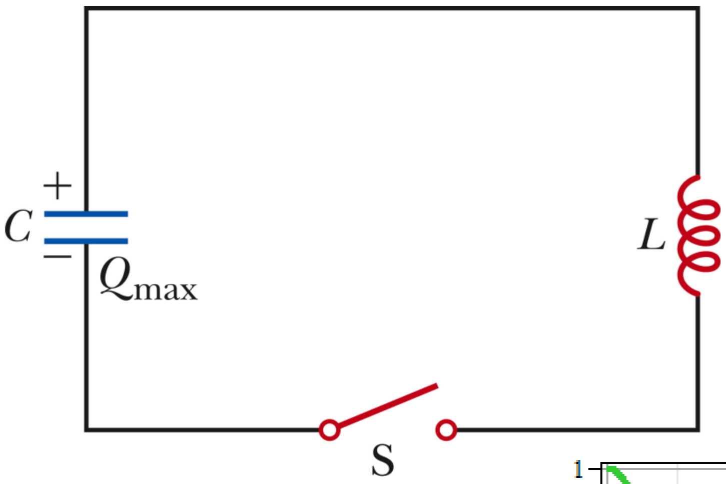
$$= \frac{1}{\sqrt{\frac{\text{Volts}}{\text{Amp/s}}\cdot\frac{\text{Coulombs}}{\text{Volts}}}} = \frac{1}{s}$$

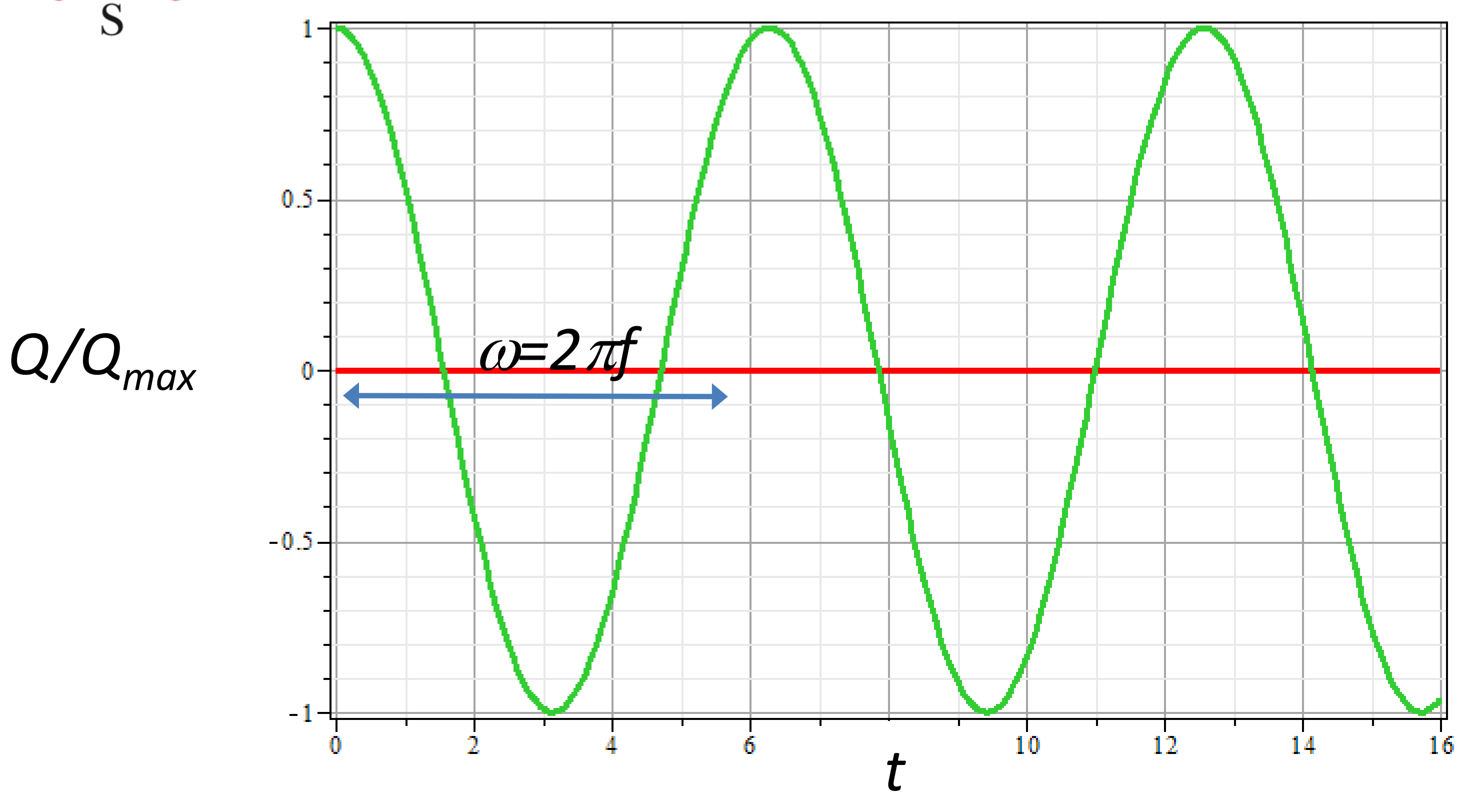
Solution assuming $Q(t=0) = Q_{\max}$:

$$Q(t) = Q_{\max}\cos(\omega t)$$

where $\omega \equiv \frac{1}{\sqrt{LC}}$

LC circuit For L=1 H, C=1 F; ω=1 rad/s





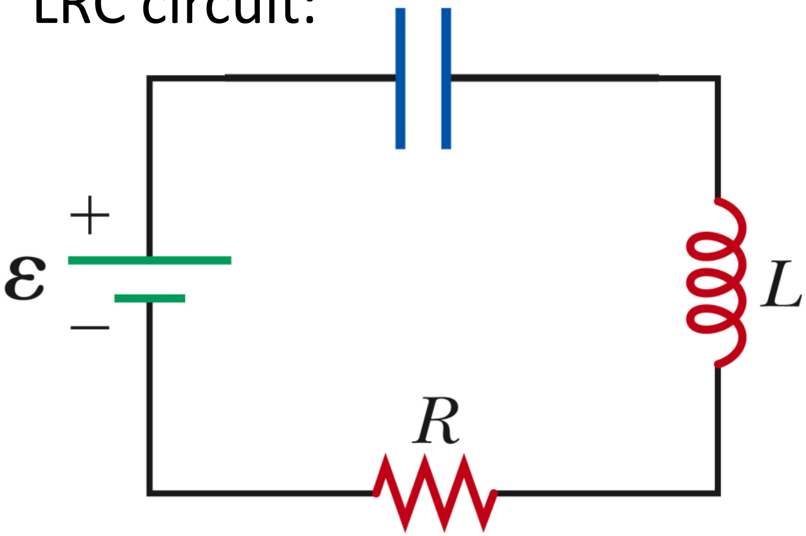
The LC circuit is mathematically analogous to:

A. Nothing seen previously seen in class
B. An elastic bouncing ball
C. A swinging pendulum
D. A mass attached to a spring
E. All above, except A

The LC circuit is useful:

A. Because of its mathematical analogies
B. Physicists like simple formulae like cos(ωt)
C. It could be useful in science experiments
D. It could be useful for toys
E. It could be useful for everyday appliances

LRC circuit:



$$\mathcal{E} - \frac{Q}{C} - L\frac{dI}{dt} - RI = 0$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q - \mathcal{E} = 0$$

Solution assuming $Q(t=0) = 0$:

$$Q(t) = C\mathcal{E}\left(1 - e^{-Rt/2L}\cos(\omega' t)\right)$$

$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$