PHY 114 A General Physics II
11 AM-12:15 PM TR Olin 101

Plan for Lecture 15 (Chapter 33):

Alternating current circuits

1. **AC EMF**
2. **R, C, L behaviors in AC circuit**
3. **RLC circuits**

Remember to send in your chapter reading questions…

10	02/23/2012	Review	28.1-28.5	(Review for exam)	
	02/28/2012	Exam			
11	03/01/2012	Magnetic fields	29.1-29.6	29.5,29.12,29.47	03/06/2012
12	03/06/2012	Magnetic field sources	30.1-30.6	30.5,30.21,30.29	03/08/2012
13	03/08/2012	Faraday's law	31.1-31.5	31.12,31.23,31.40	03/20/2012
	03/13/2012	*No class (Spring Break)*			
	03/15/2012	*No class (Spring Break)*			
14	03/20/2012	Induction and AC circuits	32.1-32.6	32.4,32.20,32.43	03/22/2012
15	03/22/2012	AC circuits	33.1-33.9	33.8,33.24,33.71	03/27/2012
16	03/27/2012	Electromagnetic waves	34.1-34.3	34.3,34.10,34.13	03/29/2012
17	03/29/2012	Electromagnetic waves	34.4-34.7	34.22,34.46,34.57	04/03/2012
18	04/03/2012	Ray optics Evening exam	35.1-35.8		
19	04/05/2012	Image formation Evening exam	36.1-36.4		

3rd exam will be scheduled for evenings during the week of 4/2/2012

Homework hint:

–/0.333 points My Notes | SerPSE8 32.P.04

In the circuit of Figure P32.48, the battery emf $\mathcal{E}$ is 40 V, the resistance R is 190 Ω, and the capacitance C is 0.500 μF. The switch S is closed for a long time, and no voltage is measured across the capacitor. After the switch is opened, the potential difference across the capacitor reaches a maximum value of 150 V. What is the value of the inductance L?

H

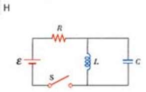

Figure P32.48

"The switch S is closed for a long time, and no voltage is measured across the capacitor." Which of the following statements is false?

A. The voltage across the inductor and the capacitor must always be the same.
B. After a long time the current through the inductor is $\mathcal{E}$/R.
C. After a long time the current through the resistor is $\mathcal{E}$/R.
D. After a long time, the current through the inductor is 0.

Homework hint:

–/0.333 points | My Notes | SerPSE8 32.P.04

In the circuit of Figure P32.48, the battery emf $\mathcal{E}$ is 40 V, the resistance R is 190 Ω, and the capacitance C is 0.500 μF. The switch S is closed for a long time, and no voltage is measured across the capacitor. After the switch is opened, the potential difference across the capacitor reaches a maximum value of 150 V. What is the value of the inductance L?

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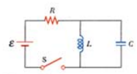
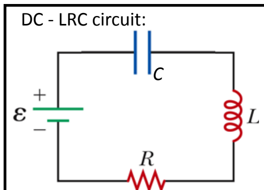


Figure P32.48

After the switch S is opened (*t=0*) which of the following statements are true?

A. The circuit is now an LC circuit with the initial condition that $Q(t=0)=C\mathcal{E}$.
B. The circuit is now an LC circuit with the initial condition that $I(t=0)=\mathcal{E}/R$.
C. The current will oscillate forever with an angular frequency of $\omega = 1/\sqrt{LC}$.

DC - LRC circuit:



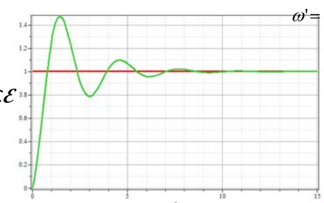
$$\mathcal{E} - \frac{Q}{C} - L\frac{dI}{dt} - RI = 0$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q - \mathcal{E} = 0$$

Solution assuming $Q(t=0)=0$:

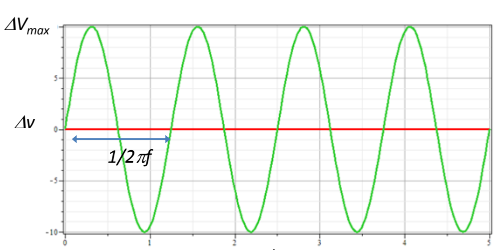
$$Q(t) = C\mathcal{E}\left(1 - e^{-Rt/2L}\cos(\omega' t)\right)$$

$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$



AC EMF source

$$\mathcal{E}(t) \equiv \Delta v(t) = \Delta V_{\max}\sin(\omega t) = \Delta V_{\max}\sin(2\pi f t)$$



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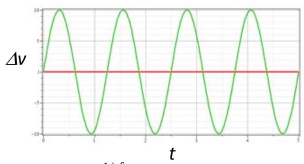


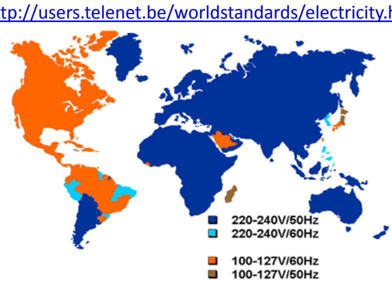
$$\langle \Delta v(t) \rangle = \int_0^{1/f} \Delta V_{\max}\sin(2\pi f t)\,dt = 0$$

$$\langle \Delta v(t) \rangle^2 = \int_0^{1/f} [\Delta V_{\max}\sin(2\pi f t)]^2 dt = \frac{1}{2}|\Delta V_{\max}|^2 \equiv |\Delta V_{rms}|^2$$

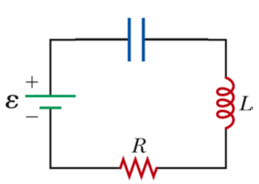
In US the standard wall voltage is ΔV_{rms} =120 V with *f* =60 Hz
In Europe the standard wall voltage is ΔV_{rms} =220 V with *f* =50 Hz

Voltage standards throughout the world:

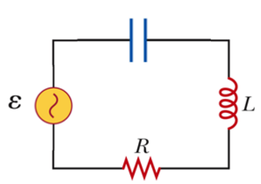
http://users.telenet.be/worldstandards/electricity.htm#voltage



DC - LRC circuit:



AC - LRC circuit:

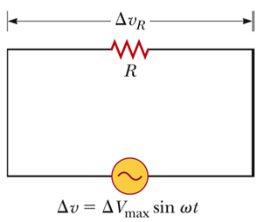


Which of the following statements most accurately reflect the relationships between the behaviors of LRC circuits with a DC versus an AC EMF source.

A. The differential equations are different and the solutions are different.
B. The differential equations are the same – but the solutions are different.
C. The differential equations are the same – so the solutions have to be the same.

In fact: The DC solutions to the differential equations are generally valid representations of transient behaviors. Often we are interested in the "steady state" solutions in the presence of AC voltage sources.

AC EMF with resistor:

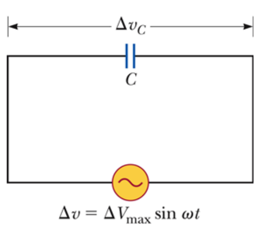


$$\Delta v - RI = 0$$

$$\Delta V_{max} \sin(\omega t) - RI(t) = 0$$

$$\Rightarrow I(t) = \frac{\Delta V_{max}}{R} \sin(\omega t)$$

AC EMF with capacitor:



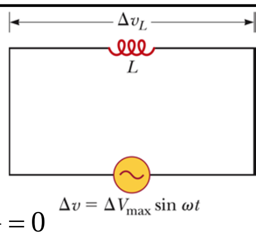
$$\Delta v - \frac{Q}{C} = 0$$

$$\Delta V_{max} \sin(\omega t) - \frac{Q(t)}{C} = 0$$

$$\Rightarrow Q(t) = C\Delta V_{max} \sin(\omega t)$$

$$\Rightarrow I(t) = \frac{dQ(t)}{dt} = \omega C\Delta V_{max} \cos(\omega t)$$

AC EMF with inductor:



$$\Delta v - L\frac{dI}{dt} = 0$$

$$\Delta V_{\max}\sin(\omega t) - L\frac{dI(t)}{dt} = 0$$

$$\Rightarrow \frac{dI(t)}{dt} = \frac{\Delta V_{\max}}{L}\sin(\omega t)$$

$$\Rightarrow I(t) = -\frac{\Delta V_{\max}}{\omega L}\cos(\omega t)$$

Summary of results for each circuit element:

General AC EMF of the form :

$$\Delta v(t) = \Delta V_{\max}\sin(\omega t - \phi)$$

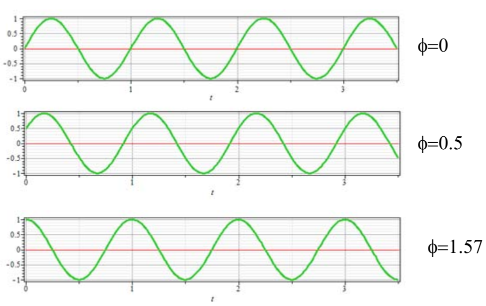
With resistor only : $I(t) = \frac{\Delta V_{\max}}{R}\sin(\omega t - \phi)$

With capacitor only : $I(t) = \omega C \Delta V_{\max}\cos(\omega t - \phi)$

With inductor only : $I(t) = -\frac{\Delta V_{\max}}{\omega L}\cos(\omega t - \phi)$

Comment about phase factor ϕ:

Plots of $\sin(2\pi t+\phi)$



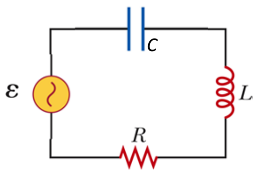
Series AC LRC circuit

$$\mathcal{E} - \frac{Q}{C} - L\frac{dI}{dt} - RI = 0$$

$$\mathcal{E} = \Delta V_{\max}\sin(\omega t)$$

Assume solution $I(t)$ to have the form :

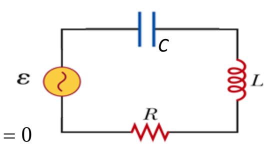
$$I(t) = I_{\max}\sin(\omega t - \phi)$$



Declare success if :

- $I(t)$ satisfies differential equations
- $I_{\max}$ has a consistent value
- ϕ has a consistent value

Series AC LRC circuit



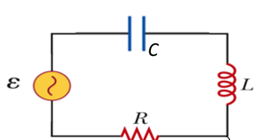
$$\Delta V_{\max}\sin(\omega t) - \frac{Q}{C} - L\frac{dI}{dt} - RI = 0$$

Assuming : $I(t) = I_{\max}\sin(\omega t - \phi)$

$$\Delta V_{\max}\sin(\omega t) + \frac{I_{\max}\cos(\omega t - \phi)}{\omega C} - \omega L I_{\max}\cos(\omega t - \phi) - R I_{\max}\sin(\omega t - \phi) = 0$$

➔ Must be true at all *t*; special values of I_{max} and ϕ

Series AC LRC circuit



$$\Delta V_{\max}\sin(\omega t) - I_{\max}\left\{\left(-\frac{1}{\omega C} + \omega L\right)\cos(\omega t - \phi) + R\sin(\omega t - \phi)\right\} = 0$$

$$\Delta V_{\max}\sin(\omega t) - I_{\max}\left\{(-X_C + X_L)\cos(\omega t - \phi) + R\sin(\omega t - \phi)\right\} = 0$$

Some trig identities (see pg. A -12 of your text)

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$A\cos\beta + B\sin\beta = \sqrt{A^2 + B^2}\sin(\alpha + \beta)$$

if $\sin\alpha \equiv \frac{A}{\sqrt{A^2 + B^2}}$ and $\cos\alpha \equiv \frac{B}{\sqrt{A^2 + B^2}}$

$$\Delta V_{\max}\sin(\omega t) - I_{\max}\left\{(-X_C + X_L)\cos(\omega t - \phi) + R\sin(\omega t - \phi)\right\} = 0$$

$$\Delta V_{\max}\sin(\omega t) - I_{\max}\sqrt{(-X_C + X_L)^2 + R^2}$$

$$\times\left\{\frac{(-X_C + X_L)}{\sqrt{(-X_C + X_L)^2 + R^2}}\cos(\omega t - \phi) + \frac{R}{\sqrt{(-X_C + X_L)^2 + R^2}}\sin(\omega t - \phi)\right\} = 0$$

Let $Z \equiv \sqrt{(-X_C + X_L)^2 + R^2}$

$$\sin\alpha \equiv \frac{(-X_C + X_L)}{\sqrt{(-X_C + X_L)^2 + R^2}}$$

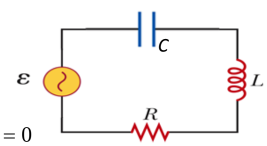
$$\cos\alpha \equiv \frac{R}{\sqrt{(-X_C + X_L)^2 + R^2}}$$

$$\Delta V_{\max}\sin(\omega t) - I_{\max} Z\left\{\sin\alpha\cos(\omega t - \phi) + \cos\alpha\sin(\omega t - \phi)\right\} = 0$$

$$\Delta V_{\max}\sin(\omega t) - I_{\max} Z\left\{\sin(\alpha + \omega t - \phi)\right\} = 0$$

$$\Rightarrow \phi = \alpha \qquad \Rightarrow \tan\phi = \frac{X_L - X_C}{R} \qquad \Rightarrow I_{\max} = \frac{\Delta V_{\max}}{Z}$$

Series AC LRC circuit



$$\Delta V_{\max}\sin(\omega t) - \frac{Q}{C} - L\frac{dI}{dt} - RI = 0$$

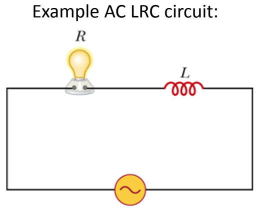
Assuming : $I(t) = I_{\max}\sin(\omega t - \phi)$

$$X_L \equiv \omega L \qquad X_C \equiv \frac{1}{\omega C}$$

$$Z \equiv \sqrt{(X_L - X_C)^2 + R^2}$$

$$\tan\phi = \frac{X_L - X_C}{R} \qquad I_{\max} = \frac{\Delta V_{\max}}{Z}$$

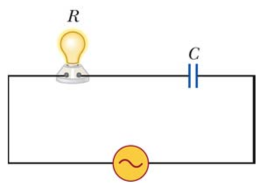
Example AC LRC circuit:



Consider the series resistor R and inductor L connected to an AC EMF. The voltage amplitude V_{max} is held fixed while the frequency ω is varied. The bulb is brightest when

A. ω is highest
B. ω is lowest
C. Bulb brightness is independent of ω

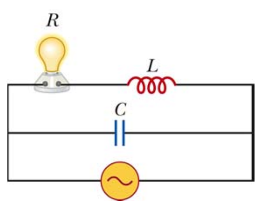
Example AC LRC circuit:



Consider the series resistor R and capacitor C connected to an AC EMF. The voltage amplitude V_{max} is held fixed while the frequency ω is varied. The bulb is brightest when

A. ω is highest
B. ω is lowest
C. Bulb brightness is independent of ω

Example AC LRC circuit:



Consider the series resistor R and capacitor C connected to an AC EMF. The voltage amplitude V_{max} is held fixed while the frequency ω is varied. The bulb is brightest when

A. ω is highest
B. ω is lowest
C. Bulb brightness is independent of ω

Power in an AC circuit

$$P = I\Delta v = I_{\max}\sin(\omega t-\phi)\,\Delta V_{\max}\sin(\omega t)$$
$$= \frac{(\Delta V_{\max})^2}{Z}\sin(\omega t-\phi)\sin(\omega t)$$
$$= \frac{(\Delta V_{\max})^2}{Z}\left(\sin^2(\omega t)\cos\phi-\cos(\omega t)\sin(\omega t)\sin\phi\right)$$
$$\langle P\rangle_{avg} = \frac{1}{2}\frac{(\Delta V_{\max})^2}{Z}\cos\phi$$

Note :

$$\int_0^{2\pi/\omega}\sin^2(\omega t)dt = \frac{1}{2} \qquad \int_0^{2\pi/\omega}\sin(\omega t)\cos(\omega t)dt = 0$$

Power in an AC circuit -- continued

$$\langle P \rangle_{avg} = \frac{1}{2}\frac{(\Delta V_{\max})^2}{Z}\cos\phi$$

$$\text{Recall}: \; Z \equiv \sqrt{(X_L - X_L)^2 + R^2}$$

$$\cos\phi = \frac{R}{\sqrt{(X_L + X_C)^2 + R^2}} = \frac{R}{Z}$$

$$\Rightarrow \langle P \rangle_{avg} = \frac{1}{2}\left(\frac{\Delta V_{\max}}{Z}\right)^2 R = \frac{1}{2}(I_{\max})^2 R$$

Frequency dependence of impedance in AC circuit

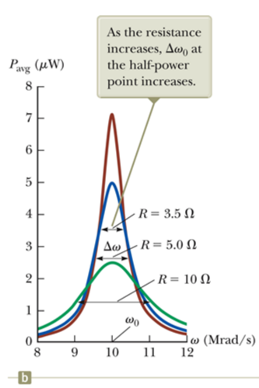
For pure LC circuit, we found a resonance frequency

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

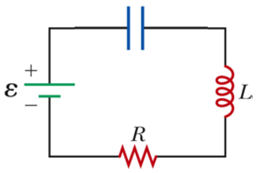
$$Z = \sqrt{(X_L - X_L)^2 + R^2} = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}$$

$$Z = \sqrt{\omega^2 L^2\left(1 - \frac{\omega_0^2}{\omega^2}\right)^2 + R^2}$$

$$\langle P \rangle_{avg} = \frac{1}{2}\left(\frac{\Delta V_{\max}}{Z}\right)^2 R = \frac{1}{2}(\Delta V_{\max})^2 \frac{R}{\omega^2 L^2\left(1 - \frac{\omega_0^2}{\omega^2}\right)^2 + R^2}$$



DC - LRC circuit:

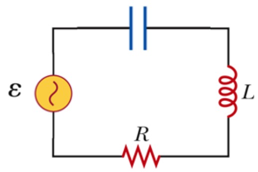


Solution assuming $Q(t=0)=0$:

$$Q(t) = C\mathcal{E}\left(1 - e^{-Rt/2L}\cos(\omega' t)\right)$$

$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

AC - LRC circuit:



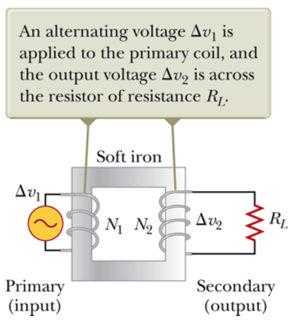
"Steady-state" solution for $\mathcal{E} = \Delta V_{max}\sin(\omega t)$

$$I(t) = \frac{\Delta V_{max}}{Z}\sin(\omega t - \phi)$$

$$X_L \equiv \omega L; \quad X_C \equiv \frac{1}{\omega C}; \quad \tan\phi = \frac{X_L - X_C}{R}$$

$$Z \equiv \sqrt{(X_L - X_C)^2 + R^2}$$

AC transformer devices:



Faraday's law for Δv_1:

$$\Delta v_1 = -N_1 \frac{d\Phi_B}{dt}$$

Faraday's law for Δv_2:

$$\Delta v_2 = -N_2 \frac{d\Phi_B}{dt}$$

To a good approximation,

$\frac{d\Phi_B}{dt}$ is the same for 1 and 2:

$$\Rightarrow \Delta v_2 = \frac{N_2}{N_1}\Delta v_1$$

AC transformer example:

$$\Delta v_2 = \frac{N_2}{N_1}\Delta v_1$$

Both Δv_1 and Δv_2 are oscillating in time;

applying the equality to the RMS amplitudes,

we can determine N_2/N_1 need for $\frac{V_{rms\,2}}{V_{rms\,1}} = \frac{12}{120}$:

$$\frac{N_2}{N_1} = \frac{V_{rms\,2}}{V_{rms\,1}} = \frac{1}{10}$$

If Δv_1 were a DC voltage source:

A. The transformer would work in the same way.
B. The transformer would not work at all.