PHY 114 A General Physics II
11 AM-12:15 PM  TR Olin 101

Plan for Lecture 16 (Chapter 33):

Electromagnetic Waves

1. Maxwell’s equations (with help from Coulomb, Ampere, Faraday, Gauss …)

2. Solutions to Maxwell’s equations and their significance.
Remember to send in your chapter reading questions...

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3rd exam will be scheduled for evenings during the week of 4/2/2012
Possible exam dates

Preferred times:
A. 5-7 PM
B. 6-8 PM
C. 7-9 PM
D. 8-10 PM

Preferred evenings:
A. Monday 4/2
B. Tuesday 4/3
C. Wednesday 4/4
D. Thursday 4/5
Comment on AC circuits:

\[ E - RI_1 - L \frac{dI_2}{dt} = 0 \]

\[ -L \frac{dI_2}{dt} + \frac{Q_3}{C} = 0 \]

\[ I_1 = I_2 + I_3 \]

Solution method:
1. Transform differential equation into algebraic equation using trig or complex functions
2. “Solve” algebra problem
3. Analyze for physical solution
Homework hint:

(a) In the figure below, find the rms current delivered by the 45.0 V (rms) power supply when the frequency is very large, $R_1 = 210 \, \Omega$, and $R_2 = 105 \, \Omega$. 

(b) In the figure below, find the rms current delivered by the 45.0 V (rms) power supply when the frequency is very small, $R_1 = 210 \, \Omega$, and $R_2 = 105 \, \Omega$. 

Recall the impedance of a series $LRC$ circuit:

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

Which components have the largest $Z$ at high frequency?

A. $R_1$ and $C$  
B. $R_2$ and $L$

Which components have the largest $Z$ at low frequency?

A. $R_1$ and $C$  
B. $R_2$ and $L$
Summary of electric and magnetic equations known before ~1860:

Gauss's laws:

Integral form: \[ \int \mathbf{E}(r) \cdot d\mathbf{A} = \frac{Q_{\text{in}}}{\varepsilon_0} \]
Differential form: \[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \]

Ampere's law:

Integral form: \[ \int \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{in}} \]
Differential form: \[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \]

Faraday's law:

\[ \int \mathbf{E}(r) \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B}(r) \cdot d\mathbf{A} \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

Couples \( \mathbf{E} \) and \( \mathbf{B} \) fields
Comments on vector fields, partial derivatives, gradient, curl ...

1. **Function of several variables and their derivatives**
2. Vector functions
3. Derivatives of vector functions

\[
f(x, y) = \left( \frac{\partial f(x, y)}{\partial x} \right)_{y=0.5}
\]

\[
f(x, y = 0.5)
\]
Comments on vector fields, partial derivatives, gradient, curl ...

1. **Function of several variables and their derivatives**
2. Vector functions
3. Derivatives of vector functions

Other examples of partial derivatives:

\[ f(x, t) = A \sin(k(x - ct)) \quad A, k, c \text{ constants} \]

\[
\frac{\partial f(x, t)}{\partial t} \equiv \left( \frac{\partial f(x, t)}{\partial t} \right)_x = -Akc \cos(k(x - ct))
\]

\[
\frac{\partial f(x, t)}{\partial x} \equiv \left( \frac{\partial f(x, t)}{\partial x} \right)_t = Ak \cos(k(x - ct))
\]
Comments on vector fields, partial derivatives, gradient, curl ...

1. Function of several variables and their derivatives
2. **Vector functions**
3. Derivatives of vector functions

**Vector in 3 - dimensions**

\[ \mathbf{V} = V_x \mathbf{i} + V_y \mathbf{j} + V_z \mathbf{k} \]

Vector function \[ \Rightarrow V_x = V_x(x, y, z, t) \]
\[ V_y = V_y(x, y, z, t) \]
\[ V_z = V_z(x, y, z, t) \]
Comments on vector fields, partial derivatives, gradient, curl ...

1. Function of several variables and their derivatives
2. Vector functions
3. Derivatives of vector functions

\[ \mathbf{V} = V_x \hat{i} + V_y \hat{j} + V_z \hat{k} = \frac{\partial V_x(x, y, z, t)}{\partial t} \hat{i} + \frac{\partial V_y(x, y, z, t)}{\partial t} \hat{j} + \frac{\partial V_z(x, y, z, t)}{\partial t} \hat{k} \]

Common notation for spatial derivatives:

\[ \nabla \equiv \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \]
Comments on vector fields, partial derivatives, gradient, curl ...

1. Function of several variables and their derivatives
2. Vector functions
3. Derivatives of vector functions

\[ \nabla \equiv \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \]

Gradient:

\[ \nabla f(x, y, z, t) \equiv \frac{\partial f(x, y, z, t)}{\partial x} \mathbf{i} + \frac{\partial f(x, y, z, t)}{\partial y} \mathbf{j} + \frac{\partial f(x, y, z, t)}{\partial z} \mathbf{k} \]

Divergence:

\[ \nabla \cdot \mathbf{V} \equiv \frac{\partial V_x(x, y, z, t)}{\partial x} + \frac{\partial V_y(x, y, z, t)}{\partial y} + \frac{\partial V_z(x, y, z, t)}{\partial z} \]

Curl:

\[ \nabla \times \mathbf{V} \equiv \left( \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z} \right) \mathbf{i} + \left( \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} \right) \mathbf{j} + \left( \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right) \mathbf{k} \]
Additional comments

Integral ↔ differential form of Gauss’s law \[ \int \mathbf{V}(\mathbf{r}) \cdot d\mathbf{A} \leftrightarrow \nabla \cdot \mathbf{V} \]

\[ x \text{ contribution to surface integral :} \]

\[ d\mathbf{A} = dy \, dz \, \hat{i} \]

\[ \int V \cdot d\mathbf{A} \bigg|_x = (V_x(x + dx, y, z) - V_x(x, y, z)) dy \, dz \]

\[ = \frac{\partial V_x(x, y, z)}{\partial x} dx \, dy \, dz \]
Incomplete set of equations:

\[ \oint E(\mathbf{r}) \cdot dA = \frac{Q_{\text{in}}}{\varepsilon_0} \]

\[ \oint B(\mathbf{r}) \cdot dA = 0 \]

\[ \int E(\mathbf{r}) \cdot ds = -\frac{d}{dt} \int B(\mathbf{r}) \cdot dA \]

\[ \oint B \cdot ds = \mu_0 I_{\text{in}} \]

\[ \nabla \cdot E = \frac{\rho}{\varepsilon_0} \]

\[ \nabla \cdot B = 0 \]

\[ \nabla \times E = -\frac{\partial B}{\partial t} \]

\[ \nabla \times B = \mu_0 J \]

A changing magnetic flux produces an electric field – can a changing electric flux produce a magnetic field?

What happens to fields between capacitor plates with time varying charge?
Slight problem with Ampere’s law applied in the vicinity of a capacitor:

$$\oint B \cdot ds = \mu_0 \int J \cdot dS_1$$

$$\oint B \cdot ds \neq \mu_0 \int J \cdot dS_2$$

The conduction current $I$ in the wire passes only through $S_1$, which leads to a contradiction in Ampère’s law that is resolved only if one postulates a displacement current through $S_2$. 
Slight problem with Ampere’s law applied in the vicinity of a capacitor:

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \int \mathbf{J} \cdot d\mathbf{S}_1 \]

From Gauss's law

\[ \oint \mathbf{E} \cdot d\mathbf{S}_2 = \frac{q}{\varepsilon_0} \]

The conduction current \( I \) in the wire passes only through \( S_1 \), which leads to a contradiction in Ampère’s law that is resolved only if one postulates a displacement current through \( S_2 \).
More details about Ampere-Maxwell equation:

\[ B 2\pi r = \mu_0 I \]

\[ B 2\pi r = \mu_0 \varepsilon_0 A \frac{dE}{dt} \]
Full Maxwell’s equations

\[ \oint E(r) \cdot dA = \frac{Q_{\text{in}}}{\varepsilon_0} \]

\[ \oint B(r) \cdot dA = 0 \]

\[ \int E(r) \cdot ds = -\frac{d}{dt} \int B(r) \cdot dA \]

\[ \oint B \cdot ds = \mu_0 I_{\text{in}} + \mu_0 \varepsilon_0 \frac{d}{dt} \int E(r) \cdot dA \]

\[ \nabla \cdot E = \frac{\rho}{\varepsilon_0} \]

\[ \nabla \cdot B = 0 \]

\[ \nabla \times E = -\frac{\partial B}{\partial t} \]

\[ \nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \]
Why are Maxwell’s equations important?

A. They are not really – physicist are weird
B. They are used to torture PHY 114 students
C. They summarize all of electricity and magnetism presently known.
D. They explain the nature of light
Full Maxwell’s equations

\[ \oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = \frac{Q_{\text{in}}}{\varepsilon_0} \]
\[ \oint \mathbf{B}(\mathbf{r}) \cdot d\mathbf{A} = 0 \]
\[ \int \mathbf{E}(\mathbf{r}) \cdot ds = -\frac{d}{dt} \int \mathbf{B}(\mathbf{r}) \cdot d\mathbf{A} \]
\[ \oint \mathbf{B} \cdot ds = \mu_0 I_{\text{in}} + \mu_0 \varepsilon_0 \frac{d}{dt} \int \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} \]
\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \]
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \]

Some conclusions:
1. Without sources, there are no \( \mathbf{E} \) or \( \mathbf{B} \) fields
2. \( \mathbf{E} \) and \( \mathbf{B} \) can exist far from the sources
Maxwell’s equations without sources

\[ \oint E(r) \cdot dA = 0 \]
\[ \oint B(r) \cdot dA = 0 \]
\[ \int E(r) \cdot ds = -\frac{d}{dt} \int B(r) \cdot dA \]
\[ \oint B \cdot ds = \mu_0 \varepsilon_0 \frac{d}{dt} \int E(r) \cdot dA \]
\[ \nabla \cdot E = 0 \]
\[ \nabla \cdot B = 0 \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} \]
\[ \nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \]

Far from the charge and current sources the \( E \) and \( B \) fields

A. Always get smaller with increasing distance
B. Always get smaller at long times
C. Can maintain a steady amplitude at all times and distances
\[ \nabla \cdot \mathbf{E} = 0 \]
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \]

Possible solution:

\[ \mathbf{E} = E_y \hat{j} \]
\[ \mathbf{B} = B_z \hat{k} \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \]

\[ \Rightarrow \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \]
\[ \Rightarrow -\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t} \]
\[ \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \]

\[ -\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t} \]

\[ \frac{\partial}{\partial x} \left( \frac{\partial E_y}{\partial x} \right) = -\frac{\partial}{\partial x} \left( \frac{\partial B_z}{\partial t} \right) = -\frac{\partial}{\partial t} \frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \frac{\partial}{\partial t} \frac{\partial E_y}{\partial t} \]

\[ \frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \]
\[
\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}
\]

\[
\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}
\]

\[
\mu_0 \varepsilon_0 = \left(4 \pi \times 10^{-7} \text{Tm/A}\right) \left(8.854 \times 10^{-12} \text{C}^2/\text{Nm}^2\right)
\]

\[
= \frac{1}{(3 \times 10^8 \text{m/s})^2} \equiv \frac{1}{c^2}
\]
The wave equation

\[ \frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2} \quad \frac{\partial^2 B_z}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 B_z}{\partial t^2} \]

Solution form:

\[ E_y = E_y(x, t) = E_{\text{max}} \cos(k(x - ct)) \]

\[ B_z = B_z(x, t) = B_{\text{max}} \cos(k(x - ct)) \]

\[ kc = \omega \]

\[ k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f \]

\[ c = \lambda f \]

\[ B_{\text{max}} = \frac{E_{\text{max}}}{c} \]
\[ E_y = E_y(x, t) = E_{\text{max}} \cos(k(x - ct)) \]

\[ B_z = B_z(x, t) = \frac{E_{\text{max}}}{c} \cos(k(x - ct)) \]
What good are electromagnetic plane waves?
   A. They are good for nothing.
   B. They are idealizations that physics students never see.
   C. They come from outer space.
   D. An example is in this room.

Which of the following states comparing sound waves to electromagnetic waves are false?
   A. They both satisfy the wave equation
   B. For both, the wave speed $c$ is related to frequency and wavelength according to $c=\lambda f$.
   C. They both travel through a medium.
   D. The Doppler effect for sound waves is different in electromagnetic waves and in sound waves.
Astronomers think that they have detected radiation from a star $15 \times 10^9$ light years away.

\[ d = 15 \times 10^9 \cdot 3 \times 10^8 \text{ m/s} \cdot 365.25 \cdot 24 \cdot 3600 = 1.4 \times 10^{26} \text{ m} \]