PHY 114 A General Physics II
11 AM-12:15 PM TR Olin 101

Plan for Lecture 16 (Chapter 33):

Electromagnetic Waves

1. **Maxwell's equations (with help from Coulomb, Ampere, Faraday, Gauss …)**
2. **Solutions to Maxwell's equations and their significance.**

Remember to send in your chapter reading questions…

10	02/23/2012	Review	28.1-28.5	(Review for exam)	
	02/28/2012	Exam			
11	03/01/2012	Magnetic fields	29.1-29.6	29.5,29.12,29.47	03/06/2012
12	03/06/2012	Magnetic field sources	30.1-30.6	30.5,30.21,30.29	03/08/2012
13	03/08/2012	Faraday's law	31.1-31.5	31.12,31.23,31.40	03/20/2012
	03/13/2012	*No class (Spring Break)*			
	03/15/2012	*No class (Spring Break)*			
14	03/20/2012	Induction and AC circuits	32.1-32.6	32.4,32.20,32.43	03/22/2012
15	03/22/2012	AC circuits	33.1-33.9	33.8,33.24,33.71	03/27/2012
16	03/27/2012	Electromagnetic waves	34.1-34.3	34.3,34.10,34.13	03/29/2012
17	03/29/2012	Electromagnetic waves	34.4-34.7	34.22,34.46,34.57	04/03/2012
18	04/03/2012	Ray optics Evening exam	35.1-35.8		
19	04/05/2012	Image formation Evening exam	36.1-36.4		
20	04/10/2012	Image formation	36.5-36.10		

3rd exam will be scheduled for evenings during the week of 4/2/2012

April 2012

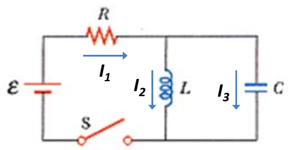
S	M	T	W	T	F	S
25	**26**	27	28	**29**	30	31
1	2	3	4	5	**6**	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	1	2	3	4	5

Possible exam dates

Preferred times:
- A. 5-7 PM
- B. 6-8 PM
- C. 7-9 PM
- D. 8-10 PM

Preferred evenings:
- A. Monday 4/2
- B. Tuesday 4/3
- C. Wednesday 4/4
- D. Thursday 4/5

Comment on AC circuits:



$$\mathcal{E} - RI_1 - L\frac{dI_2}{dt} = 0$$

$$-L\frac{dI_2}{dt} + \frac{Q_3}{C} = 0$$

$$I_1 = I_2 + I_3$$

Solution method:

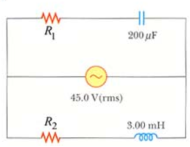
1. Transform differential equation in to algebraic equation using trig or complex functions
2. “Solve” algebra problem
3. Analyze for physical solution

Homework hint:

6. –/0.334 points My Notes | SerPSE8 33.P.071.

(a) In the figure below, find the rms current delivered by the 45.0 V (rms) power supply when the frequency is very large, $R1$ = 210 Ω, and $R2$ = 105 Ω.
mA

(b) In the figure below, find the rms current delivered by the 45.0 V (rms) power supply when the frequency is very small, $R1$ = 210 Ω, and $R2$ = 105 Ω.
mA



Recall the impedance of a series *LRC* circuit:

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

Which components have the largest Z at high frequency?
A. R_1 and C B. R_2 and L

Which components have the largest Z at low frequency?
A. R_1 and C B. R_2 and L

Summary of electric and magnetic equations known before ~1860:

Gauss's laws:

Integral form: $\oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = \frac{Q_{in}}{\varepsilon_0}$ Integral form: $\oint \mathbf{B}(\mathbf{r}) \cdot d\mathbf{A} = 0$

Differential form: $\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$ Differential form: $\nabla \cdot \mathbf{B} = 0$

Ampere's law:

Integral form: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{in}$

Differential form: $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$

Faraday's law:

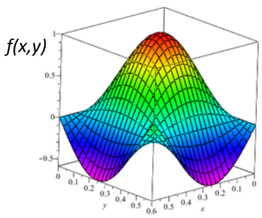
$$\oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{s} = -\frac{d}{dt}\int \mathbf{B}(r) \cdot d\mathbf{A}$$

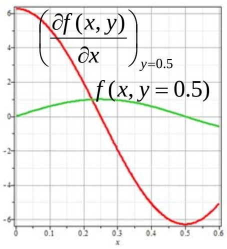
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Couples **E** and **B** fields

Comments on vector fields, partial derivatives, gradient, curl …

1. Function of several variables and their derivatives
2. Vector functions
3. Derivatives of vector functions





Comments on vector fields, partial derivatives, gradient, curl …

1. Function of several variables and their derivatives
2. Vector functions
3. Derivatives of vector functions

Other examples of partial derivatives:

$$f(x,t) = A\sin(k(x-ct)) \qquad A, k, c \text{ constants}$$

$$\frac{\partial f(x,t)}{\partial t} \equiv \left(\frac{\partial f(x,t)}{\partial t}\right)_x = -Akc\cos(k(x-ct))$$

$$\frac{\partial f(x,t)}{\partial x} \equiv \left(\frac{\partial f(x,t)}{\partial x}\right)_t = Ak\cos(k(x-ct))$$

Comments on vector fields, partial derivatives, gradient, curl …

1. Function of several variables and their derivatives
2. Vector functions
3. Derivatives of vector functions

Vector in 3-dimensions

$$\mathbf{V} = V_x\hat{\mathbf{i}} + V_y\hat{\mathbf{j}} + V_z\hat{\mathbf{k}}$$

Vector function $\Rightarrow$

$$V_x = V_x(x,y,z,t)$$
$$V_y = V_y(x,y,z,t)$$
$$V_z = V_z(x,y,z,t)$$

Comments on vector fields, partial derivatives, gradient, curl …

1. Function of several variables and their derivatives
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$$\mathbf{V} = V_x\hat{\mathbf{i}} + V_y\hat{\mathbf{j}} + V_z\hat{\mathbf{k}} = V_x(x,y,z,t)\hat{\mathbf{i}} + V_y(x,y,z,t)\hat{\mathbf{j}} + V_z(x,y,z,t)\hat{\mathbf{k}}$$

$$\frac{\partial \mathbf{V}}{\partial t} = \frac{\partial V_x(x,y,z,t)}{\partial t}\hat{\mathbf{i}} + \frac{\partial V_y(x,y,z,t)}{\partial t}\hat{\mathbf{j}} + \frac{\partial V_z(x,y,z,t)}{\partial t}\hat{\mathbf{k}}$$

$$\frac{\partial \mathbf{V}}{\partial x} = \frac{\partial V_x(x,y,z,t)}{\partial x}\hat{\mathbf{i}} + \frac{\partial V_y(x,y,z,t)}{\partial x}\hat{\mathbf{j}} + \frac{\partial V_z(x,y,z,t)}{\partial x}\hat{\mathbf{k}}$$

Common notation for spatial derivatives:

$$\nabla \equiv \frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}$$

Comments on vector fields, partial derivatives, gradient, curl …

1. Function of several variables and their derivatives
2. Vector functions
3. Derivatives of vector functions

$$\nabla \equiv \frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial x}\hat{\mathbf{j}} + \frac{\partial}{\partial x}\hat{\mathbf{k}}$$

Gradient:

$$\nabla f(x,y,z,t) \equiv \frac{\partial f(x,y,z,t)}{\partial x}\hat{\mathbf{i}} + \frac{\partial f(x,y,z,t)}{\partial y}\hat{\mathbf{j}} + \frac{\partial f(x,y,z,t)}{\partial z}\hat{\mathbf{k}}$$

Divergence:

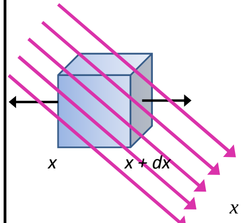
$$\nabla \cdot \mathbf{V} \equiv \frac{\partial V_x(x,y,z,t)}{\partial x} + \frac{\partial V_y(x,y,z,t)}{\partial y} + \frac{\partial V_z(x,y,z,t)}{\partial z}$$

Curl:

$$\nabla \times \mathbf{V} \equiv \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\hat{\mathbf{i}} + \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\hat{\mathbf{j}} + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\hat{\mathbf{k}}$$

Additional comments

Integral ←→ differential form of Gauss's law $\oint \mathbf{V}(\mathbf{r})\cdot d\mathbf{A} \Leftrightarrow \nabla\cdot\mathbf{V}$



x contribution to surface integral:

$$d\mathbf{A} = dy\,dz\,\hat{\mathbf{i}}$$

$$\int \mathbf{V}\cdot d\mathbf{A}\Big|_x = \left(V_x(x+dx,y,z) - V_x(x,y,z)\right)dy\,dz$$

$$= \frac{\partial V_x(x,y,z)}{\partial x}\,dx\,dy\,dz$$

Incomplete set of equations:

$$\oint \mathbf{E}(\mathbf{r})\cdot d\mathbf{A} = \frac{Q_{in}}{\varepsilon_0} \qquad \nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint \mathbf{B}(\mathbf{r})\cdot d\mathbf{A} = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

$$\int \mathbf{E}(\mathbf{r})\cdot d\mathbf{s} = -\frac{d}{dt}\int \mathbf{B}(r)\cdot d\mathbf{A} \qquad \nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

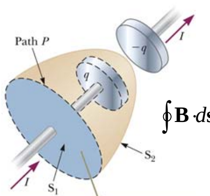
$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{in} \qquad \nabla\times\mathbf{B} = \mu_0\mathbf{J}$$

A changing magnetic flux produces an electric field – can a changing electric flux produce a magnetic field?

What happens to fields between capacitor plates with time varying charge?

Slight problem with Ampere's law applied in the vicinity of a capacitor:

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 \int \mathbf{J}\cdot d\mathbf{S}_1$$

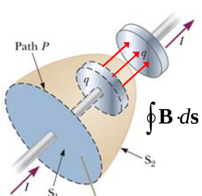


The conduction current I in the wire passes only through S_1, which leads to a contradiction in Ampère's law that is resolved only if one postulates a displacement current through S_2.

$$\oint \mathbf{B}\cdot d\mathbf{s} \neq \mu_0 \int \mathbf{J}\cdot d\mathbf{S}_2$$

Slight problem with Ampere's law applied in the vicinity of a capacitor:

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 \int \mathbf{J}\cdot d\mathbf{S}_1$$



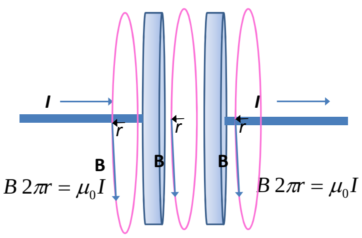
The conduction current I in the wire passes only through S_1, which leads to a contradiction in Ampère's law that is resolved only if one postulates a displacement current through S_2.

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0\varepsilon_0 \frac{d}{dt}\int \mathbf{E}\cdot d\mathbf{S}_2$$

From Gauss's law

$$\int \mathbf{E}\cdot d\mathbf{S}_2 = \frac{q}{\varepsilon_0}$$

More details about Ampere-Maxwell equation:



$$B\,2\pi r = \mu_0 I \qquad\qquad B\,2\pi r = \mu_0 I$$

$$B\,2\pi r = \mu_0 \varepsilon_0 A \frac{dE}{dt}$$

Full Maxwell's equations

$$\oint \mathbf{E}(\mathbf{r})\cdot d\mathbf{A} = \frac{Q_{in}}{\varepsilon_0} \qquad \nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint \mathbf{B}(\mathbf{r})\cdot d\mathbf{A} = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

$$\int \mathbf{E}(\mathbf{r})\cdot d\mathbf{s} = -\frac{d}{dt}\int \mathbf{B}(r)\cdot d\mathbf{A} \qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{in} + \mu_0\varepsilon_0 \frac{d}{dt}\int \mathbf{E}(r)\cdot d\mathbf{A} \qquad \nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

Why are Maxwell's equations important?

A. They are not really – physicist are weird
B. They are used to torture PHY 114 students
C. They summarize all of electricity and magnetism presently known.
D. They explain the nature of light

Full Maxwell's equations

$$\oint \mathbf{E}(\mathbf{r})\cdot d\mathbf{A} = \frac{Q_{in}}{\varepsilon_0} \qquad \nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint \mathbf{B}(\mathbf{r})\cdot d\mathbf{A} = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

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$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{in} + \mu_0\varepsilon_0\frac{d}{dt}\int \mathbf{E}(r)\cdot d\mathbf{A} \qquad \nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

Some conclusions:

1. Without sources, there are no **E** or **B** fields
2. **E** and **B** can exist far from the sources

Maxwell's equations without sources

$$\oint \mathbf{E}(\mathbf{r})\cdot d\mathbf{A} = 0 \qquad \nabla\cdot\mathbf{E} = 0$$

$$\oint \mathbf{B}(\mathbf{r})\cdot d\mathbf{A} = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

$$\int \mathbf{E}(\mathbf{r})\cdot d\mathbf{s} = -\frac{d}{dt}\int \mathbf{B}(r)\cdot d\mathbf{A} \qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0\varepsilon_0\frac{d}{dt}\int \mathbf{E}(r)\cdot d\mathbf{A} \qquad \nabla\times\mathbf{B} = \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

Far from the charge and current sources the **E** and **B** fields

A. Always get smaller with increasing distance
B. Always get smaller at long times
C. Can maintain a steady amplitude at all times and distances

$$\nabla\cdot\mathbf{E} = 0$$

$$\nabla\cdot\mathbf{B} = 0$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{B} = \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

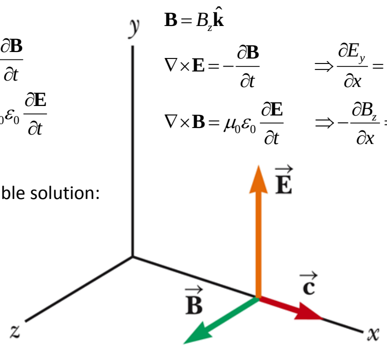
$$\mathbf{E} = E_y\hat{\mathbf{j}}$$

$$\mathbf{B} = B_z\hat{\mathbf{k}}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \quad \Rightarrow \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

$$\nabla\times\mathbf{B} = \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t} \quad \Rightarrow -\frac{\partial B_z}{\partial x} = \mu_0\varepsilon_0\frac{\partial E_y}{\partial t}$$

Possible solution:

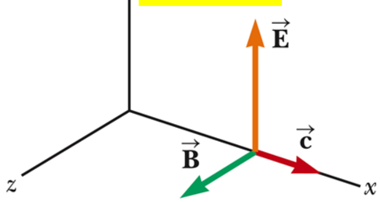


$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

$$-\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

$$\frac{\partial}{\partial x}\frac{\partial E_y}{\partial x} = -\frac{\partial}{\partial x}\frac{\partial B_z}{\partial t} = -\frac{\partial}{\partial t}\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\frac{\partial E_y}{\partial t}$$

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

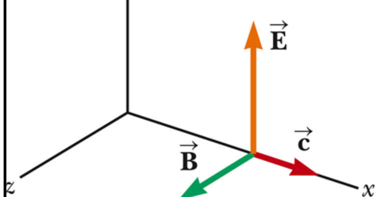


$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

$$\mu_0 \varepsilon_0 = \left(4\pi \times 10^{-7}\,\mathrm{Tm/A}\right)\left(8.854 \times 10^{-12}\,\mathrm{C^2/Nm^2}\right)$$

$$= \frac{1}{\left(3 \times 10^8\,\mathrm{m/s}\right)^2} \equiv \frac{1}{c^2}$$



The wave equation

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} \qquad \frac{\partial^2 B_z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B_z}{\partial t^2}$$

Solutionform:

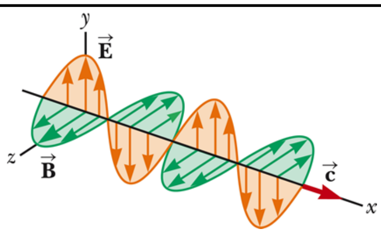
$$E_y = E_y(x,t) = E_{\max}\cos\left(k(x-ct)\right)$$

$$B_z = B_z(x,t) = B_{\max}\cos\left(k(x-ct)\right)$$

$$kc = \omega$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f \qquad B_{\max} = \frac{E_{\max}}{c}$$

$$c = \lambda f$$



$$E_y = E_y(x,t) = E_{max}\cos(k(x-ct))$$

$$B_z = B_z(x,t) = \frac{E_{max}}{c}\cos(k(x-ct))$$

What good are electromagnetic plane waves?

A. They are good for nothing.
B. They are idealizations that physics students never see.
C. They come from outer space.
D. An example is in this room.

Which of the following states comparing sound waves to electromagnetic waves are **false**?

A. They both satisfy the wave equation
B. For both, the wave speed c is related to frequency and wavelength according to $c=\lambda f$.
C. They both travel through a medium.
D. The Doppler effect for sound waves is different in electromagnetic waves and in sound waves.

Astronomers think that they have detected radiation from a star $15\text{x}10^9$ light years away.

$$d = 15\times10^9 \cdot 3\times10^8 m/s \cdot 365.25 \cdot 24 \cdot 3600 = 1.4\times10^{26} m$$