PHY 114 A General Physics II  
11 AM-12:15 PM TR Olin 101

Plan for Lecture 16 (Chapter 33):
Electromagnetic Waves

1. Maxwell’s equations (with help from Coulomb, Ampere, Faraday, Gauss ...)
2. Solutions to Maxwell’s equations and their significance.

Remember to send in your chapter reading questions...

3rd exam will be scheduled for evenings during the week of 4/2/2012

Possible exam dates

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Preferred times:  
A. 5-7 PM  
B. 6-8 PM  
C. 7-9 PM  
D. 8-10 PM

Preferred evenings:  
A. Monday 4/2  
B. Tuesday 4/3  
C. Wednesday 4/4  
D. Thursday 4/5
Comment on AC circuits:

\[ \mathcal{E} - RI_1 - L \frac{dI_2}{dt} = 0 \]
\[ -j \frac{dQ}{dt} - \frac{Q}{C} = 0 \]
\[ I_1 = I_2 + I_3 \]

Solution method:
1. Transform differential equation into algebraic equation using trig or complex functions
2. "Solve" algebra problem
3. Analyze for physical solution

Homework hint:

Recall the impedance of a series LRC circuit:
\[ Z = \sqrt{(X_L - X_C)^2 + R^2} \]

Which components have the largest Z at high frequency?
A. \( R_1 \) and \( C \)
B. \( R_2 \) and \( L \)

Which components have the largest Z at low frequency?
A. \( R_1 \) and \( C \)
B. \( R_2 \) and \( L \)

Summary of electric and magnetic equations known before ~1860:

Gauss’s laws:

Integral form: \( \int \mathbf{E} \cdot d\mathbf{A} = \frac{\mathcal{Q}}{\varepsilon_0} \)
Integral form: \( \int \mathbf{B} \cdot d\mathbf{A} = 0 \)

Differential form: \( \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \)
Differential form: \( \nabla \cdot \mathbf{B} = 0 \)

Ampere’s law:

Integral form: \( \int \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_w \)
Integral form: \( \int \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A} \)
Differential form: \( \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \)
Differential form: \( \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \)

Faraday’s law:

Couples \( \mathbf{E} \) and \( \mathbf{B} \) fields
Comments on vector fields, partial derivatives, gradient, curl ...

1. Function of several variables and their derivatives
2. Vector functions
3. Derivatives of vector functions

Other examples of partial derivatives:

\[ f(x,t) = A \sin(k(x-c\tau)) \quad A, k, c \text{ constants} \]

\[ \frac{\partial f(x,t)}{\partial t} = \left( \frac{\partial f(x,t)}{\partial t} \right)_x = -Ak \cos(k(x-c\tau)) \]

\[ \frac{\partial f(x,t)}{\partial x} = \left( \frac{\partial f(x,t)}{\partial x} \right)_t = Ak \cos(k(x-c\tau)) \]

Vector in 3-dimensions

\[ \mathbf{V} = V_x \mathbf{i} + V_y \mathbf{j} + V_z \mathbf{k} \]

Vector function \( \mathbf{V} = V_i(x,y,z,t) \)

\[ V_x = V_i(x,y,z,t) \]

\[ V_y = V_i(x,y,z,t) \]

\[ V_z = V_i(x,y,z,t) \]
Comments on vector fields, partial derivatives, gradient, curl ...
1. Function of several variables and their derivatives
2. Vector functions
3. Derivatives of vector functions

\[ \mathbf{V} = \mathbf{V}_x \mathbf{i} + \mathbf{V}_y \mathbf{j} + \mathbf{V}_z \mathbf{k} = V_x(x, y, z, t) \mathbf{i} + V_y(x, y, z, t) \mathbf{j} + V_z(x, y, z, t) \mathbf{k} \]

\[ \frac{\partial \mathbf{V}}{\partial t} = \frac{\partial V_x}{\partial t} \mathbf{i} + \frac{\partial V_y}{\partial t} \mathbf{j} + \frac{\partial V_z}{\partial t} \mathbf{k} \]

\[ \frac{\partial \mathbf{V}}{\partial x} = \frac{\partial V_x}{\partial x} \mathbf{i} + \frac{\partial V_y}{\partial x} \mathbf{j} + \frac{\partial V_z}{\partial x} \mathbf{k} \]

Common notation for spatial derivatives:

\[ \nabla = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \]

Additional comments

Integral ↔ differential form of Gauss's law \[ \int \mathbf{V}(\mathbf{r}) \cdot d\mathbf{A} \leftrightarrow \nabla \cdot \mathbf{V} \]

\[ d\mathbf{A} = dy \, dz \mathbf{k} \]

\[ \int \mathbf{V} \cdot d\mathbf{A} = \left[ V_x(x, y, z) - V_y(x, y, z) \right] dy \, dz \]

\[ = \frac{\partial V_x}{\partial x} \, dx \, dy \, dz \]
Incomplete set of equations:
\[
\oint \mathbf{E}(r) \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0} \\
\oint \mathbf{B}(r) \cdot d\mathbf{A} = 0 \\
\oint \mathbf{E}(r) \cdot ds = -\frac{d}{dt} \oint \mathbf{B}(r) \cdot d\mathbf{A} \\
\oint \mathbf{B} \cdot ds = \mu_0 J_0
\]

\[\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \\
\nabla \cdot \mathbf{B} = 0 \\
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{B} = \mu_0 \mathbf{J}
\]

A changing magnetic flux produces an electric field—can a changing electric flux produce a magnetic field?

What happens to fields between capacitor plates with time varying charge?

Slight problem with Ampere's law applied in the vicinity of a capacitor:

\[
\oint \mathbf{B} \cdot ds = \mu_0 \oint \mathbf{J} \cdot dS_1 \\
\oint \mathbf{B} \cdot ds \neq \mu_0 \oint \mathbf{J} \cdot dS_2
\]

The conduction current in the wire passes only through \(S_1\), which leads to a contradiction to Ampere's law that is resolved only if one postulates a displacement current through \(S_2\).

Slight problem with Ampere's law applied in the vicinity of a capacitor:

\[
\oint \mathbf{B} \cdot ds = \mu_0 \oint \frac{d}{dt} \left[ \mathbf{E} \cdot dS_1 \right] \\
\oint \mathbf{B} \cdot ds = \mu_0 \varepsilon_0 \oint \frac{d}{dt} \left[ \mathbf{E} \cdot dS_2 \right]
\]

From Gauss's law
\[
\oint \mathbf{E} \cdot dS_2 = \frac{Q}{\varepsilon_0}
\]
More details about Ampere-Maxwell equation:

\[ B \cdot \text{d} \mathbf{a} = \mu_0 I \]

Full Maxwell's equations

\[ \int \mathbf{E}(r) \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0} \]
\[ \int \mathbf{B}(r) \cdot d\mathbf{A} = 0 \]
\[ \int \mathbf{E}(r) \cdot d\mathbf{A} = \frac{1}{\varepsilon_0} \nabla \cdot \mathbf{B} = 0 \]
\[ \int \mathbf{B}(r) \cdot d\mathbf{A} = \frac{d}{dt} \int \mathbf{E}(r) \cdot d\mathbf{A} \]

Why are Maxwell's equations important?
A. They are not really – physicist are weird
B. They are used to torture PHY 114 students
C. They summarize all of electricity and magnetism presently known.
D. They explain the nature of light
Some conclusions:
1. Without sources, there are no E or B fields
2. E and B can exist far from the sources

Far from the charge and current sources the E and B fields
A. Always get smaller with increasing distance
B. Always get smaller at long times
C. Can maintain a steady amplitude at all times and distances
The wave equation

\[ \frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 B}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 B}{\partial t^2} \]

Solution:

\[ E_x = E_x(x,t) = E_{max} \cos(k(x - ct)) \]
\[ B_x = B_x(x,t) = B_{max} \cos(k(x - ct)) \]

\[ k = \frac{\omega}{c} \]
\[ \omega = 2\pi f \]
\[ B_{max} = \frac{E_{max}}{c} \]
What good are electromagnetic plane waves?
A. They are good for nothing.
B. They are idealizations that physics students never see.
C. They come from outer space.
D. An example is in this room.

Which of the following states comparing sound waves to electromagnetic waves are false?
A. They both satisfy the wave equation
B. For both, the wave speed c is related to frequency and wavelength according to $c=\lambda f$.
C. They both travel through a medium.
D. The Doppler effect for sound waves is different in electromagnetic waves and in sound waves.

Astronomers think that they have detected radiation from a star $15 \times 10^6$ light years away.

$$d = 15 \times 10^9 \cdot 3 \times 10^3 \text{ m} / \text{s} \cdot 365.25 \cdot 24 \cdot 3600 = 1.4 \times 10^{26} \text{ m}$$