

**PHY 114 A General Physics II**  
**11 AM-12:15 PM TR Olin 101**

**Plan for Lecture 17 (Chapter 34):**

**Electromagnetic Waves**

- 1. Maxwell's equations & their solutions**
- 2. Electromagnetic energy and their spectral distribution**
- 3. Review of Chapters 29-34**

# Remember to send in your chapter reading questions...

11	03/01/2012	Magnetic fields	<a href="#">29.1-29.6</a>	<a href="#">29.5.29.12.29.47</a>	03/06/2012
12	03/06/2012	Magnetic field sources	<a href="#">30.1-30.6</a>	<a href="#">30.5.30.21.30.29</a>	03/08/2012
13	03/08/2012	Faraday's law	<a href="#">31.1-31.5</a>	<a href="#">31.12.31.23.31.40</a>	03/20/2012
	03/13/2012	No class (Spring Break)			
	03/15/2012	No class (Spring Break)			
14	03/20/2012	Induction and AC circuits	<a href="#">32.1-32.6</a>	<a href="#">32.4.32.20.32.43</a>	03/22/2012
15	03/22/2012	AC circuits	<a href="#">33.1-33.9</a>	<a href="#">33.8.33.24.33.71</a>	03/27/2012
16	03/27/2012	Electromagnetic waves	<a href="#">34.1-34.3</a>	<a href="#">34.3.34.10.34.13</a>	03/29/2012
	03/29/2012	Electromagnetic waves	<a href="#">34.4-34.7</a>	<a href="#">34.22.34.46.34.57</a>	04/03/2012
18	04/03/2012	Ray optics Evening exam	<a href="#">35.1-35.8</a>	<a href="#">35.20.35.27.35.35</a>	04/05/2012
19	04/05/2012	Image formation Evening exam	<a href="#">36.1-36.4</a>	<a href="#">36.8.36.31.36.42</a>	04/10/2012
20	04/10/2012	Image formation	<a href="#">36.5-36.10</a>	<a href="#">36.52.36.54.36.64</a>	04/12/2012

**3<sup>rd</sup> exam (covering Chapters 29-34) is scheduled for evenings during the week of 4/2/2012.**

April 2012



S	M	T	W	T	F	S	
25	<b>26</b>	27	28	<b>29</b>	30	31	
3rd exam dates	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
	<b>8</b>	9	10	11	12	13	14
	<b>15</b>	16	17	18	19	20	21
	<b>22</b>	23	24	25	26	27	28
	29	30	1	2	3	4	5

You will be scheduled for one of these (based on email info)  
– probably in Olin 107 between 6-10 PM:

- Monday 4/2
- Tuesday 4/3
- Wednesday 4/4
- Thursday 4/5

**Several people still need to email their preferred exam times.**

# Full Maxwell's equations

$$\oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = \frac{Q_{in}}{\epsilon_0}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\oint \mathbf{B}(\mathbf{r}) \cdot d\mathbf{A} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

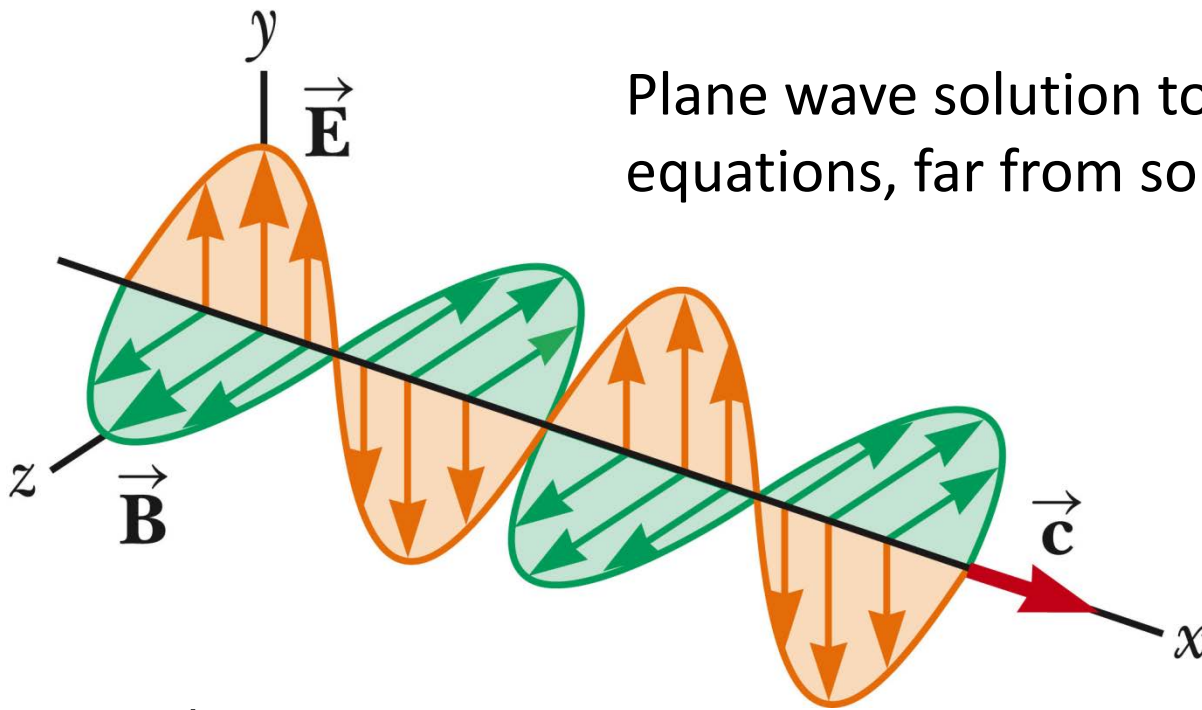
$$\int \mathbf{E}(\mathbf{r}) \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B}(r) \cdot d\mathbf{A}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{in} + \mu_0 \epsilon_0 \frac{d}{dt} \int \mathbf{E}(r) \cdot d\mathbf{A}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Plane wave solution to Maxwell's equations, far from sources:



$$\mathbf{E} = E_y \hat{\mathbf{j}} \quad \mathbf{B} = B_z \hat{\mathbf{k}}$$

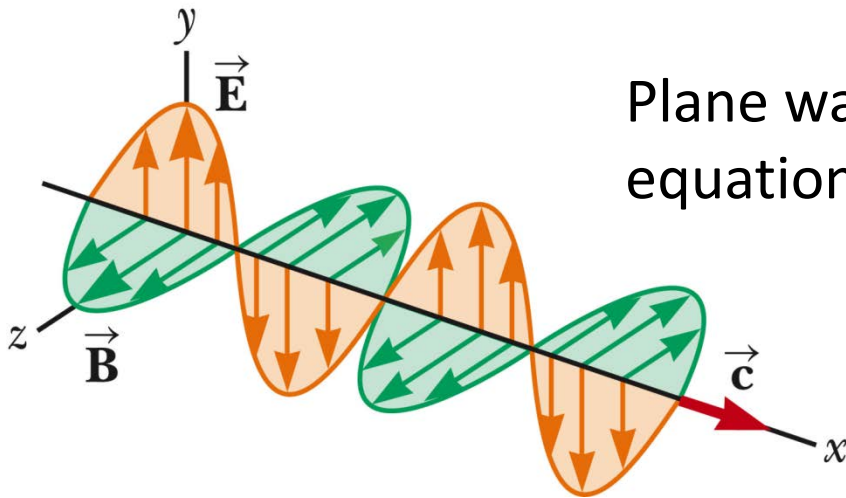
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \Rightarrow \quad \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad \Rightarrow \quad -\frac{\partial B_z}{\partial x} = \mu_0 \epsilon_0 \frac{\partial E_y}{\partial t}$$

$$E_y = E_y(x, t) = E_{\max} \cos(k(x - ct))$$

$$B_z = B_z(x, t) = \frac{E_{\max}}{c} \cos(k(x - ct))$$

Plane wave solution to Maxwell's equations, far from sources:



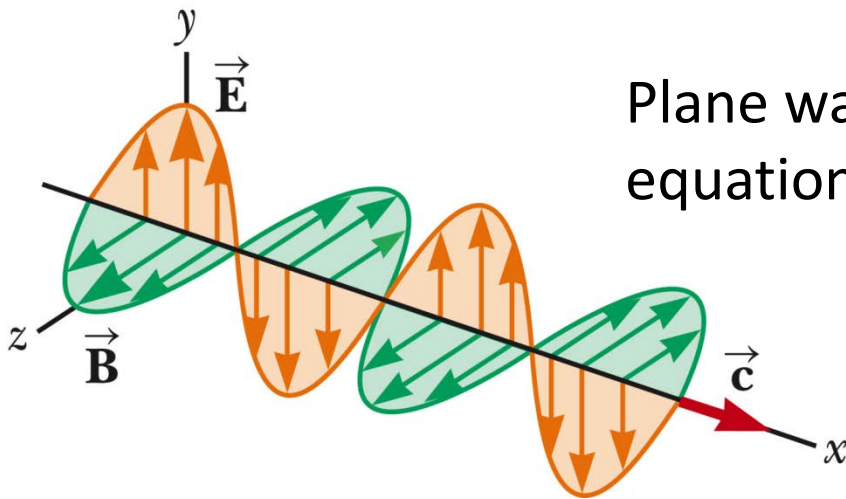
$$E_y = E_y(x, t) = E_{\max} \cos(k(x - ct))$$

$$B_z = B_z(x, t) = \frac{E_{\max}}{c} \cos(k(x - ct))$$

Which of the following changes in the above solution would **no longer** represent E-M waves:

- A.  $\cos \leftrightarrow \sin$
- B. Change value of  $E_{\max}$
- C. Change value of  $k$
- D. Change value of  $c$
- E. All of the above

Plane wave solution to Maxwell's equations, far from sources:



$$E_y = E_y(x, t) = E_{\max} \cos(k(x - ct))$$


$$B_z = B_z(x, t) = \frac{E_{\max}}{c} \cos(k(x - ct))$$


Additional comments:

For this solution, the **y** direction is called the **polarization** direction (the E field orientation)

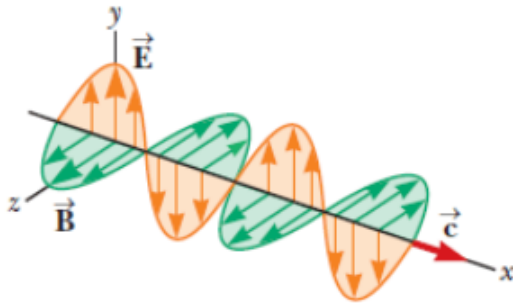
This is a periodic wave, where  $k=2\pi/\lambda$  and  $\lambda$  represents the wavelength and the frequency of the wave is  $kc=\omega=2\pi f$ .

# Homework hint:

5.  -/0.333 points

 My Notes | SerPSE8 34.P.013.MI.

The figure below shows a plane electromagnetic sinusoidal wave propagating in the  $x$  direction. Suppose the wavelength is  $58.0$  m and the electric field vibrates in the  $xy$  plane with an amplitude of  $18.0$  V/m.



(a) Calculate the frequency of the wave.

MHz

(b) Calculate the magnetic field  $\vec{B}$  when the electric field has its maximum value in the negative  $y$  direction.

magnitude  nT

direction  Select

(c) Write an expression for  $\vec{B}$  with the correct unit vector, with numerical values for  $B_{\max}$ ,  $k$ , and  $\omega$ , and with its magnitude in the form

$$B = B_{\max} \cos(kx - \omega t).$$

(Assume  $B$  is measured in nT,  $x$  is measured in m and  $t$  in s.)

magnitude   $\cos\left(\right.$    $x -$    $t\left.)\right)$  nT

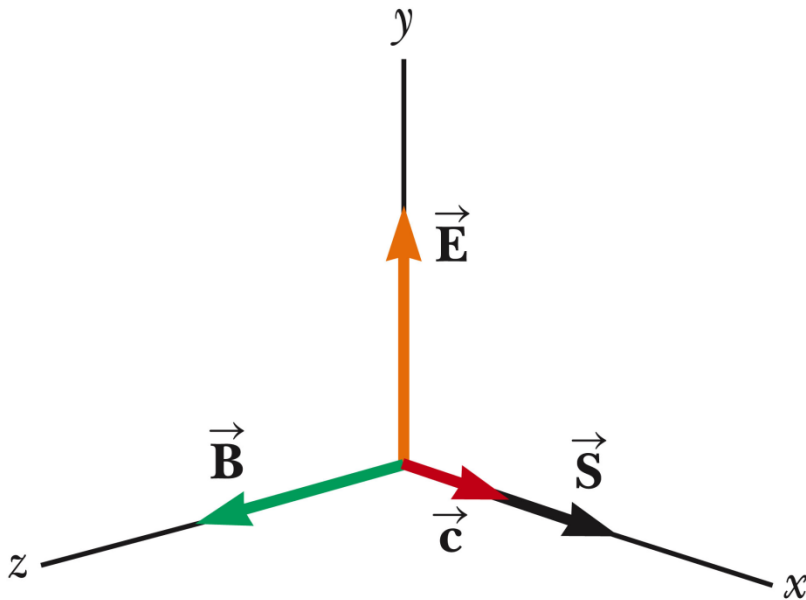
direction  Select



# Energy carried by electromagnetic waves:

Poynting vector :

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \quad \text{units : } \frac{1}{\text{T} \cdot \text{m/Amp}} \cdot \frac{\text{N}}{\text{C}} \cdot \text{T} = \frac{\text{N}}{\text{m} \cdot \text{s}}$$
$$= \frac{\text{N} \cdot \text{m}}{\text{m}^2 \cdot \text{s}} = \frac{\text{J}}{\text{m}^2 \cdot \text{s}} = \frac{\text{W}}{\text{m}^2}$$

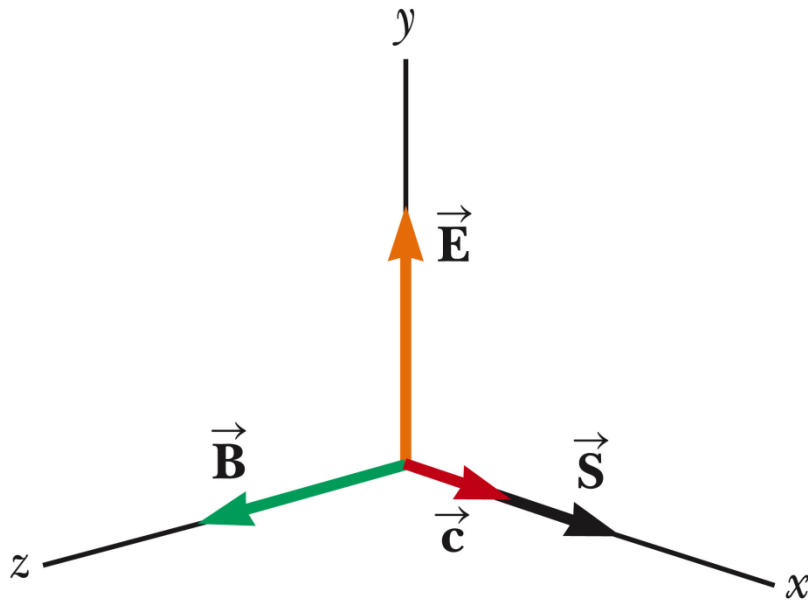


$$E_y = E_y(x, t) = E_{\max} \cos(k(x - ct))$$

$$B_z = B_z(x, t) = \frac{E_{\max}}{c} \cos(k(x - ct))$$

$$\Rightarrow \mathbf{S} = \frac{E_{\max}^2}{\mu_0 c} \cos^2(k(x - ct)) \hat{\mathbf{i}}$$

# Energy carried by electromagnetic waves – continued :

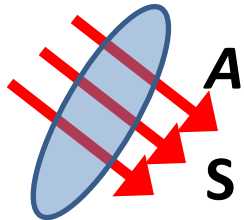


Poynting vector :

$$\mathbf{S} = \frac{E_{\max}^2}{\mu_0 c} \cos^2(k(x - ct)) \hat{\mathbf{i}}$$

Time averaged Poynting vector :

$$\mathbf{S}_{avg} = \frac{E_{\max}^2}{2\mu_0 c} \hat{\mathbf{i}}$$



Power carried by E - M wave :

$$P_{avg} = \mathbf{S}_{avg} \cdot \mathbf{A} = \frac{E_{\max}^2}{2\mu_0 c} \hat{\mathbf{i}} \cdot \mathbf{A}$$

Power carried by E - M wave :

$$P_{avg} = \mathbf{S}_{avg} \cdot \mathbf{A} = \frac{E_{max}^2}{2\mu_0 c} \hat{\mathbf{i}} \cdot \mathbf{A}$$

Example :

typical laser pointer has  $P_{avg} = 3 \times 10^{-3} W$ ,  $A = 3 \times 10^{-6} m^2$

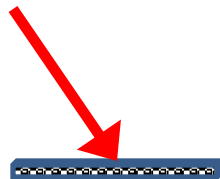
$$\Rightarrow E_{max} = \sqrt{\frac{2\mu_0 c P_{avg}}{A}} = 870 N / C$$

Radiation pressure

$$P_{pressure} \propto \frac{S}{c}$$

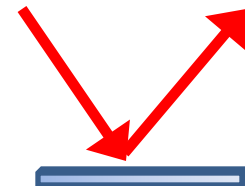
absorbing :

$$P_{pressure} = \frac{S}{c}$$



reflecting :

$$P_{pressure} = \frac{2S}{c}$$



Energy density within electromagnetic wave:

Electromagnetic energy density :

Electrical energy

Magnetic energy

$$u = \frac{1}{2} \epsilon_0 |\mathbf{E}|^2 + \frac{1}{2\mu_0} |\mathbf{B}|^2$$

$$u_{avg} = \frac{1}{4} \epsilon_0 |E_{max}|^2 + \frac{1}{4\mu_0} |B_{max}|^2$$

$$u_{avg} = \frac{1}{2} \epsilon_0 |E_{max}|^2 = \frac{1}{2\mu_0} |B_{max}|^2 = \frac{S_{avg}}{c}$$

## Sources of electromagnetic radiation

$$\oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = \frac{Q_{in}}{\epsilon_0}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\oint \mathbf{B}(\mathbf{r}) \cdot d\mathbf{A} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\int \mathbf{E}(\mathbf{r}) \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B}(\mathbf{r}) \cdot d\mathbf{A}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{in} + \mu_0 \epsilon_0 \frac{d}{dt} \int \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A}$$

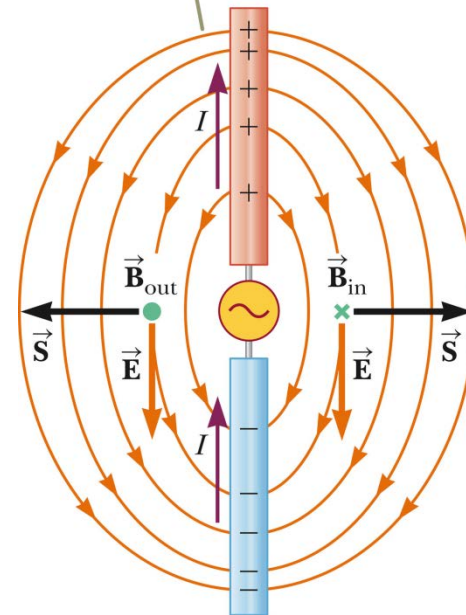
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

→ Need accelerating charges to produce E-M radiation

# Radiation from antenna's

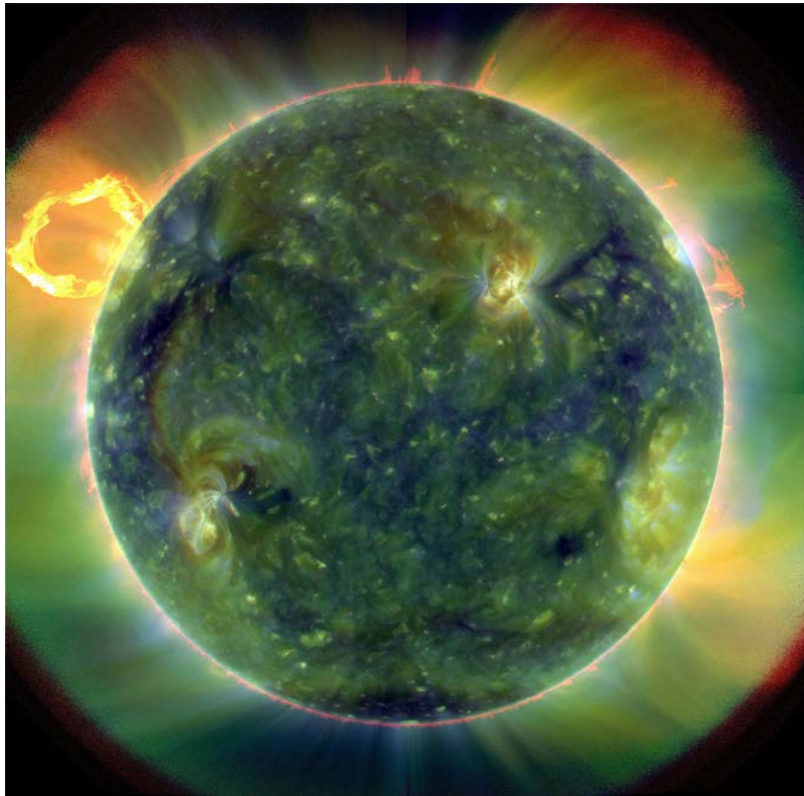


The electric field lines resemble those of an electric dipole (shown in Fig. 23.20).



# Electromagnetic radiation from the sun

[http://www.nasa.gov/mission\\_pages/sdo/news/first-light.html](http://www.nasa.gov/mission_pages/sdo/news/first-light.html)



A full-disk multiwavelength extreme ultraviolet image of the sun taken by SDO on March 30, 2010. False colors trace different gas temperatures. Reds are relatively cool (about 60,000 Kelvin, or 107,540 F); blues and greens are hotter (greater than 1 million Kelvin, or 1,799,540 F). Credit: NASA/Goddard/SDO AIA Team

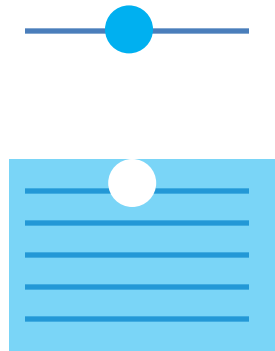
Launched on Feb. 11, 2010, SDO is the most advanced spacecraft ever designed to study the sun. During its five-year mission, it will examine the sun's magnetic field and also provide a better understanding of the role the sun plays in Earth's atmospheric chemistry and climate. Since launch, engineers have been conducting testing and verification of the spacecraft's components. Now fully operational, SDO will provide images with clarity 10 times better than high-definition television and will return more comprehensive science data faster than any other solar observing spacecraft.

# Electromagnetic radiation from quantum mechanics: Atoms, molecules, solids

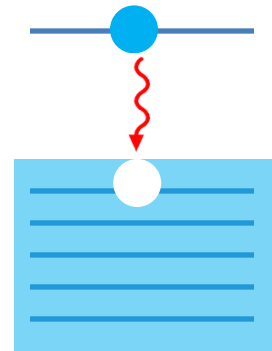
Ground state



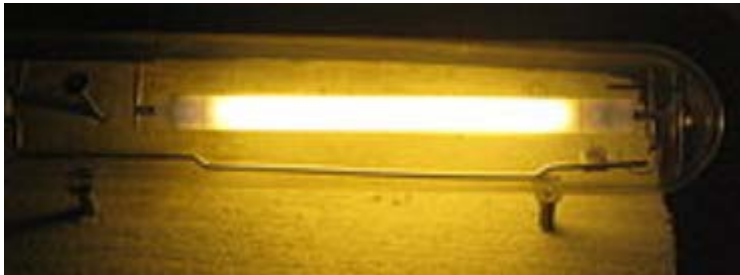
Excited state



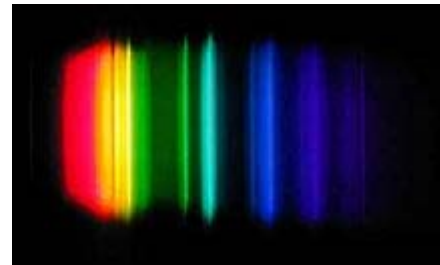
Radiation



Sodium vapor lamp

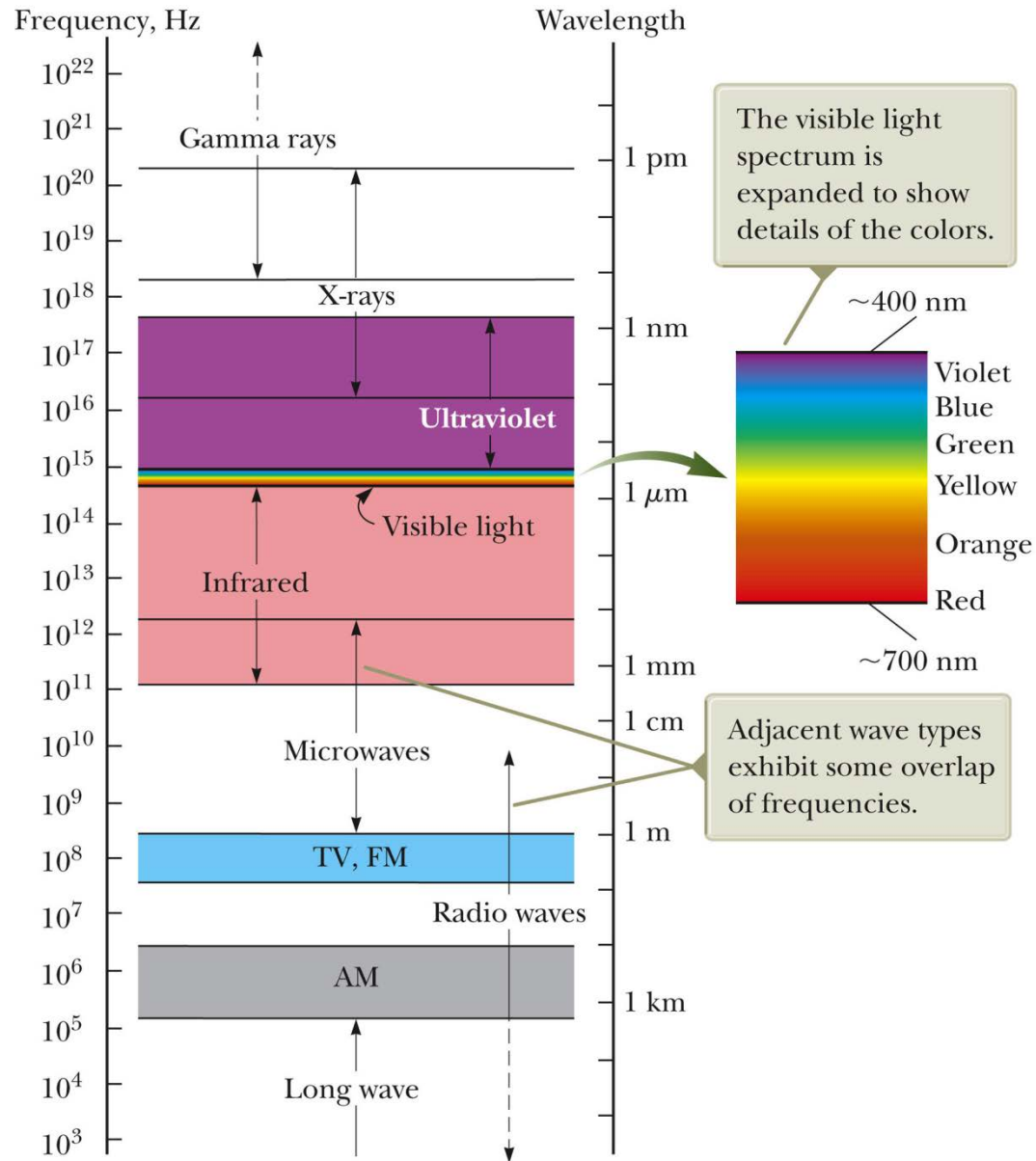


Spectrum





# Electromagnetic spectrum

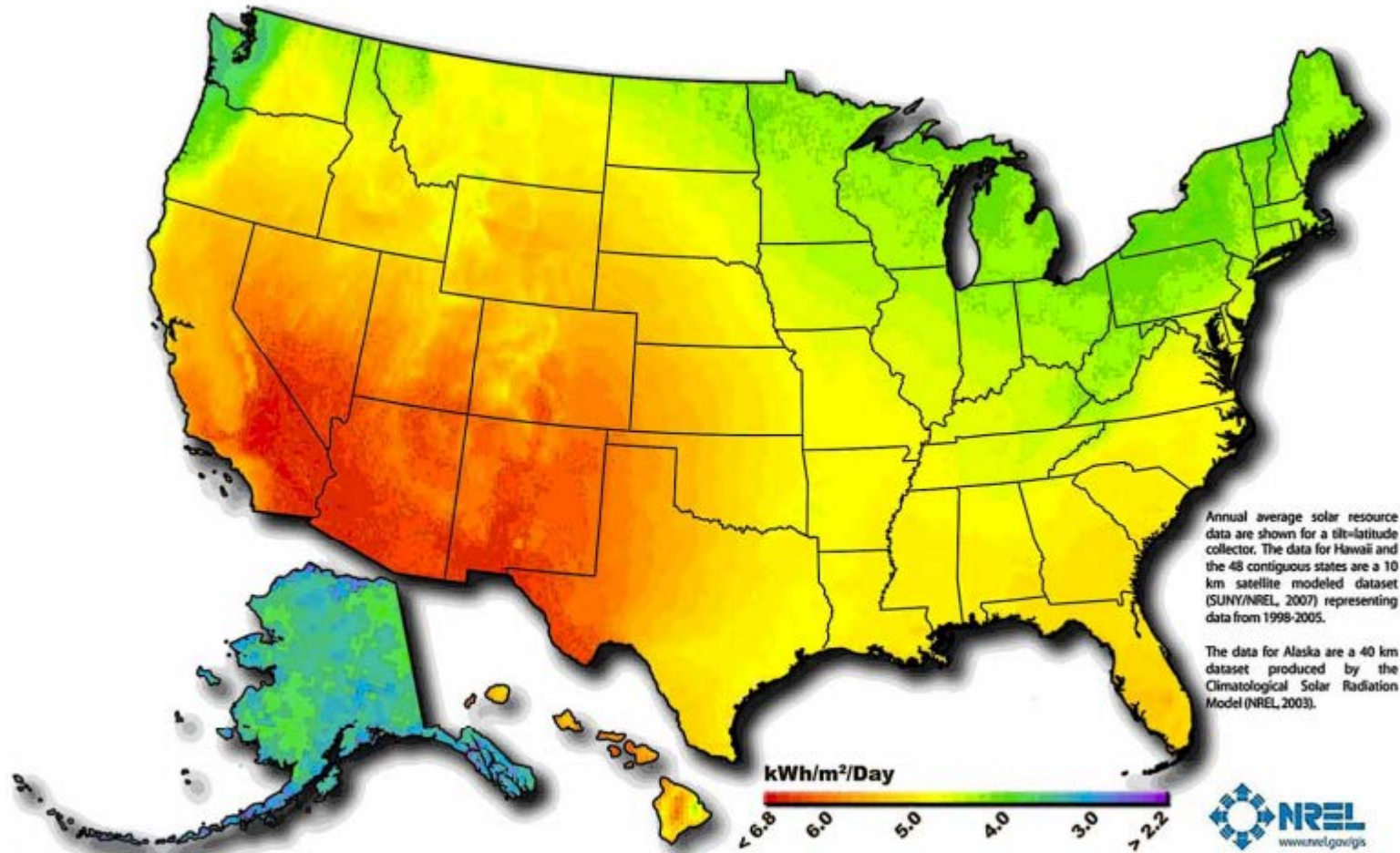


## Comment about solar energy

Technology to capture and use electromagnetic radiation from the sun and use it as a heat source or as a generator of voltage in special semiconductor devices

(see web page from Prof. Wesley Henderson from NCSU:

[http://www.che.ncsu.edu/ILEET/CHE596web\\_Fall2011/21\\_CHE596-015\\_2011-11-15\\_Renewables.pdf](http://www.che.ncsu.edu/ILEET/CHE596web_Fall2011/21_CHE596-015_2011-11-15_Renewables.pdf)



Author: Billy Roberts - October 20, 2008

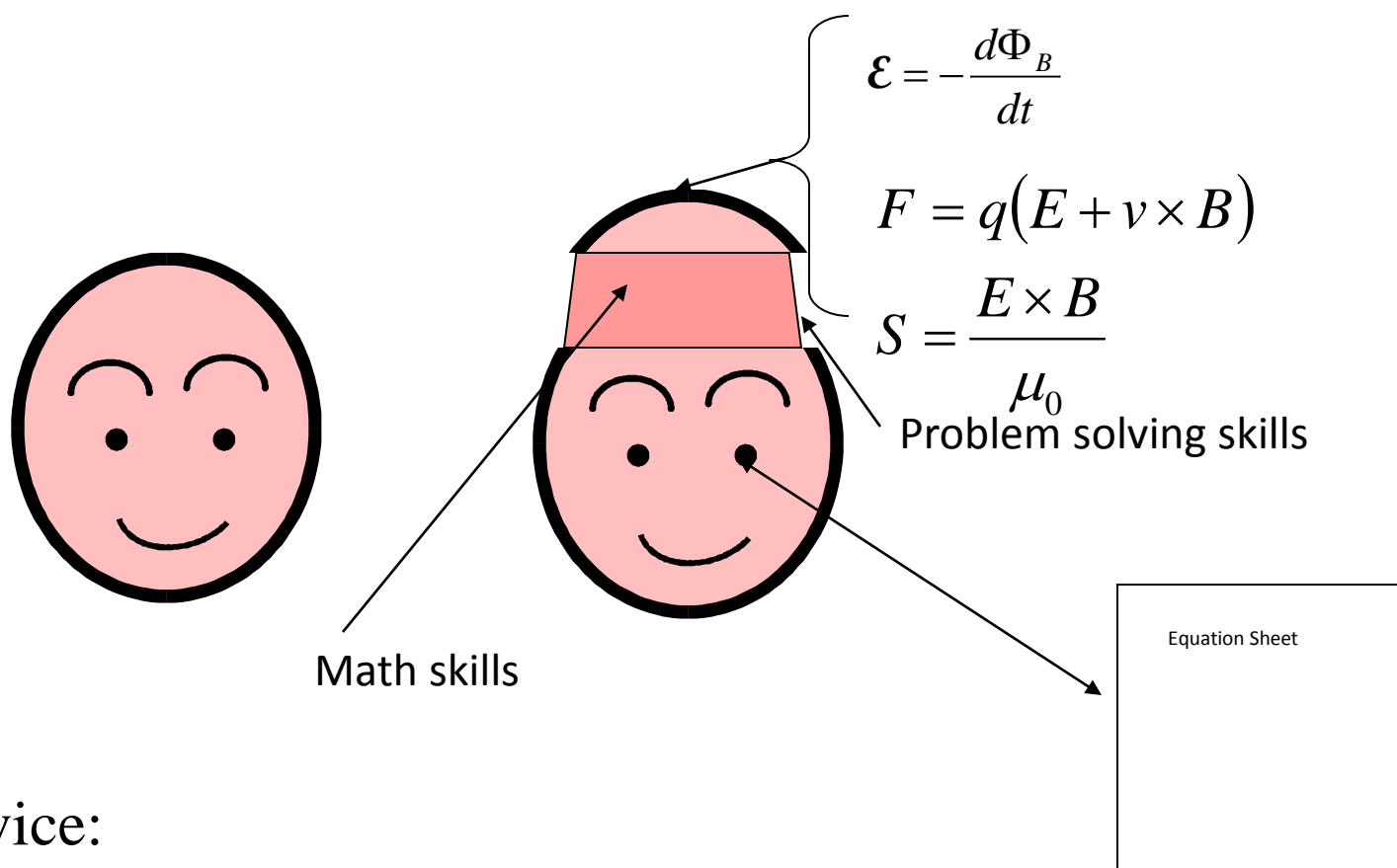
This map was produced by the National Renewable Energy Laboratory for the U.S. Department of Energy.



Reminder:

**Third exam – evenings of Mon-Thurs (4/2-4/5) –  
covering Chapters 29-34**

- ~5 problems – show your work and reasoning for possible partial credit.
- Should bring 1 8½” x 11” sheet of paper to the exam (to be turned in with your exam papers).
- Should bring calculator for numerical work. Must not use cell phones or computers during the exam.
- Exams will be in Olin 107 Mon-Wed and Olin 101 Thurs
- 6-10 PM (must schedule by email before 3/30/2012)



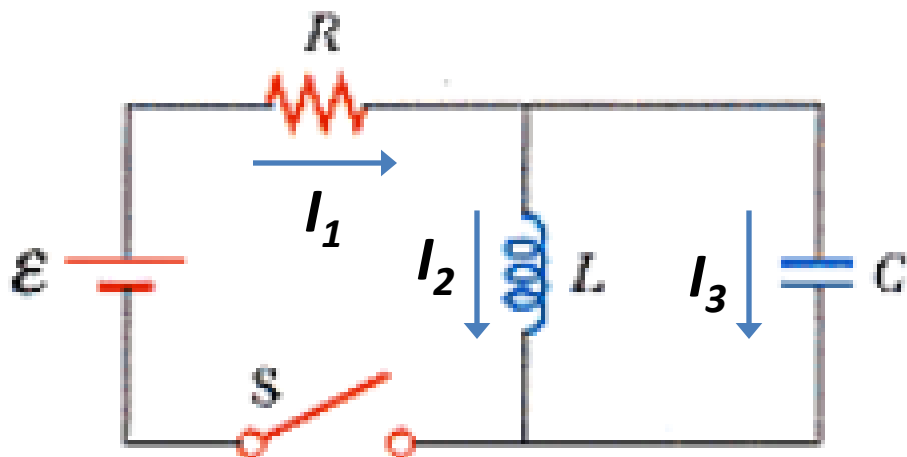
## Advice:

1. Keep basic concepts and equations at the top of your head.
2. Practice problem solving and math skills
3. Develop an equation sheet that you can consult.

## Problem solving steps

1. Visualize problem – labeling variables
2. Determine which basic physical principle(s) apply
3. Write down the appropriate equations using the variables defined in step 1.
4. Check whether you have the correct amount of information to solve the problem (same number of knowns and unknowns).
5. Solve the equations.
6. Check whether your answer makes sense (units, order of magnitude, etc.).

## Comment on AC circuits:



$$\mathcal{E} - RI_1 - L \frac{dI_2}{dt} = 0$$

$$-L \frac{dI_2}{dt} + \frac{Q_3}{C} = 0$$

$$I_1 = I_2 + I_3$$

Solution method:

1. Transform differential equation in to algebraic equation using trig or complex functions
2. "Solve" algebra problem
3. Analyze for physical solution





Example using Ampere's law and Ampere-Maxwell law:

Now consider Ampere-Maxwell's law and a uniform electric field with a constant rate of change:

$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \epsilon_0 \frac{d}{dt} \int \mathbf{E}(r) \cdot d\mathbf{A}$

$B$  at  $r_1 > R$ :

$$B 2\pi r_1 = \mu_0 \epsilon_0 \frac{dE}{dt} \pi R^2 \Rightarrow B = \frac{\mu_0 \epsilon_0 R^2}{2r_1} \frac{dE}{dt}$$

$B$  at  $r_2 < R$ :

$$B 2\pi r_2 = \mu_0 \epsilon_0 \frac{dE}{dt} \pi r_2^2 \Rightarrow B = \frac{\mu_0 \epsilon_0 r_2}{2} \frac{dE}{dt}$$

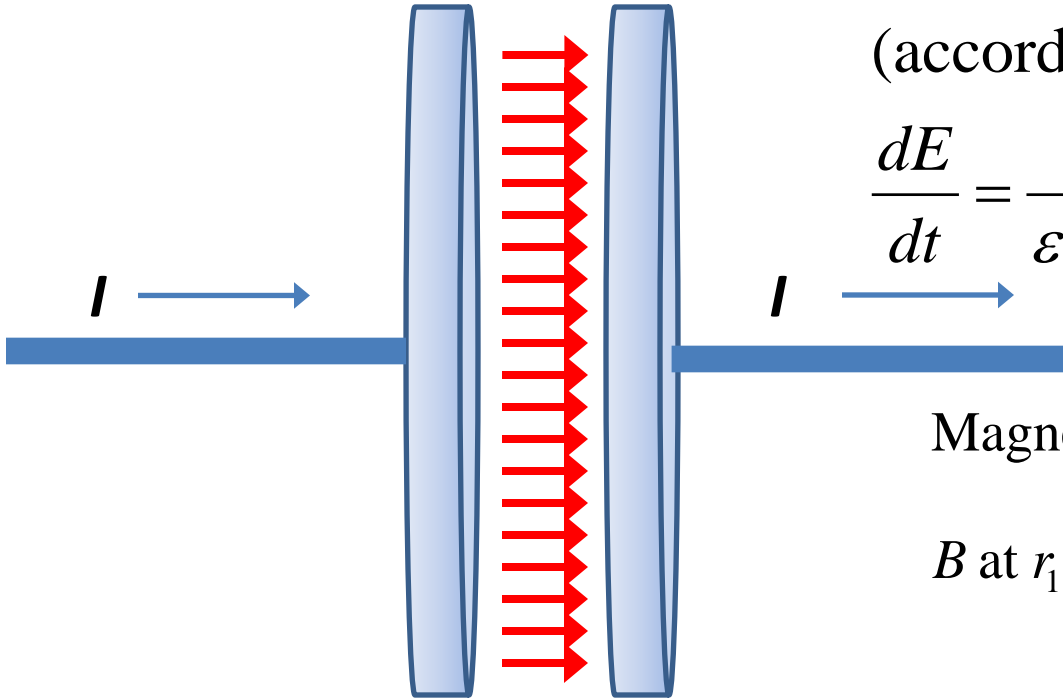
Example using Ampere's law and Ampere-Maxwell law:

Now consider Ampere-Maxwell's law and a uniform electric field with a constant rate of change -- connection to capacitance circuit where  $R$  denotes radius of plates

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \epsilon_0 \frac{d}{dt} \int \mathbf{E}(r) \cdot d\mathbf{A}$$

Electric field within capacitor plates  
(according to Gauss's law):

$$\frac{dE}{dt} = \frac{I}{\epsilon_0 \pi R^2}$$



Magnetic field between capacitor plates

$$B \text{ at } r_1 > R: \quad B = \frac{\mu_0 \epsilon_0 R^2}{2r_1} \frac{dE}{dt} = \frac{\mu_0 I}{2\pi r_1}$$

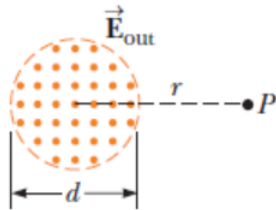
$$B \text{ at } r_2 < R: \quad B = \frac{\mu_0 \epsilon_0 r_2}{2} \frac{dE}{dt} = \frac{\mu_0 I r_2}{2\pi R^2}$$

# Homework hint:

1. + -0.334 points

[My Notes](#) | SerPSE8 34.P.003.

Consider the situation shown in the figure below. An electric field of 300 V/m is confined to a circular area  $d = 10.5$  cm in diameter and directed outward perpendicular to the plane of the figure. Consider that the field is increasing at a rate of  $18.8$  V/m  $\cdot$  s.



(a) What is the direction of the magnetic field at the point  $P$ ,  $r = 13.6$  cm from the center of the circle?

- upwards
- downwards

(b) What is the magnitude of the magnetic field at the point  $P$ ,  $r = 13.6$  cm from the center of the circle?

T

## Full Maxwell's equations

$$\oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = \frac{Q_{in}}{\epsilon_0}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\oint \mathbf{B}(\mathbf{r}) \cdot d\mathbf{A} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\int \mathbf{E}(\mathbf{r}) \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B}(\mathbf{r}) \cdot d\mathbf{A}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{in} + \mu_0 \epsilon_0 \frac{d}{dt} \int \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Lorentz force law:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$