PHY 114 A General Physics II  
11 AM-12:15 PM TR Olin 101  

Plan for Lecture 17 (Chapter 34):  
Electromagnetic Waves  
1. Maxwell’s equations & their solutions  
2. Electromagnetic energy and their spectral distribution  
3. Review of Chapters 29-34  

Remember to send in your chapter reading questions…  

3rd exam (covering Chapters 29-34) is scheduled for evenings during the week of 4/2/2012.  

You will be scheduled for one of these (based on email info)  
– probably in Olin 107 between 6-10 PM:  
  o Monday 4/2  
  o Tuesday 4/3  
  o Wednesday 4/4  
  o Thursday 4/5  

Several people still need to email their preferred exam times.
Full Maxwell's equations
\[ \int \mathbf{E}(r) \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}, \quad \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}, \]
\[ \int \mathbf{B}(r) \cdot d\mathbf{A} = 0, \quad \nabla \cdot \mathbf{B} = 0, \]
\[ \int \mathbf{E}(r) \cdot \frac{d}{dt}\mathbf{A} = -\frac{1}{\mu_0} \frac{d}{dt} \int \mathbf{B}(r) \cdot d\mathbf{A}, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \]
\[ \int \mathbf{B}(r) \cdot \frac{d}{dt}\mathbf{A} = \mu_0 j + \mu_0 \varepsilon_0 \frac{d}{dt} \frac{\partial \mathbf{E}}{\partial t}. \]

Plane wave solution to Maxwell's equations, far from sources:
\[ \mathbf{E} = E_0 \hat{\mathbf{j}} \quad \mathbf{B} = B_0 \hat{k} \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \Rightarrow \frac{\partial E_y}{\partial t} = \frac{\partial B_z}{\partial x}, \]
\[ \nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \Rightarrow \frac{\partial B_x}{\partial t} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial z}. \]

E_j = E_j(x,t) = E_0 \cos(\omega(t-x/c))
B_j = B_j(x,t) = E_0 / c \cos(\omega(t-x/c))

Plane wave solution to Maxwell's equations, far from sources:
\[ \mathbf{E}_j = \mathbf{E}_j(x,t) = E_0 \cos(\omega(t-x/c)) \]
\[ \mathbf{B}_j = \mathbf{B}_j(x,t) = \frac{E_0}{c} \cos(\omega(t-x/c)) \]

Which of the following changes in the above solution would no longer represent E-M waves:
A. \( \cos \rightarrow \sin \)
B. Change value of \( E_0 \)
C. Change value of \( k \)
D. Change value of \( c \)
E. All of the above
Plane wave solution to Maxwell's equations, far from sources:

\[ E_y = E_{ym} \cos(k(x-ct)) \]
\[ B_z = B_{zm} \cos(k(x-ct)) \]

Additional comments:
For this solution, the \( y \) direction is called the \textit{polarization} direction (the \( E \) field orientation).

This is a periodic wave, where \( k = \frac{2\pi}{\lambda} \) and \( \lambda \) represents the wavelength and the frequency of the wave is \( \omega = 2\pi f \).

Homework hint:

Energy carried by electromagnetic waves:

\[ \mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \]

units: \[ \text{N} \cdot \text{m} / \text{Amp} \cdot \text{m} \]
\[ \mathbf{N} \cdot \mathbf{m} / \text{m}^2 \cdot \text{s} = \text{W} / \text{m}^2 \]

\[ E_y = E_{ym} \cos(k(x-ct)) \]
\[ B_z = B_{zm} \cos(k(x-ct)) \]
\[ \Rightarrow \mathbf{S} = \frac{E_{ym}^2}{\mu_0 c} \text{cos}^2(k(x-ct)) \hat{\mathbf{z}} \]
Energy carried by electromagnetic waves – continued:

Poynting vector:
\[ \mathbf{S} = \frac{\mathbf{E} \times \mathbf{B}}{\mu_0 c} \]

Time averaged Poynting vector:
\[ \mathbf{S}_{\text{avg}} = \frac{E_{\text{max}}^2}{2 \mu_0 c} \mathbf{i} \]

Power carried by E - M wave:
\[ P_{\text{avg}} = \mathbf{S}_{\text{avg}} \cdot \mathbf{A} = \frac{E_{\text{max}}^2}{2 \mu_0 c} \mathbf{i} \cdot \mathbf{A} \]

Power carried by E - M wave:
\[ P_{\text{avg}} = \mathbf{S}_{\text{avg}} \cdot \mathbf{A} = \frac{E_{\text{max}}^2}{2 \mu_0 c} \mathbf{i} \cdot \mathbf{A} \]

Example:

typical laser pointer has \( P_{\text{avg}} = 3 \times 10^9 W \), \( A = 3 \times 10^{-6} m^2 \)

\[ \Rightarrow E_{\text{rms}} = \frac{2 \mu_0 c P_{\text{avg}}}{A} = 870N/C \]

Radiation pressure
\[ P_{\text{pressure}} \propto \frac{S}{c} \]

absorbing: \[ P_{\text{pressure}} = \frac{S}{c} \]

reflecting: \[ P_{\text{pressure}} = \frac{2S}{c} \]

Energy density within electromagnetic wave:

Electromagnetic energy density:

\[ u = \frac{1}{2} \varepsilon_0 \left| \mathbf{E} \right|^2 + \frac{1}{2 \mu_0} \left| \mathbf{B} \right|^2 \]

\[ u_{\text{avg}} = \frac{1}{4} \varepsilon_0 \left| E_{\text{max}} \right|^2 + \frac{1}{4 \mu_0} \left| B_{\text{max}} \right|^2 \]

\[ u_{\text{avg}} = \frac{1}{2} \varepsilon_0 \left| E_{\text{max}} \right|^2 = \frac{1}{2 \mu_0} \left| B_{\text{max}} \right|^2 = \frac{S_{\text{avg}}}{c} \]
Sources of electromagnetic radiation

\[
\begin{align*}
\oint \mathbf{E}(r) \cdot d\mathbf{A} &= \frac{Q}{\varepsilon_0} \\
\oint \mathbf{B}(r) \cdot d\mathbf{A} &= 0 \\
\int \mathbf{B}(r) \cdot d\mathbf{A} &= \frac{\rho}{\varepsilon_0} \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\end{align*}
\]

» Need accelerating charges to produce E-M radiation

Radiation from antenna’s

Electromagnetic radiation from the sun


A full-disk multiband extreme ultraviolet image of the sun taken by SDO on March 30, 2010. False colors trace different gas temperatures. Reds are relatively cool (about 60,000 Kelvin, or 107,540 F), blues and greens are hotter (greater than 1 million Kelvin, or 1,799,540 F). Credit: NASA/Goddard/SDO AIA Team

Launched on Feb. 11, 2010, SDO is the most advanced spacecraft ever designed to study the sun. During its five-year mission, it will examine the sun’s magnetic field and also provide a better understanding of the role the sun plays in Earth’s atmospheric chemistry and climate. Since launch, engineers have been conducting testing and verification of the spacecraft’s components. Now fully operational, SDO will provide images with clarity 30 times better than high-definition television and will return more comprehensive science data faster than any other solar observing spacecraft.
Electromagnetic radiation from quantum mechanics: Atoms, molecules, solids

- **Ground state**
- **Excited state**
- **Radiation**

<table>
<thead>
<tr>
<th>Sodium vapor lamp</th>
<th>Spectrum</th>
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Electromagnetic spectrum

Comment about solar energy
Technology to capture and use electromagnetic radiation from the sun and use it as a heat source or as a generator of voltage in special semiconductor devices
(see web page from Prof. Wesley Henderson from NCSU: http://www.che.ncsu.edu/ILEET/CHE596web_Fall2011/21_CHE596-015_2011-11-15_Renewables.pdf)
Reminder:

Third exam – evenings of Mon-Thurs (4/2-4/5) – covering Chapters 29-34

- 5 problems – show your work and reasoning for possible partial credit.
- Should bring 1 8½” x 11” sheet of paper to the exam (to be turned in with your exam papers).
- Should bring calculator for numerical work. Must not use cell phones or computers during the exam.
- Exams will be in Olin 107 Mon-Wed and Olin 101 Thurs
- 6-10 PM (must schedule by email before 3/30/2012)

Advice:
1. Keep basic concepts and equations at the top of your head.
2. Practice problem solving and math skills
3. Develop an equation sheet that you can consult.
Problem solving steps

1. Visualize problem — labeling variables
2. Determine which basic physical principle(s) apply
3. Write down the appropriate equations using the variables defined in step 1.
4. Check whether you have the correct amount of information to solve the problem (same number of knowns and unknowns).
5. Solve the equations.
6. Check whether your answer makes sense (units, order of magnitude, etc.).

Comment on AC circuits:

\[ \mathcal{E} - RI - L \frac{dI}{dt} = 0 \]
\[ -L \frac{dI}{dt} - \frac{Q}{C} = 0 \]
\[ I_1 = I_2 + I_3 \]

Solution method:
1. Transform differential equation in to algebraic equation using trig or complex functions
2. "Solve" algebra problem
3. Analyze for physical solution

Example using Ampere’s law and Ampere-Maxwell law:

First consider Ampere’s law and a wire with uniform current I

\[ \int B \, dl = \mu_0 I_\text{m} \]

\[ B \text{ at } r > R: \]
\[ B \times 2\pi r = \mu_0 \frac{\pi R^2}{\pi R} \Rightarrow B = \frac{\mu_0 I}{2\pi r} \]

\[ B \text{ at } r < R: \]
\[ B \times 2\pi r = \mu_0 \frac{\pi r^2}{\pi R} \Rightarrow B = \frac{\mu_0 I r}{2\pi R^2} \]
Example using Ampere's law and Ampere-Maxwell law:

Now consider Ampere-Maxwell's law and a uniform electric field with a constant rate of change:

\[ \oint B \cdot dS = \mu_0 \int \mathbf{E}(r) \cdot dA \]

- For \( r > R \): \[ B = \mu_0 \varepsilon_0 \frac{dE}{dt} r \pi R^2 \]
- For \( r < R \): \[ B = \mu_0 \varepsilon_0 \frac{dE}{dt} r \pi r^2 \]

Electric field within capacitor plates (according to Gauss's law):

\[ \frac{dE}{dt} = \frac{I}{\varepsilon_0 \pi R^2} \]

Magnetic field between capacitor plates

\[ B = \frac{\mu_0 \varepsilon_0 I}{2 \pi_1} \frac{dE}{dt} = \frac{\mu_0 I}{2 \pi_1} \]

\[ B = \frac{\mu_0 \varepsilon_0 I}{2 \pi_2} \frac{dE}{dt} = \frac{\mu_0 I}{2 \pi_2} \]

Homework hint:

Consider the situation shown in the figure below. The electric field of 100 V/m is uniform in a circular plate of radius \( R = 0.6 \) m. Consider the plates to be charged to a constant potential difference of 100 V. Calculate the magnetic field at the point \( r = 0.8 \) m from the center of the circle.

1. What is the direction of the magnetic field at the point \( r = 0.8 \) m from the center of the circle?
   - spiral
   - perpendicular

2. What is the magnitude of the magnetic field at the point \( r = 0.8 \) m from the center of the circle?
Full Maxwell’s equations

\[ \oint \vec{E}(r) \cdot d\vec{A} = \frac{\partial \vec{D}}{\partial t} \]
\[ \oint \vec{B}(r) \cdot d\vec{A} = 0 \]
\[ \oint \vec{E}(r) \cdot d\vec{A} = -\frac{d}{dt} \oint \vec{B}(r) \cdot d\vec{A} \]
\[ \oint \vec{B} \cdot d\vec{A} = \mu_0 I + \mu_0 \frac{d}{dt} \oint \vec{E}(r) \cdot d\vec{A} \]

\[ \vec{F} = q \left( \vec{E} + \vec{v} \times \vec{B} \right) \]