Plan for Lecture 18 (Chapter 35):

Optical properties of light

1. Speed of light in vacuum and in materials
2. Refraction and reflection of light
3. Spectrum of light and dispersion

Plane wave solution to Maxwell's equations in vacuum:

\[ E_y = E_y(x, t) = E_{\text{max}} \cos(k(x-ct)) \]
\[ B_y = B_y(x, t) = \frac{E_{\text{max}}}{c} \cos(k(x-ct)) \]

\[ c = \sqrt{\frac{1}{\varepsilon_0 \mu_0}} = \frac{2.99792458 \times 10^8 \text{ m/s}}{3 \times 10^8 \text{ m/s}} \]

Additional comments:

For this solution, the y direction is called the polarization direction (the E field orientation)

This is a periodic wave, where \( k = 2\pi/\lambda \) and \( \lambda \) represents the wavelength and the frequency of the wave is \( kc = \omega = 2\pi f \).
Webassign hint:

In a region of free space, the electric field at an instant of time is \( E = 30(\hat{i} + \hat{j}) \text{ V/m} \), and the magnetic field is \( B = 20(\hat{k}) \text{ T} \). Calculate the following quantities:

\[
\begin{align*}
\epsilon_0 E &= \frac{N \text{ v/m}}{N \text{ v/m}} \quad \hat{E} = 30(\hat{i} + \hat{j}) \text{ V/m} \\
\mu_0 B &= \frac{N \text{ T/m}}{N \text{ T/m}} \quad \hat{B} = 20(\hat{k}) \text{ T} \\
\epsilon_0 E \times \mu_0 B &= \frac{N \text{ v/m} \text{ T/m}}{N \text{ T/m}} \\
\end{align*}
\]

(b) Determine the component representation of the Poynting vector for these fields. Use these derived planes.

\[
\begin{align*}
\epsilon &= \frac{1}{\mu_0} \\
S &= \frac{1}{\mu_0} \vec{E} \times \vec{B} \\
\vec{B} &= \hat{a} + \epsilon \hat{j} + \mu \hat{k} \\
\hat{i} \times \hat{j} &= \hat{k} = -\hat{j} \times \hat{i} \\
\hat{j} \times \hat{k} &= \hat{i} = \hat{k} \times \hat{i} = \hat{j}
\end{align*}
\]

Index of refraction \( n \):

<table>
<thead>
<tr>
<th>In vacuum</th>
<th>In medium</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_0 )</td>
<td>( \epsilon \geq \epsilon_0 )</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>( \mu \geq \mu_0 )</td>
</tr>
<tr>
<td>( c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} )</td>
<td>( v = \frac{1}{\sqrt{\epsilon \mu}} = \frac{c}{n} )</td>
</tr>
</tbody>
</table>

What happens to an electromagnetic wave traveling when it encounters a different medium?

A. It usually passes through the medium without any change
B. It usually passes through the medium but the velocity is changed
C. It usually passes through the medium with a different velocity; the \( E \) and \( B \) fields are also changed
D. It usually cannot pass through the different medium

What happens to the propagation of an electromagnetic wave traveling when it encounters a different medium?

A. The propagation vector continues the medium without change
B. The propagation vector is reflected before passing the second medium
C. The propagation vector changes when it passes through the medium
D. More than one possibility
Indices of refraction for various media:

<table>
<thead>
<tr>
<th>Substance</th>
<th>Index of Refraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sodium at 25°C</td>
<td>2.30</td>
</tr>
<tr>
<td>Cubic zirconia</td>
<td>2.410</td>
</tr>
<tr>
<td>Fluorine (CaF₂)</td>
<td>1.434</td>
</tr>
<tr>
<td>Fused quartz (SiO₂)</td>
<td>1.456</td>
</tr>
<tr>
<td>Gallium phosphate</td>
<td>3.50</td>
</tr>
<tr>
<td>Glass, crown</td>
<td>1.52</td>
</tr>
<tr>
<td>Glass, flint</td>
<td>1.66</td>
</tr>
<tr>
<td>Ice (H₂O)</td>
<td>1.500</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>1.49</td>
</tr>
<tr>
<td>Sodium chloride (NaCl)</td>
<td>1.544</td>
</tr>
</tbody>
</table>

Table 35.1: Indices of Refraction

Note: All values are for light having a wavelength of 0.55 μm in vacuum.

Suppose that you observe, at an angle of \( \theta_1 = 45^\circ \), a fish in a pond with refractive index \( n_1 = 1.333 \). How does the fish appear to you relative to its actual location?

A. Further to the right?
B. Further to the left?
C. At the same location.
Suppose you observe, at an angle of $\theta_1 = 45^\circ$, a fish in a pond with refractive index $n_1 = 1.333$. How does the fish appear to you relative to its actual location?

A. Further to the right?
B. Further to the left?
C. At the same location.

\[
n_1 \sin \theta_1 = n_2 \sin \theta_2
\]

\[
\theta_2 = \sin^{-1}\left(\frac{\sin 45^\circ}{1.333}\right)
\]

\[\approx 32^\circ\]

The picture to the left

A. is consistent with Snell’s law
B. shows that Snell’s law is false
C. shows that water bends pencils

Light through a slab

\[
n_1 \sin \theta_1 = n_2 \sin \theta_2
\]

\[
\Rightarrow \theta_2 = \sin^{-1}\left(\frac{n_1 \sin \theta_1}{n_2}\right)
\]

\[
\gamma = \theta_1 - \theta_2
\]

\[
d = a \sin \gamma
\]

\[
a = \frac{r}{\cos \theta_1}
\]

What is the value of $\theta_2$?

A. $\theta_1$
B. $\theta_2$
C. $\theta_1 + \theta_2$
More geometrical optics

\[ n \sin 30^\circ = \sin \theta \]

For \( n = 1.333 \)

\[ \theta = 41.8^\circ \]

Which of the following statements is true:

A. \( 0 < \theta_2 < 90^\circ \)
B. \( 0 < \theta_1 < \theta_c \)
C. \( \theta_1 > \theta_2 \)

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

\[ \Rightarrow \text{If } \theta_1 = 90^\circ \quad \sin \theta_2 = \frac{n_1}{n_2} \]

For \( n_1 = 1 \) and \( n_2 = 1.333 \)

\[ \theta_2 = 48.6^\circ \]
Total internal reflection for $\theta_2 > \theta_c$

$$\sin \theta_c = \frac{n_2}{n_1}$$

For $n_i = 1$ and $n_i = 1.333$, $\theta_c = 48.6^\circ$

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Uses for total internal reflection – fiber optic cables

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General case – reflection and refraction

For $E$ polarized in scattering plane

$$\frac{E_x}{E_0} = \frac{2n_i \cos \theta}{n_2 \cos \theta_1 + n_1 \cos \theta_2}$$

$$\frac{E_y}{E_0} = \frac{n_2 \cos \theta_1 + n_1 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2}$$

For $E$ polarized out of scattering plane

$$\frac{E_x}{E_0} = \frac{2n_i \cos \theta}{n_2 \cos \theta_1 + n_1 \cos \theta_2}$$

$$\frac{E_y}{E_0} = \frac{n_2 \cos \theta_1 - n_1 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2}$$
General case – reflection and refraction and multiple surfaces

Dispersion

$n = n(\lambda)$

The colors in the refracted beam are separated because dispersion in the prism causes different wavelengths of light to be refracted through different angles.

Dispersion from prism

Visible light

Deviation of red light

Angular spread

Screen

B
Where is the pot of gold?
A. Primary rainbow
B. Secondary rainbow
C. Both

Imagine you are viewing this scene. Where is the sun?
A. To your right
B. To your left
C. Behind you
D. In front of you