Plan for Lecture 19 (Chapter 36):

Optical properties of light

1. Mirror reflections
2. Images in flat and spherical mirrors

Plane wave solution to Maxwell's equations in dielectric medium with $v=c/n$:

$$E_y = E_{y0}(x,t) = E_{y\infty}\cos(\lambda (x-vt))$$

$$B_z = B_{z0}(x,t) = \frac{E_{y\infty}}{v}\cos(\lambda (x-vt))$$

Additional comments:

For this solution, the $y$ direction is called the polarisation direction (the $E$ field orientation)

This is a periodic wave, where $k=2\pi/\lambda$ and $\lambda$ represents the wavelength and the frequency of the wave is $kc/n=\omega/2\pi$. 
Index of refraction $n$:

In vacuum:
\[ c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \]

In medium:
\[ c \geq c_0 \]
\[ \varepsilon \geq \varepsilon_0 \]
\[ \mu \geq \mu_0 \]
\[ \nu = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{n} \]

\[ n = \frac{\varepsilon}{\varepsilon_0} \]
\[ n = \frac{\mu}{\mu_0} \]
\[ \frac{n_c}{n_v} = \frac{\varepsilon}{\varepsilon_0} \frac{\mu}{\mu_0} = \frac{\epsilon\mu}{\varepsilon_0\mu_0} \]

\[ \frac{n_{c v}}{\varepsilon_{c v}} \geq 1 \]

\[ \frac{n_{c v}}{\varepsilon_{c v}} = 1 \]

\[ \frac{n_{c v}}{\varepsilon_{c v}} \leq 1 \]

\[ \frac{n_{c v}}{\varepsilon_{c v}} \geq 0 \]

\[ n_{c v} = 0 \]

\[ n_{c v} = 1 \]

\[ n_{c v} = \infty \]

Snell's law:
\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

General case – reflection and refraction:

For $E$ polarized in scattering plane:
\[ \frac{E_2}{E_0} = \frac{2n_1 \cos \theta_1}{n_1^c \cos \theta_1 + n_1 \cos \theta_1} \]
\[ \frac{E_{2h}}{E_0} = \frac{n_1^c \cos \theta_1 - n_1 \cos \theta_1}{n_1 \cos \theta_1 + n_2 \cos \theta_2} \]

For $E$ polarized out of scattering plane:
\[ \frac{E_2}{E_0} = \frac{2n_1 \cos \theta_1}{n_1 \cos \theta_1 + n_1 \cos \theta_2} \]
\[ \frac{E_{2h}}{E_0} = \frac{n_1 \cos \theta_1 + n_2 \cos \theta_1}{n_1 \cos \theta_1 + n_2 \cos \theta_2} \]

If $n_2 \to \infty$, then:
\[ \frac{E_{2h}}{E_0} \to 0 \quad \text{and} \quad \frac{E_{2h}}{E_0} \to 1 \]
Images formed from reflected light:

Notation for image position:

\[ i \leftrightarrow q \]

Analysis of mirror image

Using geometry:

\[ i = p \quad h = h' \]

Mirror symmetry:

Terminology:

Virtual image – perceived image but no light can be detected at the location of the virtual image

Real image -- light can be detected at the location of the real image
Summary of geometric optics of plane mirror

General equation describing object and image positions:

Mirror equation:

\[ \frac{1}{p} + \frac{1}{i} = \frac{1}{f} \]

In this case: \( i = -p; \quad f = \infty \)

Analysis of image from plane mirror

Geometrical relationships:

\[ |i| = p \quad h = h' \]

Magnification:

\[ M = \frac{\text{Image height}}{\text{Object height}} = \frac{h'}{h} \]

Some details:

By convention,

\( i < 0 \) for virtual image

\[ \frac{1}{i} + \frac{1}{p} = 0 = \frac{1}{\infty} \]
Spherical mirrors — concave

Reflection of parallel light rays:

Why does this satellite-dish look like a concave mirror?
A. Because it is.
B. It doesn’t — not shiny enough.

Where is the receive placed relative to the radius of curvature R?
A. Placed at R.
B. Placed at R/2.
Image formed in concave mirror:

Plane mirror:

\[
\frac{1}{i} + \frac{1}{p} = \frac{1}{f}
\]

Example: \( f = 4 \text{ cm} \)

\[
p = 1 \text{ cm}
\]

\[
i = -1.33 \text{ cm}
\]

\[
M = \frac{i}{p} = \frac{1.33}{1} = 1.33
\]

"Proof" of mirror equation:

Similar triangles:

\[
\frac{h'}{h} = \frac{-i}{p}
\]

\[
\Rightarrow \frac{h'}{h} = \frac{-i}{p}
\]

Image formed by concave mirror:

General result for virtual image formed by concave mirror

\( p < f \)

Image is upright and increased in size
Where the object is located between the focal point and a concave mirror surface, the image is virtual, upright, and enlarged.

Image formed by concave mirror:

Example: \( f = 4 \text{ cm} \)
\( p = 10 \text{ cm} \)
\( i = 6.67 \text{ cm} \)

\[
\frac{1}{p} + \frac{1}{i} = \frac{1}{f}
\]

\[
M = \frac{-i}{p} = \frac{-6.67}{10} = -0.667
\]

When the object is located so that the center of curvature lies between the object and a concave mirror surface, the image is real, inverted, and reduced in size.
Image formed by concave mirror:

General result for real image formed by concave mirror:

\[ p > f \]

image is upside down

Is image always reduced in size?

(A) yes  (B) no

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Convex mirror

Example:

\[ f = -4 \text{ cm} \]
\[ p = 16 \text{ cm} \]
\[ i = -3.2 \text{ cm} \]

\[ \frac{1}{p} + \frac{1}{i} = \frac{1}{f} \]
\[ M = -\frac{i}{p} = -\frac{3.2}{16} = 0.2 \]

Image formed by concave mirror:

General result for virtual image formed by convex mirror:

image is upright and decreased in size
Can the image formed by a convex mirror ever be increased in size \(|M| > 1|\)?

(A) yes  (B) no

Is it possible to form a real image with a convex mirror?

(A) yes  (B) no

Convex mirror used for surveillance:

http://www.physicsclassroom.com/class/refln/u13l4a.cfm
Suppose that you were behind the steering wheel and saw this image in your rear-view mirror. Which of these is likely to be true?

A. The truck is closer to you than it appears.
B. The truck is further from you than it appears.
C. Don’t change lanes just in case.