

PHY 114 A General Physics II

11 AM-12:15 PM TR Olin 101

Plan for Lecture 2:

- 1. Review of Coulomb's law**
- 2. Introduction to the notion of Electric Fields**
- 3. Electric Fields for various charge configurations**

➔ PHY 114 Labs started Monday Jan. 23rd and run through the week of April 16th.

➔ PHY 114 Tutorial sessions in Olin 103

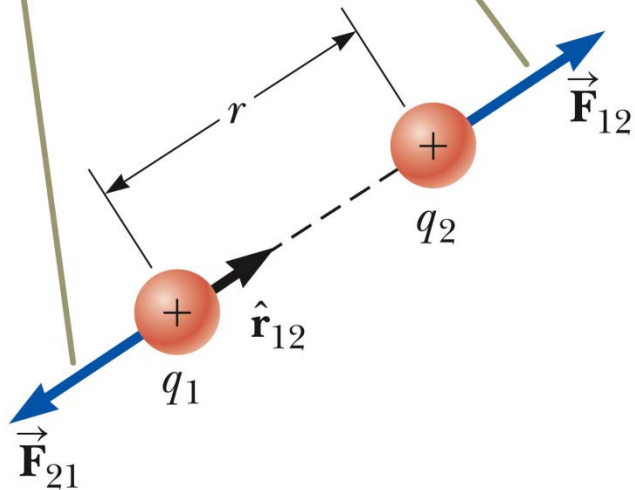
Sunday	5:30-7:30 PM	Jie Liu
Monday	5:30-7:30 PM	Jie Liu
Tuesday	6:00-8:00 PM	Loah Stevens
Wednesday	5:30-7:30 PM	Jie Liu
Thursday	5:30-7:30 PM	Loah Stevens

Recap -- Coulomb's Law

Force on particle 2 due to particle 1:

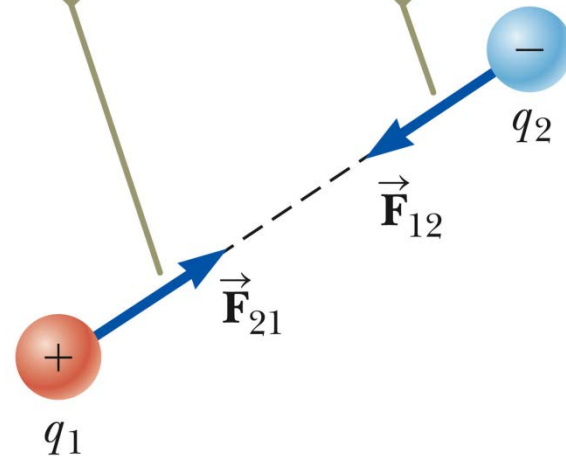
$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{|\mathbf{r}_1 - \mathbf{r}_2|^2} \hat{\mathbf{r}}_{12}$$

When the charges are of the same sign, the force is repulsive.



a

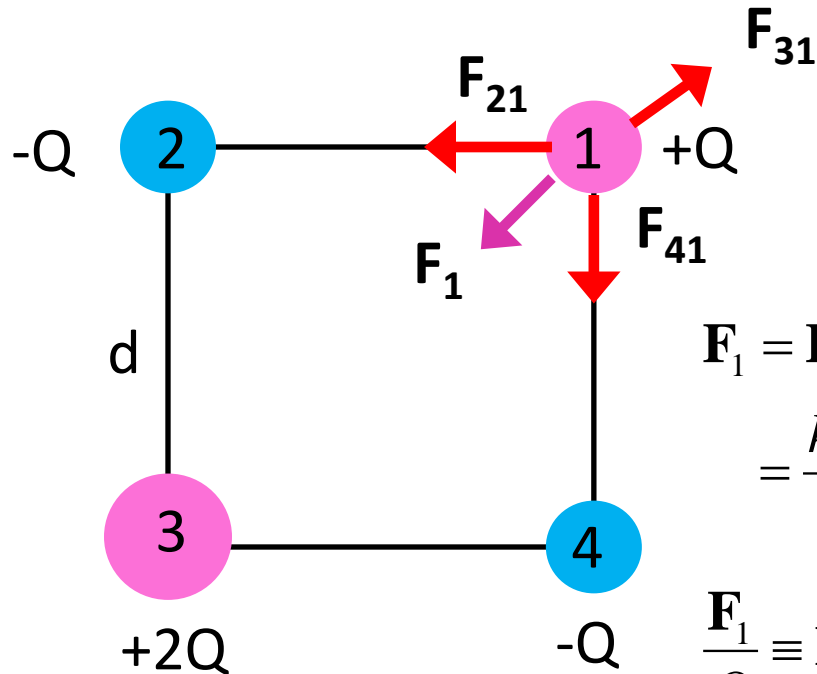
When the charges are of opposite signs, the force is attractive.



b

Example from last lecture:

Consider the configuration of 4 charges at the 4 corners of a square shown in the diagram. What is the direction of the force on the charge $+Q$ in the right corner?

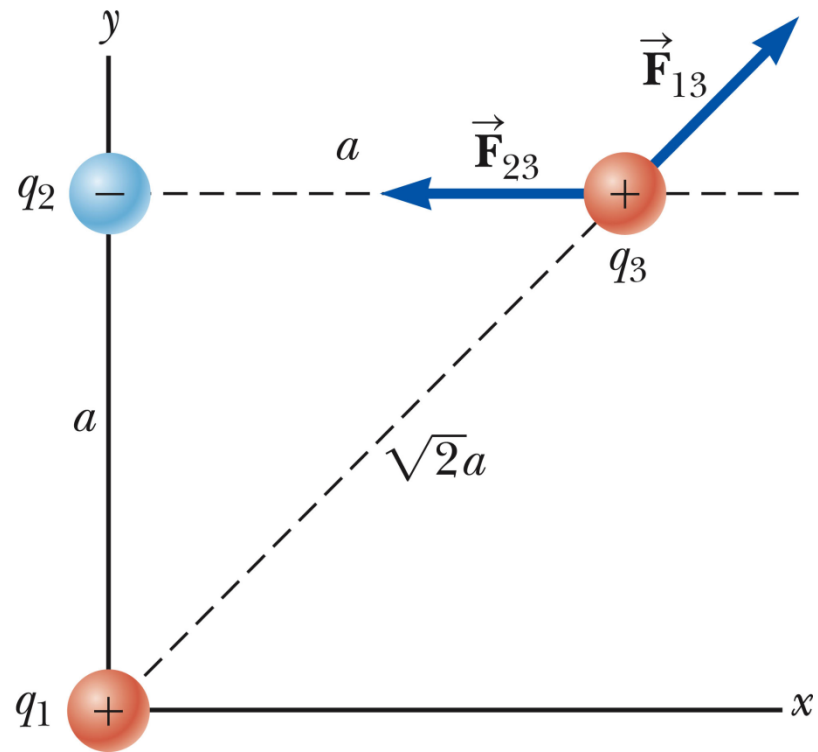


$$\mathbf{F}_1 = \mathbf{F}_{21} + \mathbf{F}_{41} + \mathbf{F}_{31}$$

$$= \frac{k_e Q}{d^2} \left(-Q\hat{\mathbf{x}} - Q\hat{\mathbf{y}} + \frac{2Q}{2} (\cos 45^\circ \hat{\mathbf{x}} + \sin 45^\circ \hat{\mathbf{y}}) \right)$$

$$\frac{\mathbf{F}_1}{Q} \equiv \mathbf{E}_1 = -\frac{k_e Q}{d^2} \left(1 - \sqrt{\frac{1}{2}} \right) (\hat{\mathbf{x}} + \hat{\mathbf{y}})$$

Another example:



$$\mathbf{F}_3 = \mathbf{F}_{13} + \mathbf{F}_{23}$$

$$= \frac{k_e q_1 q_3}{(\sqrt{2}a)^2} \hat{\mathbf{r}}_{13} + \frac{k_e q_2 q_3}{a^2} \hat{\mathbf{r}}_{23}$$

$$\frac{\mathbf{F}_3}{q_3} \equiv \mathbf{E}_3 = \frac{k_e q_1}{(\sqrt{2}a)^2} \hat{\mathbf{r}}_{13} + \frac{k_e q_2}{a^2} \hat{\mathbf{r}}_{23}$$

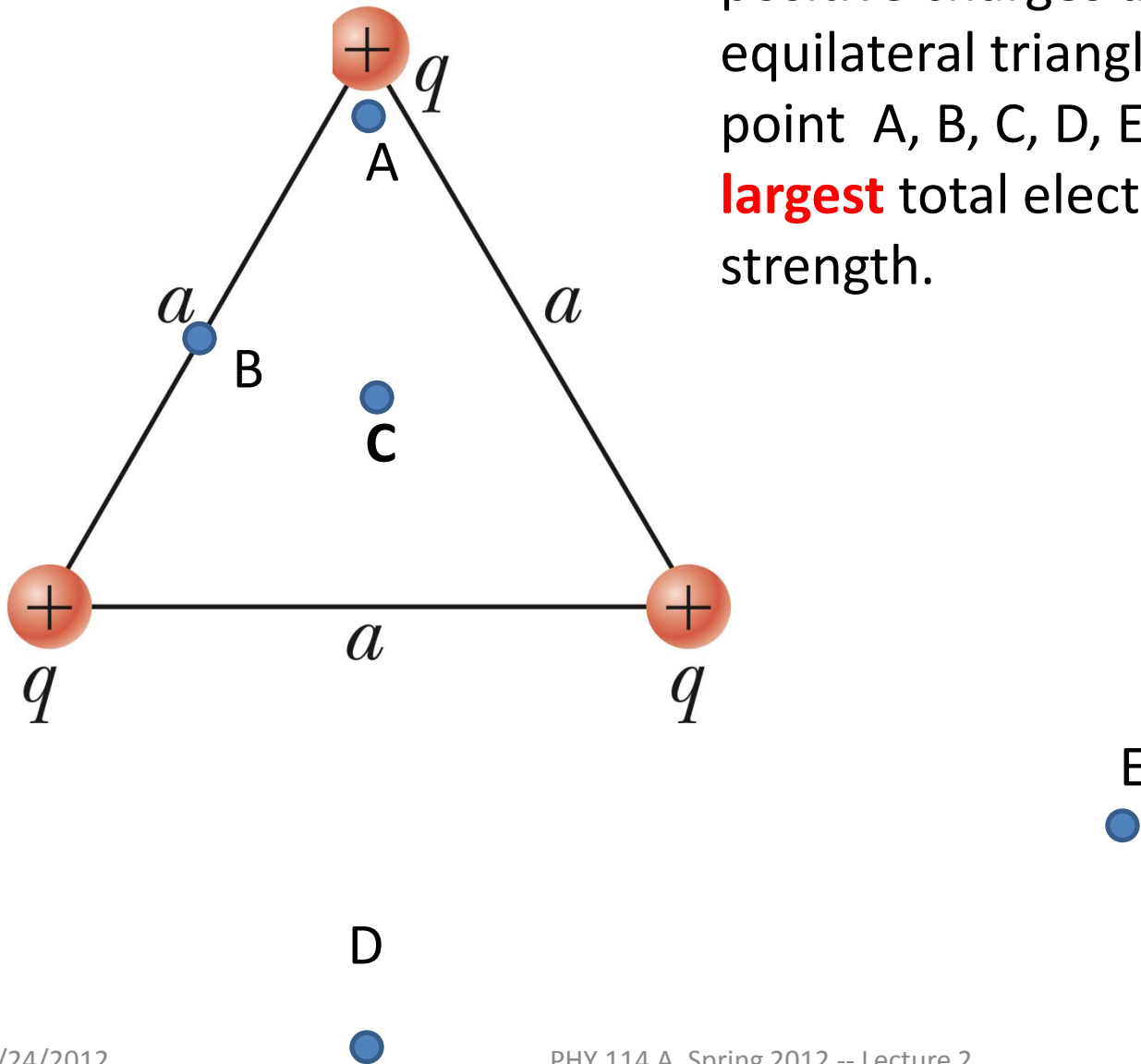
Electric field:

- An electric field $\mathbf{E}(\mathbf{r})$ is the Coulomb's law force at the position \mathbf{r} on a positive charge of $q=1$ Coulomb due to all other sources.
- The electric force on a point charge q at the position \mathbf{r} is

$$\mathbf{F}(\mathbf{r}) = q\mathbf{E}(\mathbf{r})$$

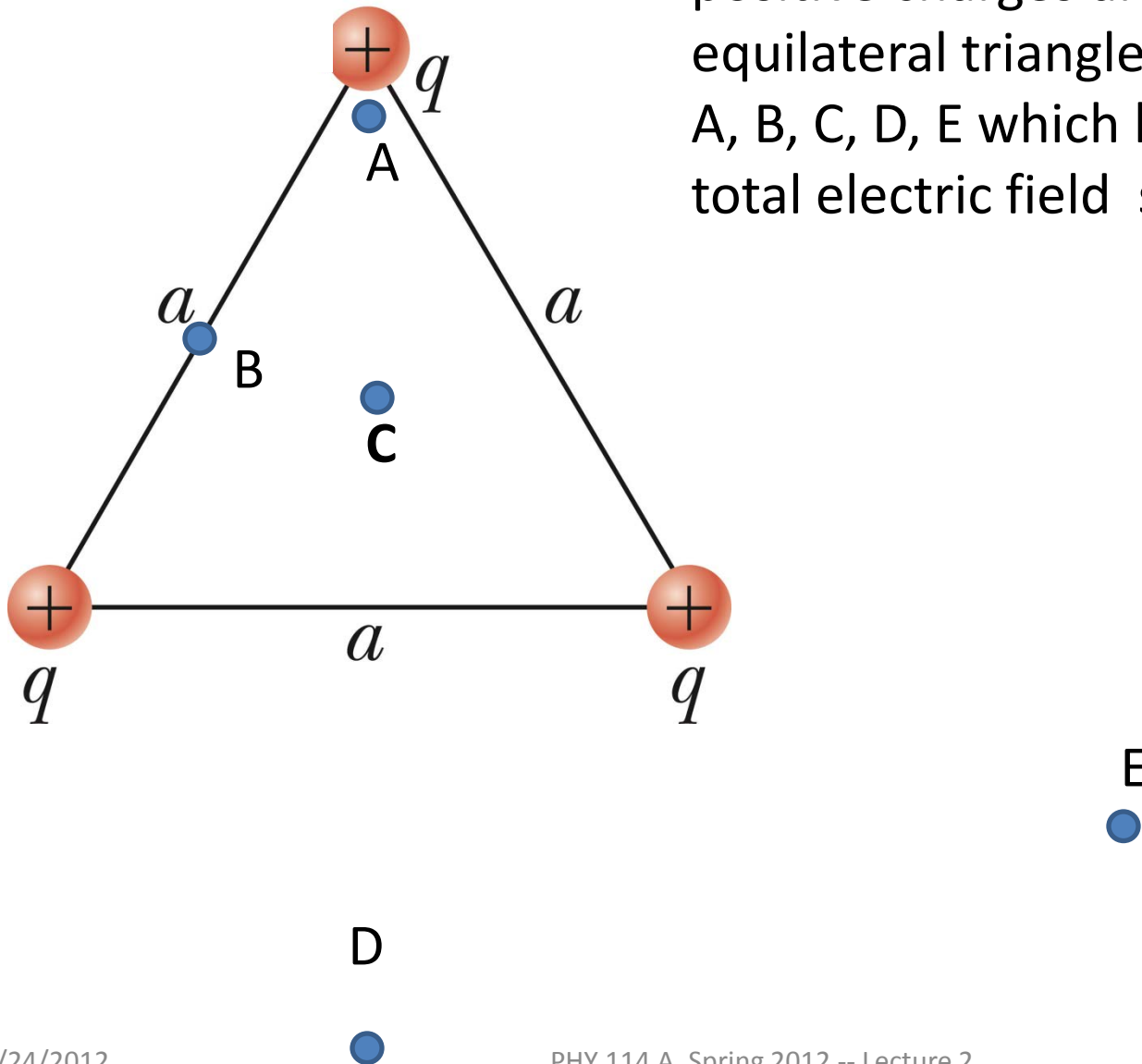
i-clicker exercise

The diagram below shows 3 equal positive charges arranged on an equilateral triangle. Choose the point A, B, C, D, E which has the **largest** total electric field strength.

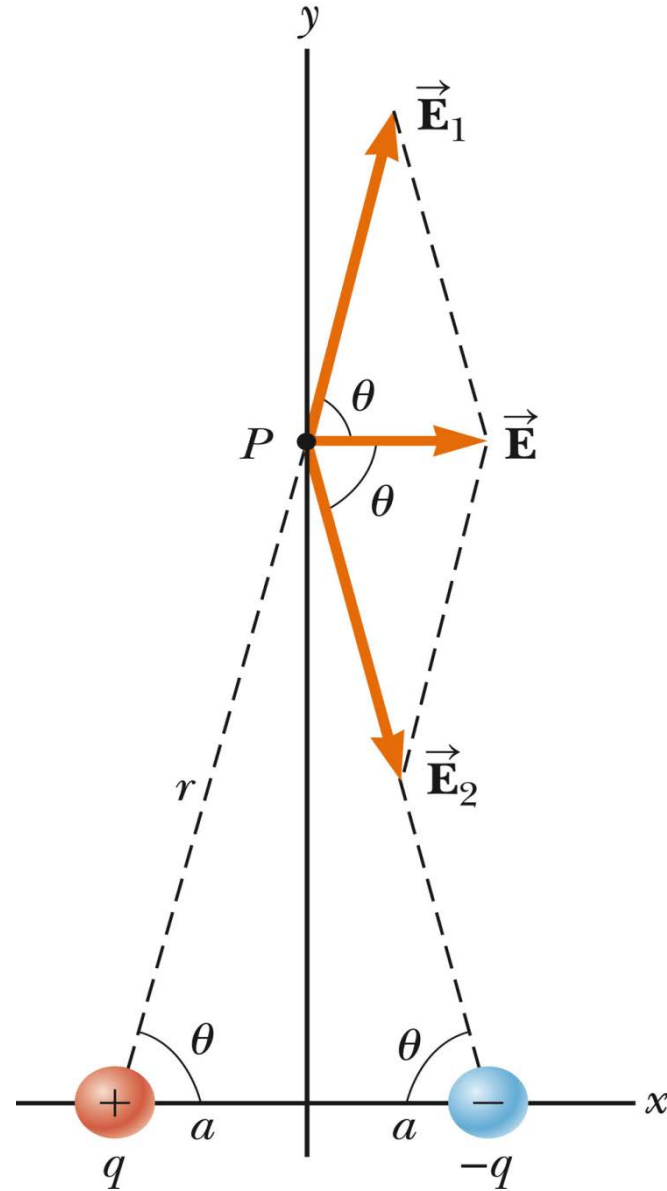
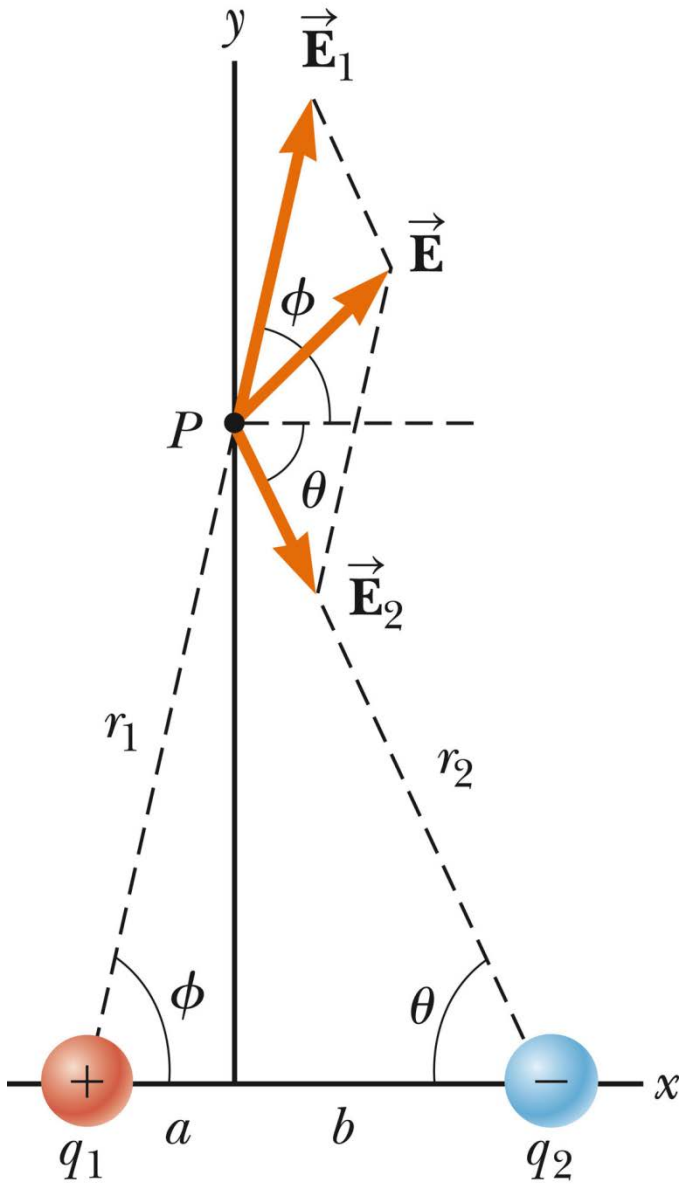


i-clicker exercise

The diagram below shows 3 equal positive charges arranged on an equilateral triangle. Choose the point A, B, C, D, E which has the **smallest** total electric field strength.



Electric field due to 2 point charges:



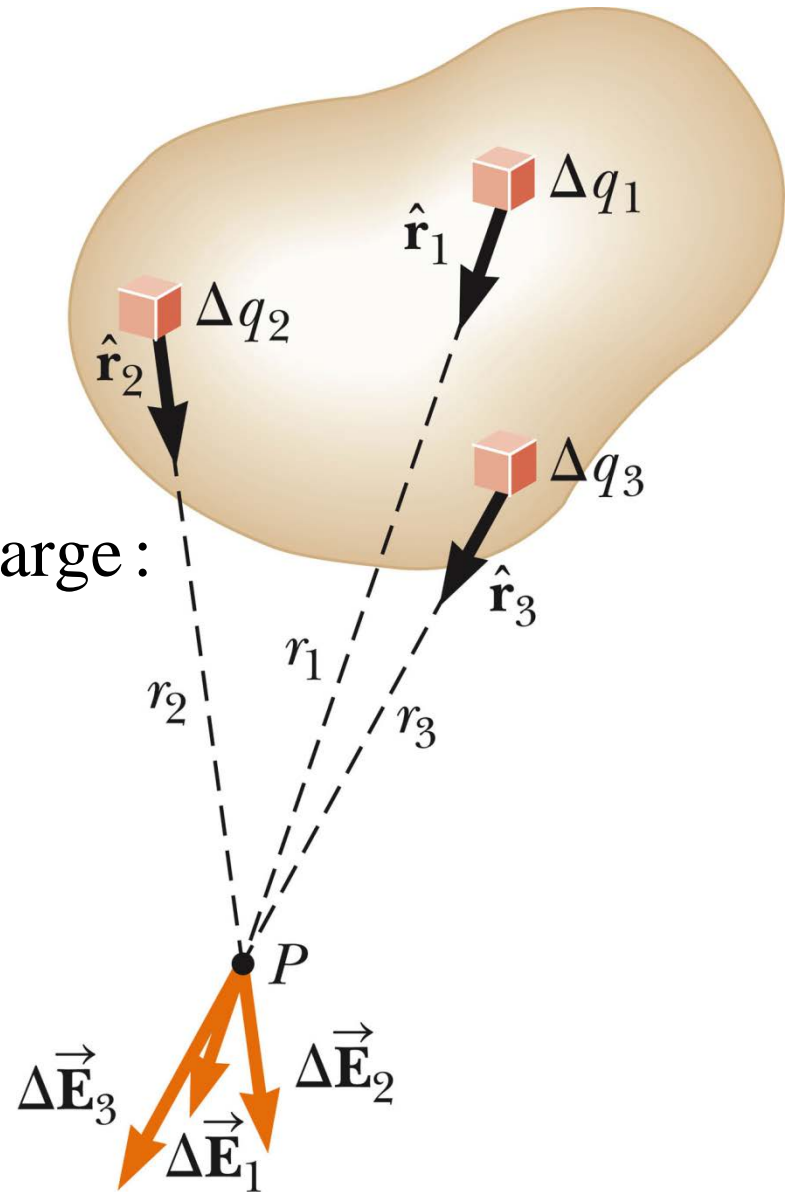
Electric field due to many charges

$$\mathbf{E}(\mathbf{r}) = k_e \sum_i \frac{q_i}{|\mathbf{r} - \mathbf{r}_i|^2} \hat{\mathbf{r}}_i$$

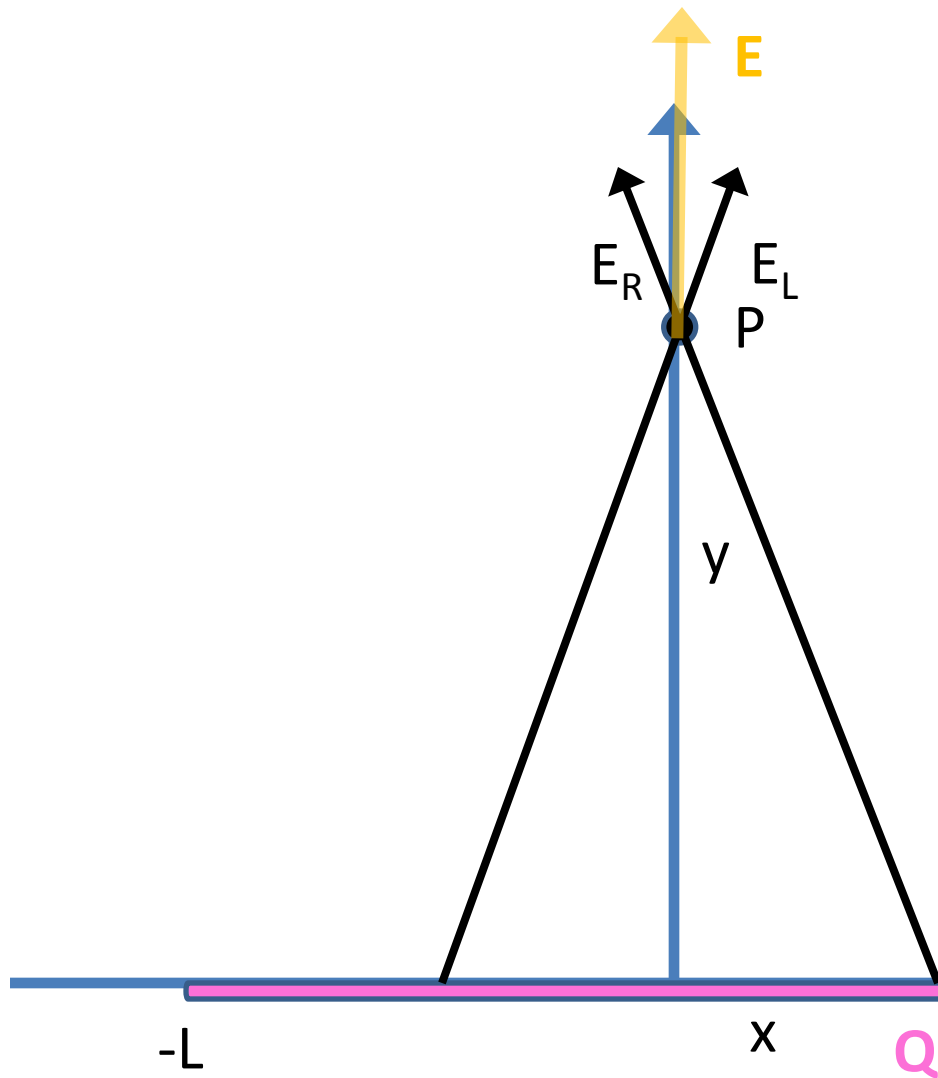
For a continuous distribution of charge:

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= k_e \lim_{\Delta q \rightarrow 0} \sum_i \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i \\ &= k_e \int \frac{dq}{r^2} \hat{\mathbf{r}} \end{aligned}$$

→ In general this integral is very difficult; Gauss's law makes it easier to evaluate.



Example – consider a long thin uniformly charged rod:



$$dq = \frac{Q}{2L} dx$$

$$E_y = k_e \int \frac{y dq}{(x^2 + y^2)^{3/2}}$$

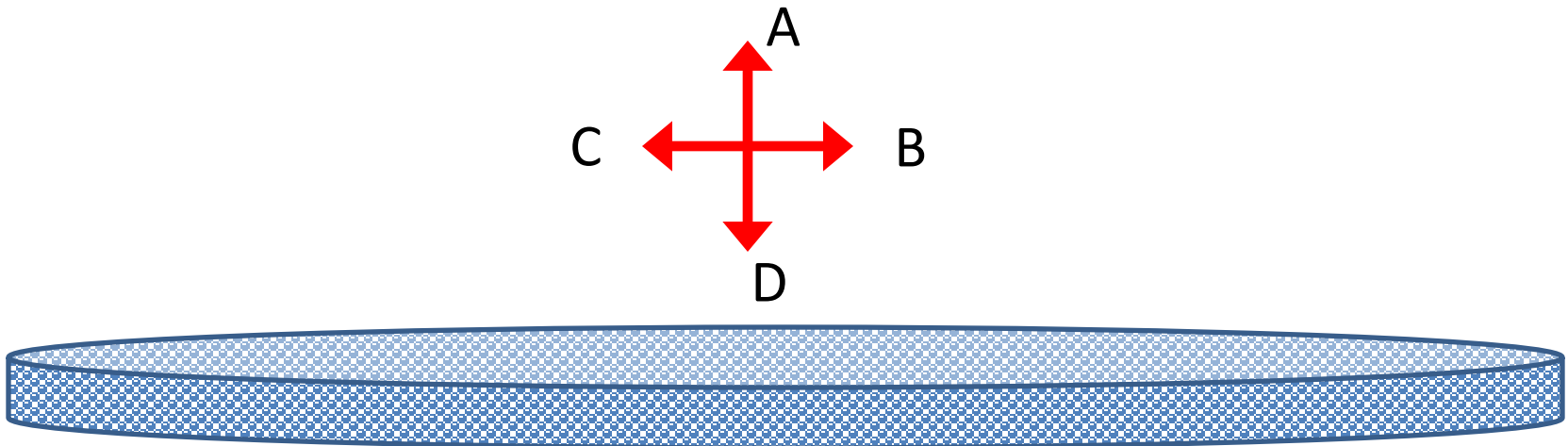
$$= \frac{k_e Q y}{2L} \int_{-L}^L \frac{dx}{(x^2 + y^2)^{3/2}}$$

$$= \frac{k_e Q y}{2L y^2} \frac{x}{(x^2 + y^2)^{1/2}} \Bigg|_{-L}^L$$

$$= \frac{k_e Q}{2L y} \frac{2L}{(L^2 + y^2)^{1/2}} \approx \frac{k_e Q}{L y}$$

i-clicker exercise

Consider a large flat plate of area A made of a material which has a uniformly distributed positive charge $q = \sigma A$. Which vector A, B, C, or D represents the direction of the electric field a small distance above the center of the plate?



i-clicker exercise

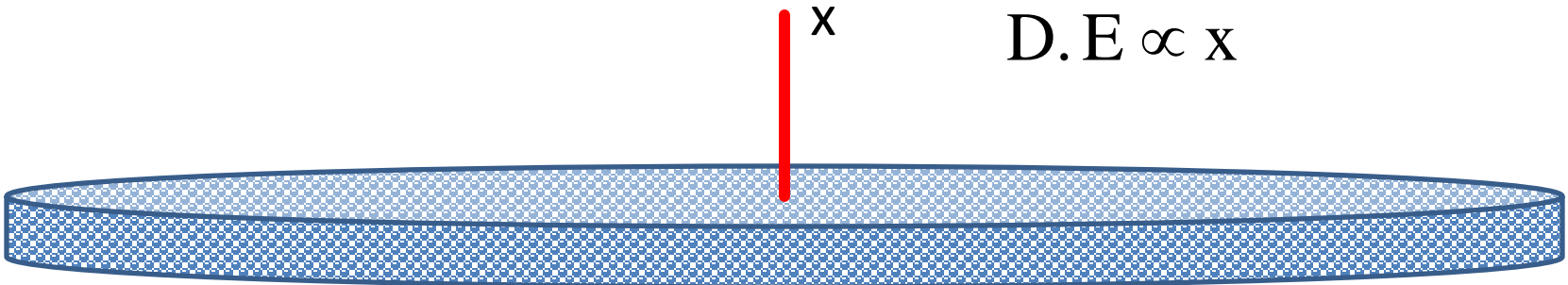
Consider a large flat plate of area A made of a material which has a uniformly distributed positive charge $q = \sigma A$. This question concerns the electric field strength E measured at a distance x from the center of the plate ($x \ll \text{radius of plate}$).

A. $E \propto 1/x^2$

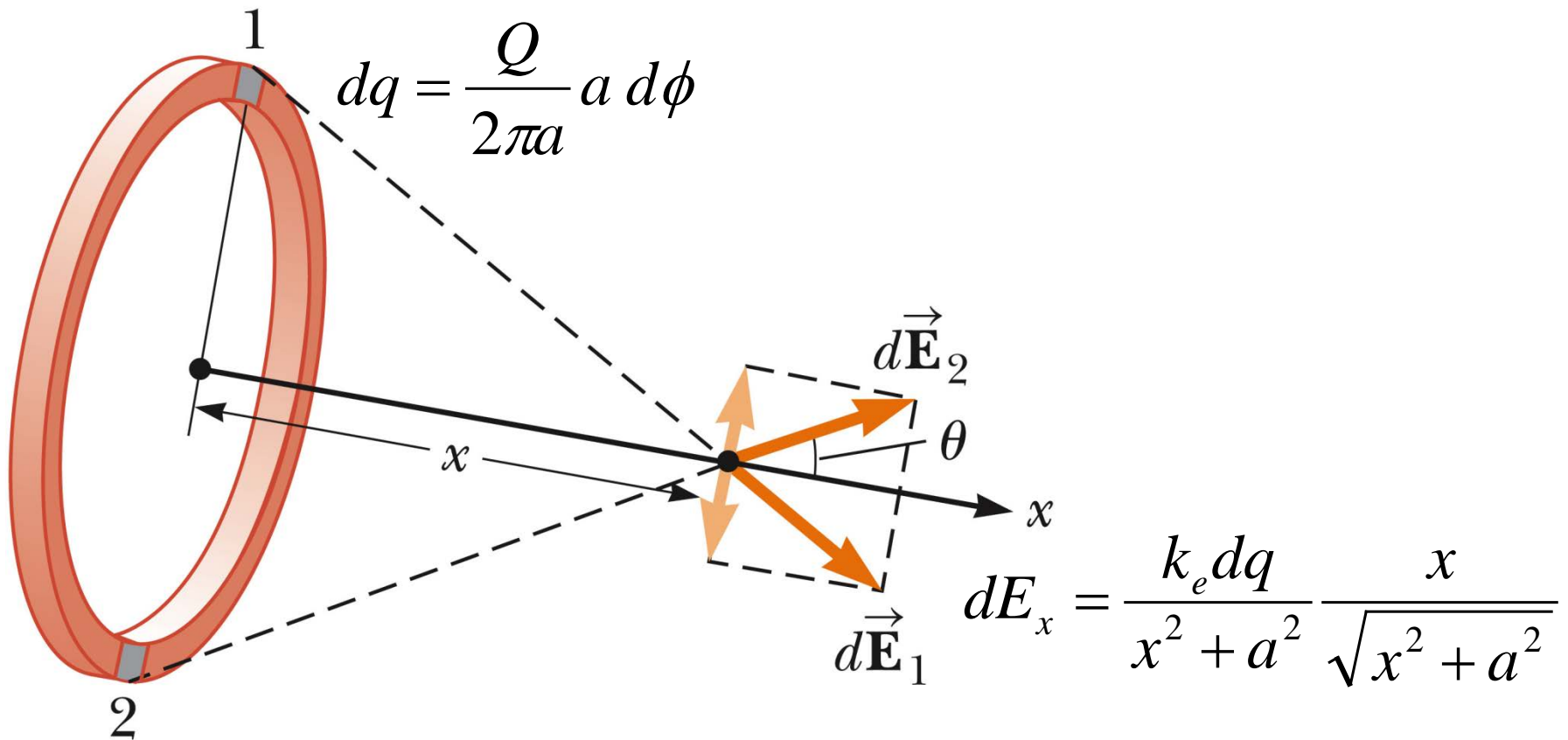
B. $E \propto 1/x$

C. $E \propto 1$

D. $E \propto x$

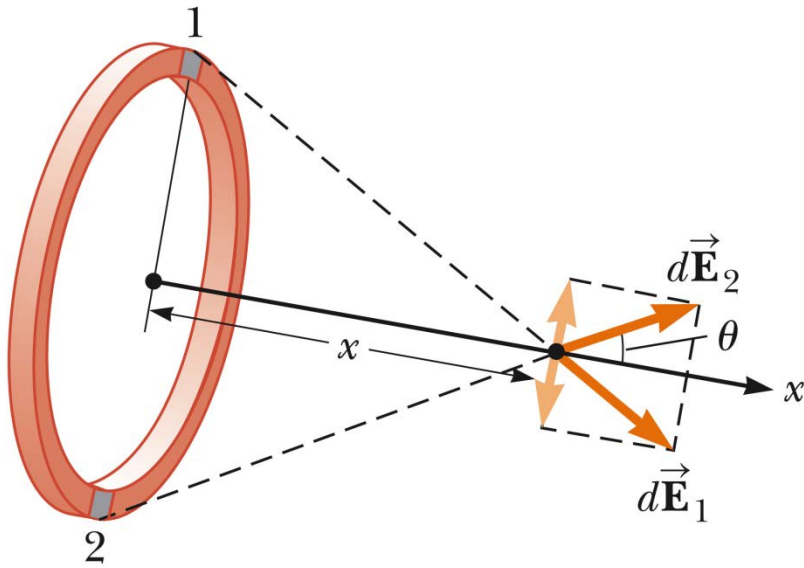


Electric field generated by a ring of charge



b

Electric field generated by a ring of charge



b

$$dq = \frac{Q}{2\pi a} a d\phi$$

$$dE_x = \frac{k_e dq}{x^2 + a^2} \frac{x}{\sqrt{x^2 + a^2}}$$

$$E_x = \int dE_x = \frac{k_e x}{(x^2 + a^2)^{3/2}} \int dq$$
$$= \frac{k_e Q x}{(x^2 + a^2)^{3/2}}$$

Electric field generated by a plate of charge

$$dq = Q^{ring}(r)dr = \frac{Q}{\pi R^2} 2\pi r dr \equiv \sigma 2\pi r dr$$

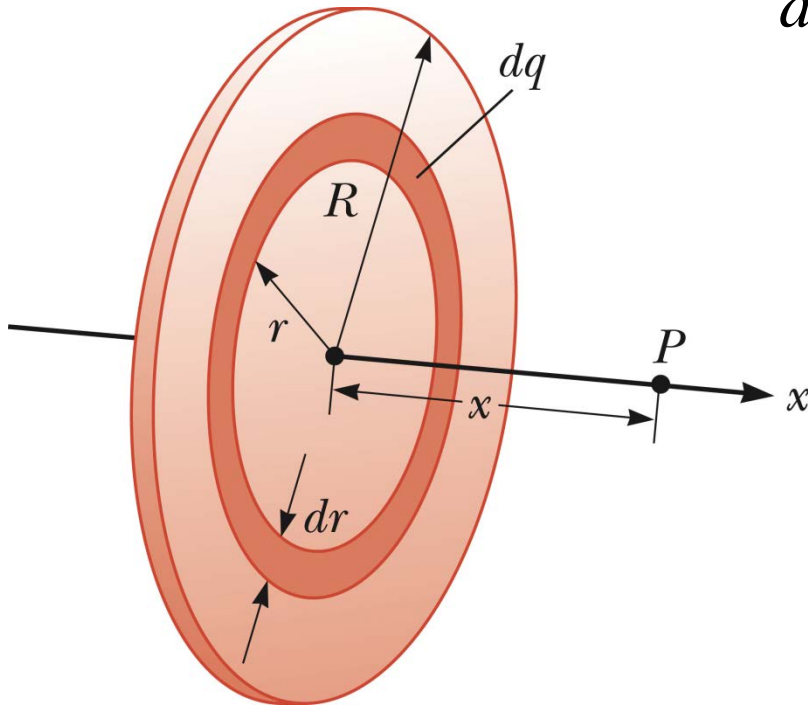
$$E_x^{ring}(r) = \frac{k_e x Q^{ring}(r)}{(x^2 + r^2)^{3/2}}$$

$$E_x^{plate} = \int_0^R \frac{k_e x \sigma 2\pi r dr}{(x^2 + r^2)^{3/2}}$$

For $R \gg x$:

$$\Rightarrow E_x^{plate} \approx 2\pi k_e \sigma \equiv \frac{\sigma}{2\epsilon_0}$$

$$= 2\pi k_e \sigma \left(1 - \frac{x}{(x^2 + R^2)^{1/2}} \right)$$



i-clicker question

What is the magnitude and direction (+ \rightarrow up, - \rightarrow down) of the electric field between oppositely charged plates?

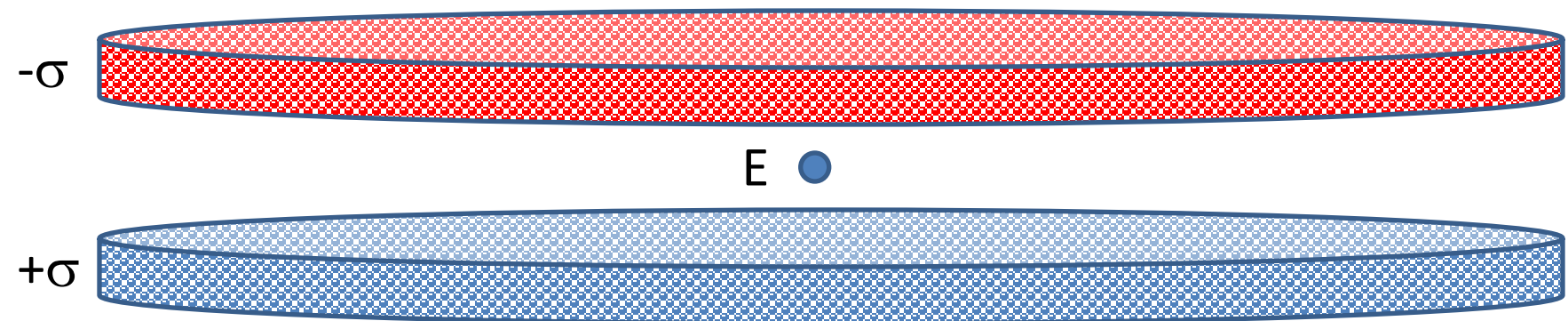
A. 0

B. $+\sigma/2\epsilon_0$

C. $-\sigma/2\epsilon_0$

D. $+\sigma/\epsilon_0$

E. $-\sigma/\epsilon_0$



Summary

$$\mathbf{F}(\mathbf{r}) = q\mathbf{E}(\mathbf{r})$$

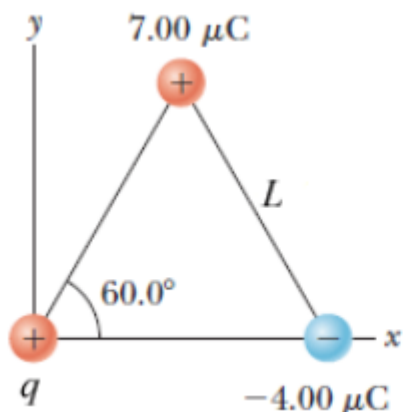
$$\mathbf{E}(\mathbf{r}) = k_e \sum_i \frac{q_i}{|\mathbf{r} - \mathbf{r}_i|^2} \hat{\mathbf{r}} - \mathbf{r}_i = k_e \int \frac{dq(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^2} \hat{\mathbf{r}} - \mathbf{r}'$$

$$E_x^{plate} \approx 2\pi k_e \sigma \equiv \frac{\sigma}{2\epsilon_0}$$

Three charged particles are located at the corners of an equilateral triangle as shown in the figure below (let $q = 2.20 \mu\text{C}$, and $L = 0.850 \text{ m}$). Calculate the total electric force on the $7.00\text{-}\mu\text{C}$ charge.

magnitude N

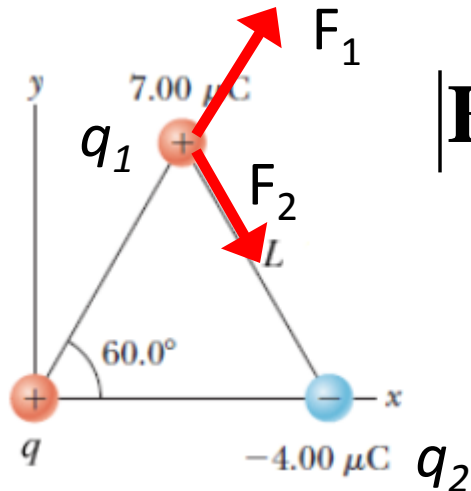
direction ° (counterclockwise from the $+x$ axis)



Three charged particles are located at the corners of an equilateral triangle as shown in the figure below (let $q = 2.20 \mu\text{C}$, and $L = 0.850 \text{ m}$). Calculate the total electric force on the $7.00\text{-}\mu\text{C}$ charge.

magnitude N

direction ° (counterclockwise from the +x axis)



$$|\mathbf{F}_1| = \frac{k_e q q_1}{L^2}$$

$$|\mathbf{F}_2| = \frac{k_e q_1 q_2}{L^2}$$

$$\mathbf{F}_1 = |\mathbf{F}_1| (\cos 60^\circ \hat{\mathbf{x}} + \sin 60^\circ \hat{\mathbf{y}})$$

$$\mathbf{F}_2 = |\mathbf{F}_2| (\cos 60^\circ \hat{\mathbf{x}} - \sin 60^\circ \hat{\mathbf{y}})$$

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$$

Some of your questions:

- How to understand and handle continuous charge distributions
 - At the a microscopic (classical) viewpoint elementary charges are discrete; at a macroscopic viewpoint it is often convenient to consider a distribution of charges.

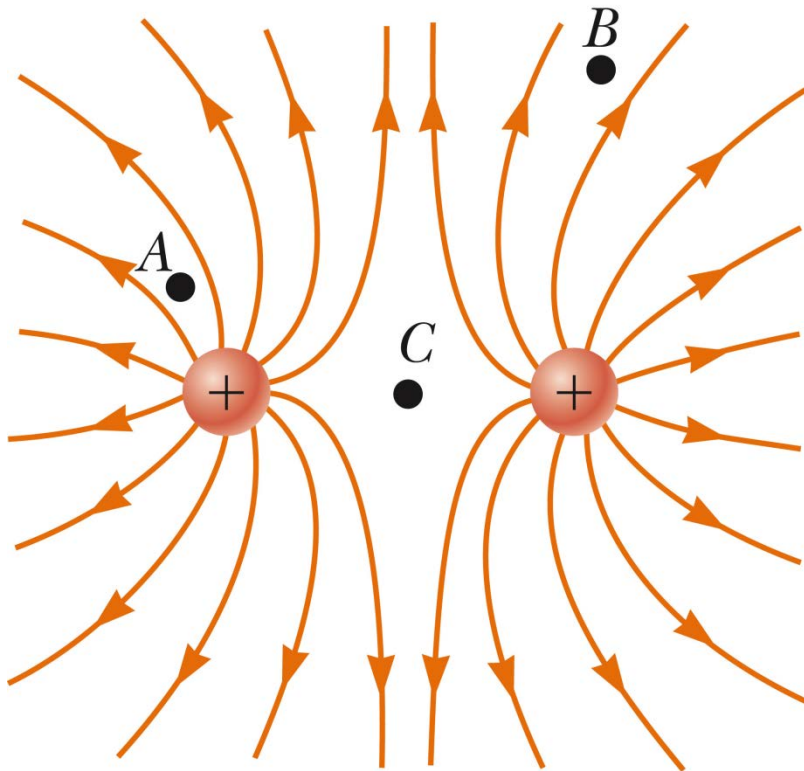
$$\text{Volume charges : } dq = \frac{Q}{V} d^3 r$$

$$\text{Surface charges : } dq = \frac{Q}{A} d^2 r$$

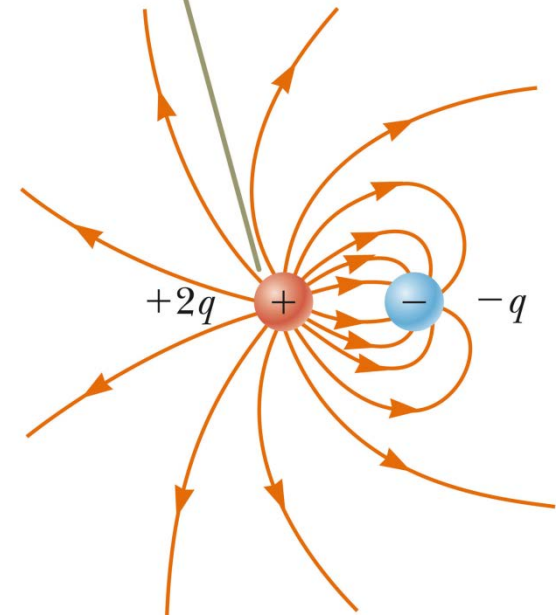
$$\text{Line charges : } dq = \frac{Q}{L} dr$$

Some of your questions:

➤ Some questions about field lines



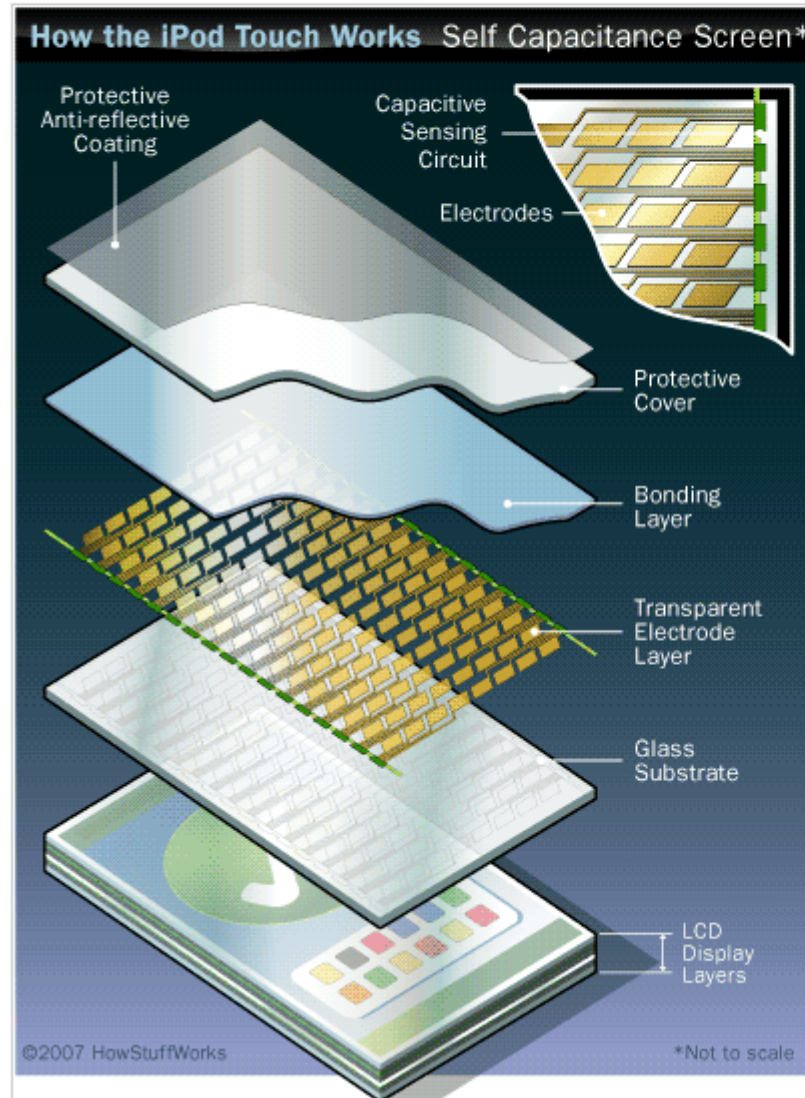
Two field lines leave $+2q$ for every one that terminates on $-q$.



Questions about usefulness of electric fields outside of physics class.

Example: ipad touch screen (from HowThingsWork web page)

- **Self capacitance:** Circuitry monitors changes in an array of electrodes.
- **Mutual capacitance:** A layer of **driving lines** carries current. A separate layer of **sensing lines** detects changes in the electrical charge when you place your finger on the screen.



Regardless of which method the screen uses, you change the electrical properties of the screen every time you touch it. The iPod records this change as data, and it uses mathematical algorithms to translate the data into an understanding of where your fingers are. In the next section, we'll explore what the iPod touch does with this data and how to navigate through its features.