Plan for Lecture 20 (Chapter 36):

Optical properties of light

1. Images formed by thin lenses
2. Optical devices
   a. Eyeglasses
   b. Cameras, microscopes, telescopes
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Note: HW 17 slightly altered
Virtual image \( \Rightarrow \textbf{not really there} \)

When the object is located between the focal point and a concave mirror surface, the image is virtual, upright, and enlarged.

Mirror equation:

\[
\frac{1}{p} + \frac{1}{q} = \frac{1}{f}
\]
Real image $\Rightarrow$ really there

Mirror equation:
\[
\frac{1}{p} + \frac{1}{q} = \frac{1}{f}
\]

When the object is located so that the center of curvature lies between the object and a concave mirror surface, the image is real, inverted, and reduced in size.
Summary of results for mirrors:

The mirror equation:

\[ \frac{1}{p} + \frac{1}{i} = \frac{1}{f} \]

Magnification:

\[ m = \frac{(\text{sign})h'}{h} = -\frac{i}{p} \]

+ signs for: \( p, f \) (concave mirror), \( i \) (real image), \( m \) (upright image)

– signs for: \( f \) (convex mirror), \( i \) (virtual image), \( m \) (inverted image)
A laser beam is incident on a 45°–45°–90° prism perpendicular to one of its faces as shown in the figure below. The transmitted beam that exits the hypotenuse of the prism makes an angle of $\theta = 28.8^\circ$ with the direction of the incident beam. Find the index of refraction of the prism.

**Basic principle:** Snell's law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
Homework hint: (HW 17; note this problem is now optional)

Basic principle: Snell's law

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

For small angles:

\[ \frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \]

For HW problem \( R \to \infty \):

\[ \frac{n_1}{p} + \frac{n_2}{q} = 0 \]

\[ \frac{n_1}{p} + \frac{n_2}{q} = 0 \]

\[ n_0 = 1 \]

\[ n_1 = 1.33 \]

\[ n_2 = 1.66 \]
Thin lens equation:

\[ \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \]

**Image formation by refraction -- lenses**

- **Front, or virtual, side**
  - \( p \) positive
  - \( q \) negative

- **Back, or real, side**
  - \( p \) negative
  - \( q \) positive

Incident light → Refracted light

Converging or diverging lens
Summary of thin lens configurations

When the object is in front of and outside the focal point of a converging lens, the image is real, inverted, and on the back side of the lens.

When the object is between the focal point and a converging lens, the image is virtual, upright, larger than the object, and on the front side of the lens.

When an object is anywhere in front of a diverging lens, the image is virtual, upright, smaller than the object, and on the front side of the lens.

Lens makers’ equation

\[ \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]
Basic physics of lenses:

Snell’s law: $n_1 \sin \theta_1 = n_2 \sin \theta_2$

Refraction at a spherical surface:

Notation note: $q \leftrightarrow i$
Refraction at a spherical surface

\[ \theta_1 = \alpha + \beta \]
\[ \theta_2 = \beta - \gamma \]
\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \Rightarrow \quad n_1 \theta_1 \approx n_2 \theta_2 \]
\[ \tan \alpha = \frac{d}{a} \approx \alpha \quad \tan \beta = \frac{d}{R} \approx \beta \quad \tan \gamma = \frac{d}{b} \approx \gamma \]
\[ \frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_2}{R} \]
Refractive at a spherical surface – continued

\[ \frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_2}{R} \]

In the small angle approximation, result is *independent* of angle.

Application to 2 surfaces in thin lens geometry

“lens makers’ equation”

\[ \frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

Sign convention:

- \( R_i \) is positive if it is convex relative to object and negative if it is concave relative to object.
Lens makers' equation – continued:

\[ \frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

Sign convention:

- \( R_i \) is positive if it is convex relative to the object and negative if it is concave relative to the object.

Object side

- \( R_1 > 0, R_2 < 0 \) \( f > 0 \) \( \Rightarrow \) converging lens

- \( R_1 < 0, R_2 > 0 \) \( f < 0 \) \( \Rightarrow \) diverging lens
Lens makers’ equation can be proven using

- Snell’s law: \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \)
- small angle approximation: \( \sin \theta_1 \approx \tan \theta_1 \approx \theta_1 \)
- thin lens approximation: thickness \( \ll f, p, i \)

\[ \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

Sign convention:

- \( R_i \) is positive if it is convex relative to object and negative if it is concave relative to object.

Lens equation:

\[ \frac{1}{p} + \frac{1}{i} = \frac{1}{f} \]
Example of thin lenses:

Converging lens: $f > 0$

Diverging lens: $f > 0$
Example:

Forming a real image using a converging lens

This could represent, for example the lens system of a camera.

Example:

\[ \frac{1}{f} + \frac{1}{p} = \frac{1}{i} \]

\[ f = 2 \text{ cm}, \ p = 5 \text{ cm} \]

\[ \Rightarrow i = 3.33 \text{ cm} \]

(\text{real image})

\[ M = -\frac{i}{p} \]

\[ = \frac{-3.33}{5} = -0.67 \]
Thin lens equation: \[ \frac{1}{p} + \frac{1}{i} = \frac{1}{f} \]
assuming sign convention:

\[ f = 2 \text{ cm}, \quad p = 5 \text{ cm} \]
\[ \Rightarrow i = 3.33 \text{ cm} \]
(Real image)
Thin lens refraction -- continued

Example:

\[ f = 2 \text{ cm, } p = 1.2 \text{ cm} \]

\[ \Rightarrow i = -3 \text{ cm} \]

(virtual image)

\[ M = \frac{-i}{p} = 2.5 \]
Thin lens refraction -- continued

Example:

\[ f = -2 \text{ cm}, \ p = 4 \text{ cm} \]
\[ \Rightarrow i = -1.333 \text{ cm} \]

(virtual image)
Sherlock Holmes is apparently examining some evidence.

Which ray diagram most closely describes this situation:
Which picture shows diverging corrective lenses? (A or B)
Which corrective lens is appropriate for nearsighted person? (A or B)
View of the eye from

http://science.howstuffworks.com/eye1.htm
Vision problems and corrective lenses

Ideal vision:

Near sighted vision – problem with “Far point”

Far sighted vision – problem with “Near point”
More details about eye:

$$i \approx 2.5\text{cm}$$

\[ \frac{1}{p} + \frac{1}{i} = \frac{1}{f} \]

\[ \frac{1}{25} + \frac{1}{2.5} = \frac{1}{f} \]

$$f = \frac{1}{\frac{1}{25} + \frac{1}{2.5}} = 2.27\text{cm}$$

“near point” ≡ closest point that the eye can focus

25 cm standard value

7-200 cm depending on person
Refraction from two or more lenses

→ Successive use of lens equation

\[
\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1}
\]

\[
p_2 = -i_1 + d
\]

\[
\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2}
\]

Example:

\[
p_1 = 50 \text{ cm}; \quad f_1 = -25 \text{ cm}; \quad f_2 = 1.4 \text{ cm}; \quad d = 1.0 \text{ cm}
\]

We find:

\[
i_1 = -16.667 \text{ cm}; \quad i_2 = 1.52 \text{ cm}
\]

Without the diverging lens:

\[
i_2 = 1.44 \text{ cm} \text{ (short of retina)}
\]
Thin lens equation:

\[ \frac{1}{p} + \frac{1}{i} = \frac{1}{f} \]

\( f > 0; i > 0 \)  

\( f > 0; i < 0 \)  

\( f < 0; i < 0 \)
Physics of the camera

Example: $f=50\text{mm}$ $\implies i=50\text{mm}$ for $p=\infty$

$\implies i=51.3\text{mm}$ for $p=2\text{m}$
Compound microscope

The objective lens forms an image here.

The eyepiece lens forms an image here.

The three-objective turret allows the user to choose from several powers of magnification.
Suppose you want to record an microscope image. Where would you place the camera?

A. Where eye is
B. Not at eye position
Optics in the Hubble Space Telescope – two convex mirrors


Mirror surfaces are coated with $5 \times 10^{-8}$ m Al and $3 \times 10^{-8}$ m MgF$_2$ (early version had $10^{-6}$ curvature error which was subsequently corrected)
When Hubble took this photograph of Mars in 1999, Mars was 87 million km (54 million miles) away, or more than 200 times the distance from Earth to the Moon.

The photo was made at the height of Martian summer in the northern hemisphere. The carbon dioxide (dry ice) portion of the north polar ice cap