PHY 114 A General Physics II
11 AM-12:15 PM TR Olin 101

Plan for Lecture 21 (Chapter 37):

Wave properties of light

1. **Interference of two electromagnetic waves**
2. **Interference of electromagnetic waves in thin films**

14	03/20/2012	Induction and AC circuits	32.1-32.6	32.4,32.20,32.43	03/22/2012
15	03/22/2012	AC circuits	33.1-33.9	33.8,33.24,33.71	03/27/2012
16	03/27/2012	Electromagnetic waves	34.1-34.3	34.3,34.10,34.13	03/29/2012
17	03/29/2012	Electromagnetic waves	34.4-34.7	34.22,34.46,34.57	04/03/2012
18	04/03/2012	Ray optics Evening exam	35.1-35.8	35.20,35.27,35.35	04/10/2012
19	04/05/2012	Image formation Evening exam	36.1-36.4	36.8,36.31,36.42	04/10/2012
20	04/10/2012	Image formation	36.5-36.10	36.52,36.54,36.64	04/12/2012
21	04/12/2012	Wave interference	37.1-37.6	37.2,37.19,37.29	04/17/2012
22	04/17/2012	Diffraction	38.1-38.6		
23	04/19/2012	Quantum Physics	40.1-42.10		
24	04/24/2012	Molecules and solids	43.1-43.8		
25	04/26/2012	Nuclear reactions	45.1-45.4		
26	05/01/2012	Nuclear radiation	45.5-45.7		
	05/08/2012	Final exam 9 AM			

4th exam will be offered during the week of April 23rd.

Upcoming event:
Society of Physics Students Keynote address
Friday April 20, 2012 8 PM in Brendle recital Hall

Professor William Phillips from NIST and UMD

"Time, Einstein, and the Coolest Stuff in the Universe"

Dr. Phillips was awarded the 1997 Nobel Prize in Physics : "for development of methods to cool and trap atoms with laser light" The 1997 prize was shared with Steven Chu of Stanford University and Claude Cohen-Tannoudji of the Ecole Normale Superieure, Paris.

3rd exam solutions
- Solutions posted on web
- Exam review session??
  Would you like to attend an exam review session?
  (A) yes (B) no

  If you would like a review session, can you meet
  (A) Today at 2 PM (here)
  (B) Tomorrow at 1 PM Olin 107
  (C) Tomorrow at 2 PM Olin 107
  (D) Other?
- Similar problems may appear on final exam

Comment about functions and derivatives:

Current as a function of time (in units of Amps):

$$I(t) = 0.2(1 - 0.01t)$$

$$\frac{dI}{dt} = -0.002$$

Charge as a function of time:

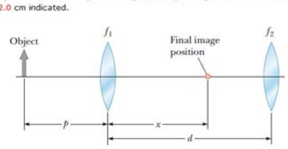
$$q(t) = q_0 e^{-t/\tau}$$

$$\frac{dq}{dt} = -\frac{q_0 e^{-t/\tau}}{\tau}$$

Webassign hint:

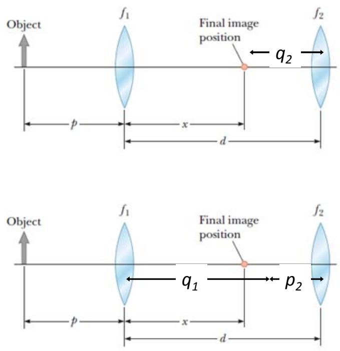
3. 0/0.333 points SerPSE8 36.P.064. [1453034]

Two converging lenses having focal lengths of f_1 = 12.5 cm and f_2 = 18.0 cm are placed a distance d = 49.0 cm apart as shown in the figure below. The image due to light passing through both lenses is to be located between the lenses at the position x = 32.0 cm indicated.



(a) At what value of p should the object be positioned to the left of the first lens?
18.1 cm

(b) What is the magnification of the final image?
-4.32



Working backwards:

$$\frac{1}{p_2}+\frac{1}{q_2}=\frac{1}{f_2}$$

$\Rightarrow$ Solve for p_2

Note: $q_2 < 0$

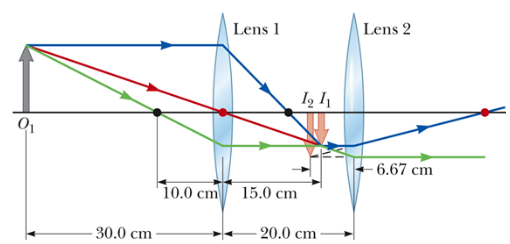
Continuing analysis:

$q_1 = d - p_2$

$$\frac{1}{p_1}+\frac{1}{q_1}=\frac{1}{f_1}$$

$\Rightarrow$ Solve for $p_1 \equiv p$

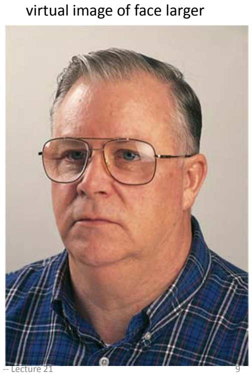
Another example (#36.10 in your text):



From previous lecture:

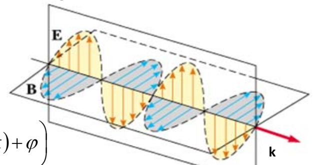
Diverging lens:
virtual image of face smaller

Converging lens:
virtual image of face larger



Wave phenomena associated with light

Plane polarized electromagnetic wave at an instant of time:



$$E_y(x,t) = E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi\right)$$

Superposition of two electromagnetic waves (electric field portion)

$$E_y^{tot}(x,t) = E_y^1(x,t) + E_y^2(x,t)$$

$$E_y^{tot}(x,t) = E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt)\right) + E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi\right)$$

$$= 2E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt) + \frac{1}{2}\varphi\right)\cos\left(\frac{\varphi}{2}\right)$$

Superposition (continued)

$$E_y^{tot}(x,t) = E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt)\right) + E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi\right)$$

$$= 2E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt) + \frac{1}{2}\varphi\right)\cos\left(\frac{\varphi}{2}\right)$$

Note that this result follows from the trigonometric identity:

$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

Squared magnitude: $|\sin(A) + \sin(B)|^2 = 4\sin^2\left(\frac{A+B}{2}\right)\cos^2\left(\frac{A-B}{2}\right)$

Constant in time

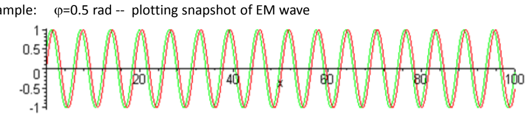
Time average = 1/2

Intensity of the EM waves:

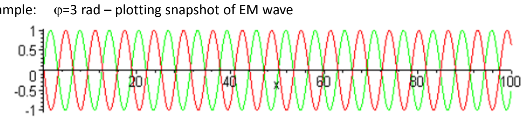
$$I^1 \equiv S_{avg} = \frac{1}{2c\mu_0}|E_{\max}|^2 = I^2$$

$$I^{tot} = \frac{4}{2c\mu_0}\left|E_{\max}\cos\left(\frac{\varphi}{2}\right)\right|^2 = 4I^1\cos^2\left(\frac{\varphi}{2}\right)$$

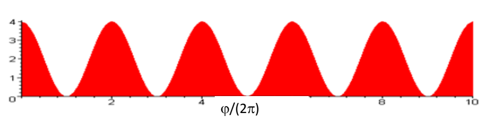
Example: φ=0.5 rad -- plotting snapshot of EM wave



Example: φ=3 rad – plotting snapshot of EM wave



Intensity as a function of φ:

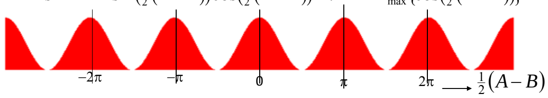


Summary of interference phenomena due to two or more electromagnetic waves which combine at a give point P with path lengths r_1 and r_2 and fixed frequency f.

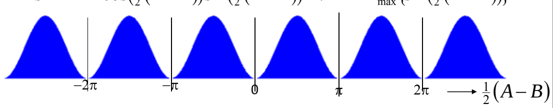
$$E(P,t) = E_{\max}\sin\left(\frac{2\pi r_1}{\lambda} - 2\pi f t\right) \pm E_{\max}\sin\left(\frac{2\pi r_2}{\lambda} - 2\pi f t\right)$$

Trig identity :

$$\sin A + \sin B = 2\sin\left(\tfrac{1}{2}(A+B)\right)\cos\left(\tfrac{1}{2}(A-B)\right) \Rightarrow I = 4I_{\max}\left\{\cos\left(\tfrac{1}{2}(A-B)\right)\right\}^2$$

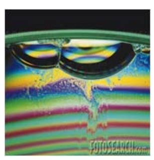


$$\sin A - \sin B = 2\cos\left(\tfrac{1}{2}(A+B)\right)\sin\left(\tfrac{1}{2}(A-B)\right) \Rightarrow I = 4I_{\max}\left\{\sin\left(\tfrac{1}{2}(A-B)\right)\right\}^2$$



Two examples of superposed radiation:

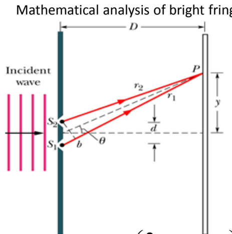
Interference from refraction and reflection of thin films

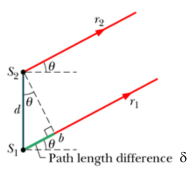


Young's double slit

Young's double slit geometry:

Mathematical analysis of bright fringes:





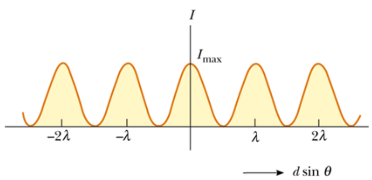
$$r_1 - r_2 = \delta = d\sin\theta$$

$$E(P,t) = E_{\max}\sin\left(\frac{2\pi r_1}{\lambda} - 2\pi f t\right) + E_{\max}\sin\left(\frac{2\pi r_2}{\lambda} - 2\pi f t\right)$$

$$= 2E_{\max}\sin\left(\frac{\pi(r_1+r_2)}{\lambda} - 2\pi f t\right)\cos\left(\frac{\pi(r_1-r_2)}{\lambda}\right)$$

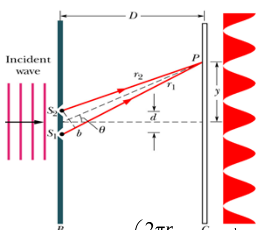
→ intensity maxima occur for $\frac{\pi(r_1-r_2)}{\lambda} = m\pi \quad \Rightarrow d\sin\theta = m\lambda$

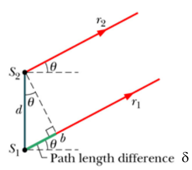
Intensity pattern at screen for double slit:



$$I = |\mathbf{S}|_{av} = \underbrace{\frac{4E_{\max}^2}{2\mu_0 c}}_{I_{max}} \cos^2\left(\frac{\pi(r_1 - r_2)}{\lambda}\right) = \frac{4E_{\max}^2}{2\mu_0 c}\cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

Diffraction pattern from a plane wave incident on a double slit:





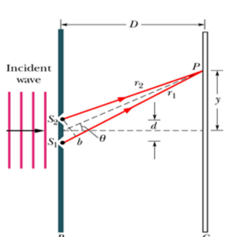
$$r_1 - r_2 = \delta = d\sin\theta$$

$$E(P,t) = E_{\max}\sin\left(\frac{2\pi r_1}{\lambda} - 2\pi f t\right) + E_{\max}\sin\left(\frac{2\pi r_2}{\lambda} - 2\pi f t\right)$$

$$= 2E_{\max}\sin\left(\frac{\pi(r_1 + r_2)}{\lambda} - 2\pi f t\right)\cos\left(\frac{\pi(r_1 - r_2)}{\lambda}\right)$$

➔ intensity maxima occur for $\frac{\pi(r_1 - r_2)}{\lambda} = m\pi \Rightarrow d\sin\theta = m\lambda$

Summary of results:

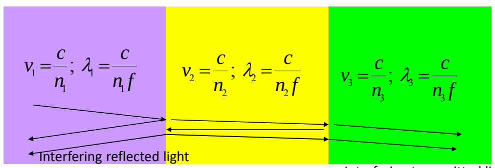


Constructive interference:
$d\sin\theta = m\lambda$
Image on screen:
$y \approx m\lambda\frac{D}{d}$

Interference in thin films

$$E_y(x,t) = E_{\max}\sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi\right)$$

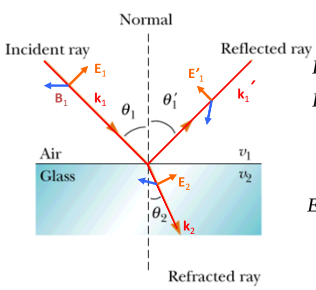
$$v = \frac{c}{n}; \qquad \lambda = \frac{c}{nf}$$



Interfering transmitted light

Interference between reflected waves:

Recall -- the behavior of a plane-polarized electromagnetic wave near the surface of two materials:



Periodic waves:

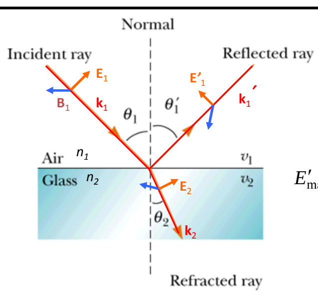
$$E_1 = E_{\max_1}\sin(\mathbf{k}_1 \cdot \mathbf{r} - \omega_1 t)$$

$$E_1' = E_{\max_1}'\sin(\mathbf{k}_1' \cdot \mathbf{r} - \omega_1' t)$$

$$\frac{\omega_1}{k_1} = \frac{\omega_1'}{k_1'} = v_1$$

$$E_2 = E_{\max_2}\sin(\mathbf{k}_2 \cdot \mathbf{r} - \omega_2 t)$$

$$\frac{\omega_2}{k_2} = v_2$$



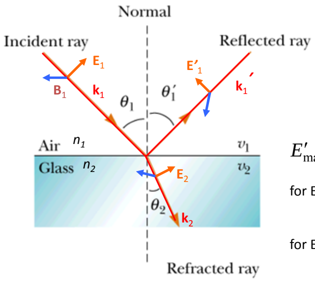
Matching electric and magnetic fields at boundary:

$$E_{\max_1}' = E_{\max_1}\frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}$$

or

$$E_{\max_1}' = E_{\max_1}\frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

Plane waves reflected and refracted at surface:



Matching electric and magnetic fields at boundary:

For reflected waves:

$$E'_{\max_1} = E_{\max_1} \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

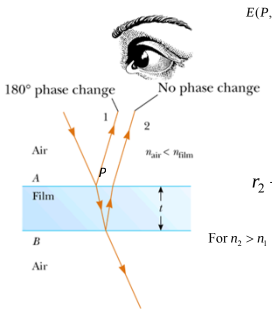
for E in plane of incidence

or

for E out of plane of incidence:

$$E'_{\max_1} = E_{\max_1} \frac{n_2 \cos\theta_2 - n_1 \cos\theta_1}{n_2 \cos\theta_2 + n_1 \cos\theta_1}$$

Multiple refractions and reflections in a thin film



$$E(P,t) = \pm E_{\max} \sin\left(\frac{2\pi r_1}{\lambda_1} - 2\pi f t\right) \pm E'_{\max} \sin\left(\frac{2\pi r_2}{\lambda_2} - 2\pi f t\right)$$

$$\approx 2E_{\max} \begin{Bmatrix}\sin\\ \cos\end{Bmatrix}\left(\frac{\pi(r_1+r_2)}{\lambda_{av}} - 2\pi f t\right)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\left(\frac{\pi(r_2-r_1)}{\lambda_2}\right)$$

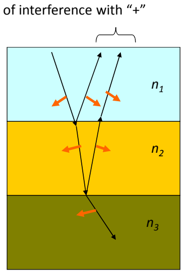
$$r_2 - r_1 \approx 2t$$

For $n_2 > n_1$ $\quad E_{\max_1} = -; \quad E'_{\max_1} = +$

$$\Rightarrow \sin\left(\frac{\pi(2t)}{\lambda_2}\right) \Rightarrow \text{max at } 2nt = (m + \tfrac{1}{2})\lambda$$

$$\Rightarrow \text{min at } 2nt = m\lambda$$

Example of interference with "+"



$n_1 < n_2 < n_3$

For each surface with $n_1 < n_2$

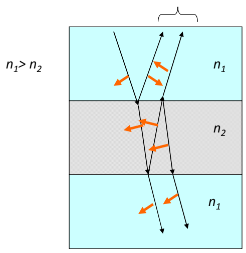
$E_2 = -E_1$ for reflected beam

$$\Rightarrow I = I_{\max} \{\cos(\tfrac{1}{2}(A-B))\}^2$$

peaks at

$$\tfrac{1}{2}(A-B) \equiv \frac{\pi(r_2 - r_1)}{\lambda} = m\pi$$

Example of interference with "–"



$$\Rightarrow I = I_{max}\left\{\sin\left(\tfrac{1}{2}(A-B)\right)\right\}^2$$

peaks at

$$\tfrac{1}{2}(A-B) \equiv \frac{\pi(r_2 - r_1)}{\lambda} = \left(m + \tfrac{1}{2}\right)\pi$$

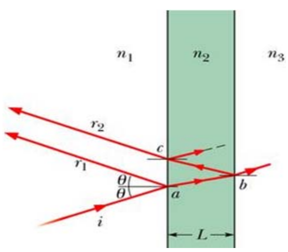
Example of interference with "+"

$$\Rightarrow I = I_{max}\left\{\cos\left(\tfrac{1}{2}(A-B)\right)\right\}^2$$

peaks at

$$\tfrac{1}{2}(A-B) \equiv \frac{\pi(r_2 - r_1)}{\lambda} = m\pi$$

Summary



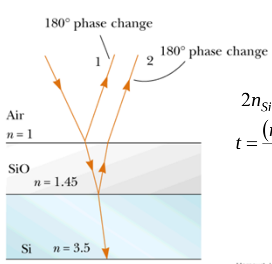
$$\varphi \approx 2\pi \frac{2L}{\lambda_2} = 2\pi \frac{2L}{\lambda} n_2$$

$$I = 4I_0 \cos^2\left(\frac{\varphi}{2}\right) \quad \text{=> maxima at} \quad 2L = m\frac{\lambda}{n_2}$$

$$I = 4I_0 \sin^2\left(\frac{\varphi}{2}\right) \quad \text{=> maxima at} \quad 2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2}$$

Example:

Suppose you want to prepare a surface so that it has minimal reflection such as in a solar cell where it is desirable to optimize refraction and minimize reflection at λ=550nm.



$$2n_{SiO}t = \left(m + \tfrac{1}{2}\right)\lambda$$

$$t = \frac{\left(m + \tfrac{1}{2}\right)\lambda}{2n_{SiO}} = \frac{\left(m + \tfrac{1}{2}\right)550\ \text{nm}}{2 \cdot 1.45}$$

$$= 94.8\ \text{nm}\ \ (\text{for } m = 0)$$

Example:

Suppose you see a rainbow pattern for oil on a pavement. What is the the approximate thickness of the oil in the red (λ=700nm) region. Assume that n_{oil}=1.4.

$$2n_{oil}t = (m + \tfrac{1}{2})\lambda$$

$$t = \frac{(m + \tfrac{1}{2})\lambda}{2n_{oil}} = \frac{(m + \tfrac{1}{2})700\text{ nm}}{2\cdot 1.4} = 125\text{ nm},\ 375\text{ nm, etc.}$$