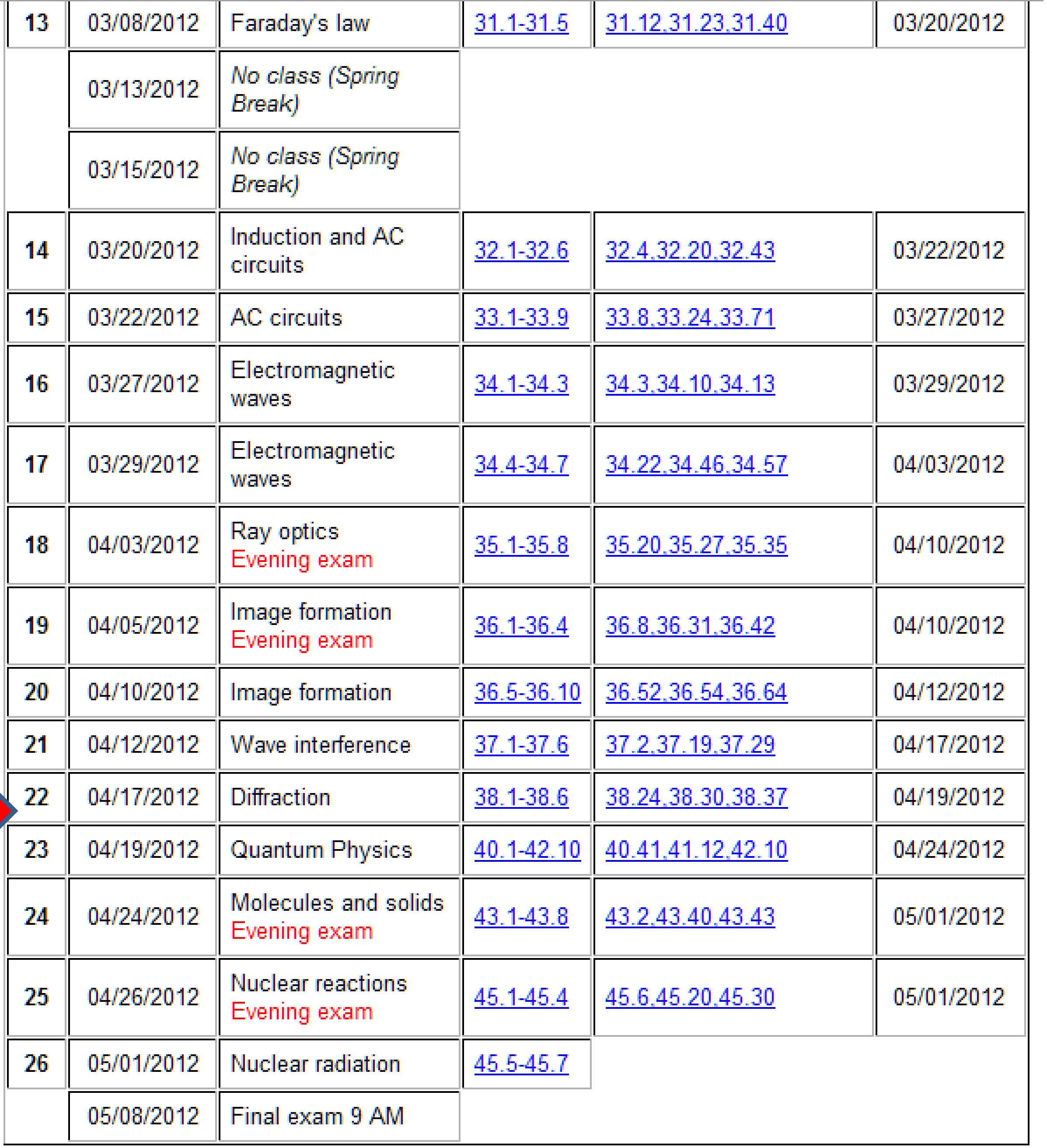
PHY 114 A General Physics II
11 AM-12:15 PM TR Olin 101

Plan for Lecture 22 (Chapter 38):

Diffraction of light

1. Diffraction gratings
2. X-ray diffraction
3. Other properties of light



13	03/08/2012	Faraday's law	31.1-31.5	31.12,31.23,31.40	03/20/2012
	03/13/2012	*No class (Spring Break)*			
	03/15/2012	*No class (Spring Break)*			
14	03/20/2012	Induction and AC circuits	32.1-32.6	32.4,32.20,32.43	03/22/2012
15	03/22/2012	AC circuits	33.1-33.9	33.8,33.24,33.71	03/27/2012
16	03/27/2012	Electromagnetic waves	34.1-34.3	34.3,34.10,34.13	03/29/2012
17	03/29/2012	Electromagnetic waves	34.4-34.7	34.22,34.46,34.57	04/03/2012
18	04/03/2012	Ray optics Evening exam	35.1-35.8	35.20,35.27,35.35	04/10/2012
19	04/05/2012	Image formation Evening exam	36.1-36.4	36.8,36.31,36.42	04/10/2012
20	04/10/2012	Image formation	36.5-36.10	36.52,36.54,36.64	04/12/2012
21	04/12/2012	Wave interference	37.1-37.6	37.2,37.19,37.29	04/17/2012
22	04/17/2012	Diffraction	38.1-38.6	38.24,38.30,38.37	04/19/2012
23	04/19/2012	Quantum Physics	40.1-42.10	40.41,41.12,42.10	04/24/2012
24	04/24/2012	Molecules and solids Evening exam	43.1-43.8	43.2,43.40,43.43	05/01/2012
25	04/26/2012	Nuclear reactions Evening exam	45.1-45.4	45.6,45.20,45.30	05/01/2012
26	05/01/2012	Nuclear radiation	45.5-45.7		
	05/08/2012	Final exam 9 AM			

Time, Einstein, and the Coolest Stuff in the Universe

A free public lecture by Nobel Laureate

Dr. William Phillips

National Institute of Standards and Technology

8:00 PM Friday, April 20

Brendle Recital Hall
Wake Forest University

Part of SPS zone 5 conference
April 20-21, 2012

Offer 1 point extra credit for attendance*

*After the lecture, email me that you attended. In the following email exchange you will be asked to answer one question about the lecture.

www.wfu.edu/physics/sps/spszone52012conf/welcome.html

3rd exam solutions

- Solutions posted on web
- Exam review session??

  Would you like to attend an exam review session?

  (A) yes (B) no

  If you would like a review session, can you meet

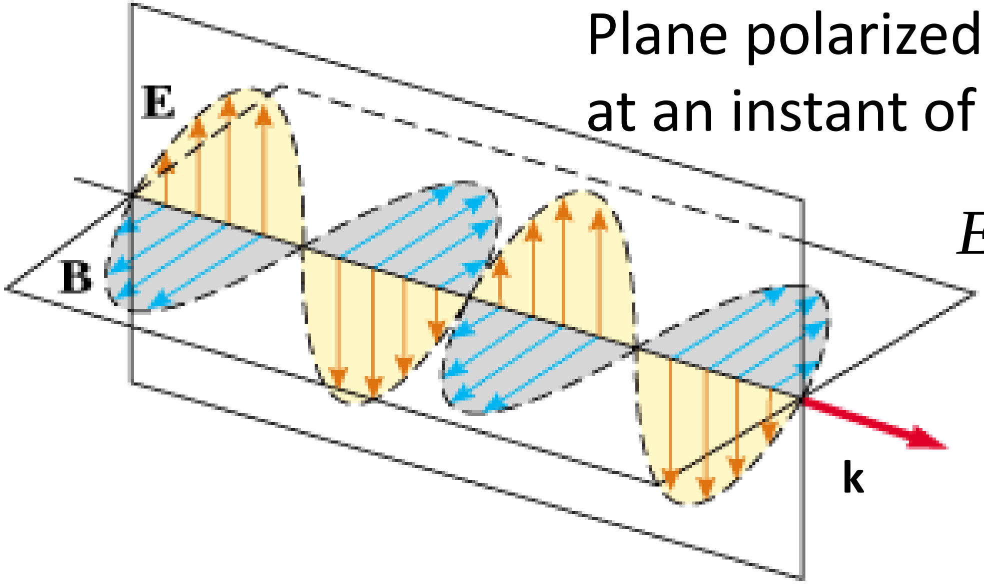
  (A) Today Tuesday at 2 PM (here)
  (B) Today Tuesday at 5 PM Olin 107
  (C) Tomorrow Wed. at 1 PM Olin 107
  (D) Tomorrow Wed. at 2 PM Olin 107
  (E) Other?
- Similar problems may appear on final exam

Fourth exam – scheduled during the week of April 23 (evenings 6-10 PM) Note: Content mostly on material in Chapters 35-38, optics, diffraction, plus possibly some ideas of Quantum Mechanics and possibly some topics from Exam 3

Which of these times are you likely to prefer:

A. Monday 4/23
B. Tuesday 4/24
C. Wednesday 4/25
D. Thursday 4/26
E. Friday 4/27

Wave phenomena associated with light



Plane polarized electromagnetic wave at an instant of time:

$$E_y(x,t) = E_{\max}\sin\left(\frac{2\pi}{\lambda}(x-vt)+\varphi\right)$$

Superposition of two electromagnetic waves (electric field portion)

$$E_y^{tot}(x,t) = E_y^1(x,t) + E_y^2(x,t)$$

$$E_y^{tot}(x,t) = E_{\max}\sin\left(\frac{2\pi}{\lambda}(x-vt)\right) + E_{\max}\sin\left(\frac{2\pi}{\lambda}(x-vt)+\varphi\right)$$

$$= 2E_{\max}\sin\left(\frac{2\pi}{\lambda}(x-vt)+\frac{1}{2}\varphi\right)\cos\left(\frac{\varphi}{2}\right)$$

Note that this result follows from the trigonometric identity :

$$\sin(A)+\sin(B)=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

Squared magnitude : $\left|\sin(A)+\sin(B)\right|^2=4\sin^2\left(\frac{A+B}{2}\right)\cos^2\left(\frac{A-B}{2}\right)$

In our case :

$$E_y^{tot}(x,t)=E_{\max}\sin\left(\frac{2\pi}{\lambda}(x-vt)\right)+E_{\max}\sin\left(\frac{2\pi}{\lambda}(x-vt)+\varphi\right)$$

$$=2E_{\max}\sin\left(\frac{2\pi}{\lambda}(x-vt)+\frac{1}{2}\varphi\right)\cos\left(\frac{\varphi}{2}\right)$$

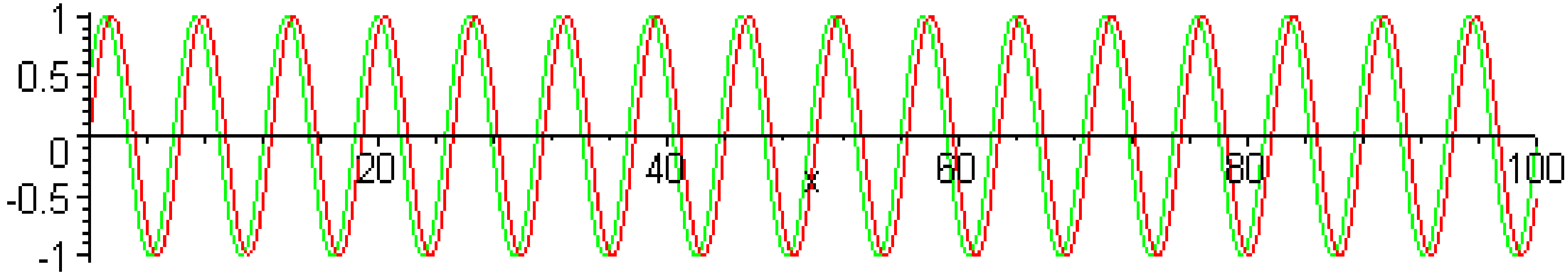
Constant in time

Rapidly changing in time

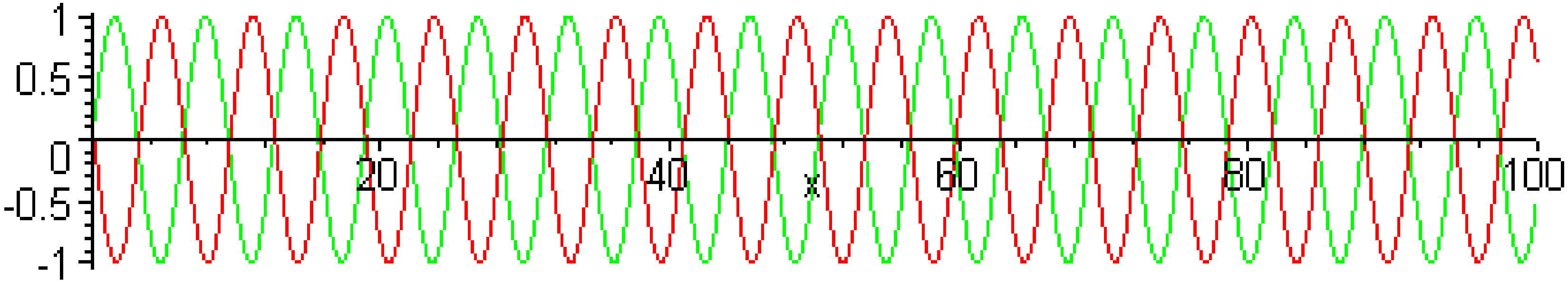
Intensity of the EM waves :

$$I\equiv S_{avg}=\frac{1}{c\mu_0}\left\langle\left|E_y^{tot}\right|^2\right\rangle_{avg}=\frac{4}{2c\mu_0}\left|E_{\max}\cos\left(\frac{\varphi}{2}\right)\right|^2=I_{\max}\cos^2\left(\frac{\varphi}{2}\right)$$

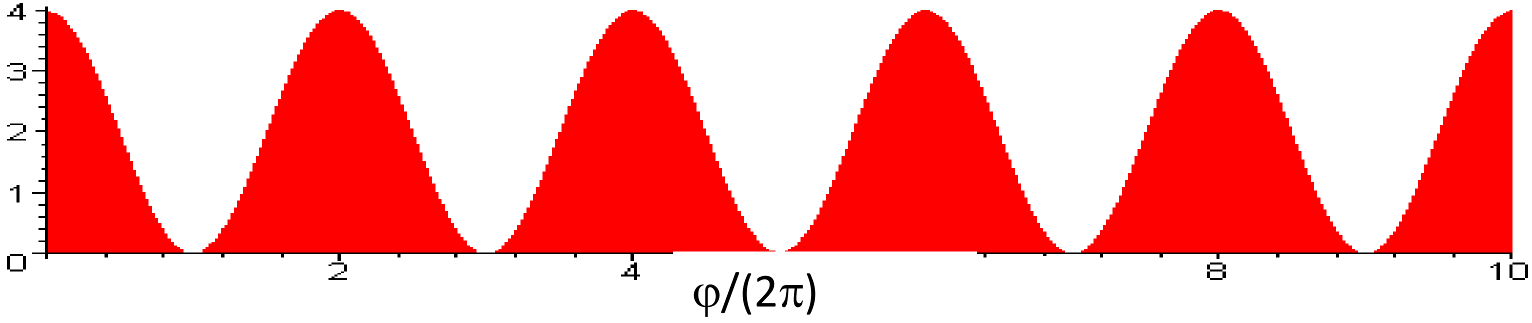
Example: φ=0.5 rad -- plotting snapshot of EM wave



Example: φ=3 rad – plotting snapshot of EM wave



Intensity as a function of φ:



$$E_y^{tot}(x,t) = E_{\max}\sin\left(\frac{2\pi}{\lambda}(x - vt)\right) + E_{\max}\sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi\right)$$

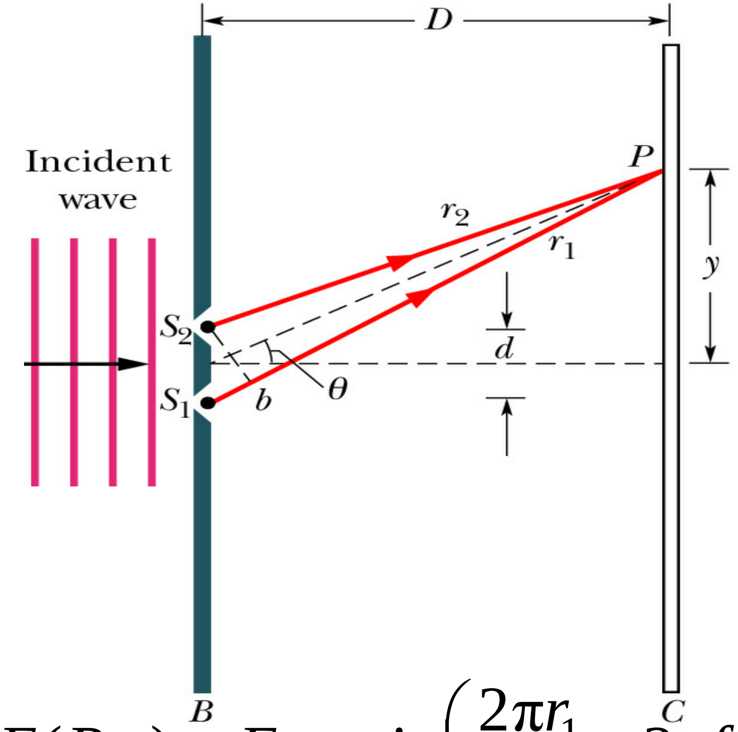
$$= 2E_{\max}\sin\left(\frac{2\pi}{\lambda}(x - vt) + \frac{1}{2}\varphi\right)\cos\left(\frac{\varphi}{2}\right)$$

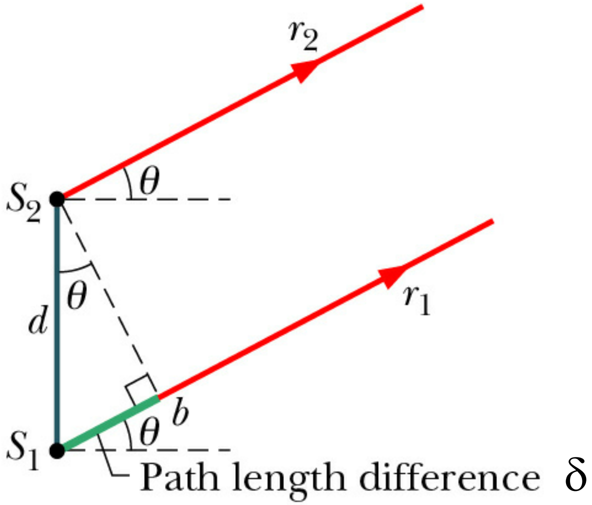
What is the significance of this result?

A. No significance – only an evil physics professor could love such a result.
B. Shows that any two electromagnetic waves can interfere
C. Shows that two electromagnetic wave with the same amplitude, wavelength, and velocity can interfere if they have different phases ϕ

Young's double slit geometry:

Mathematical analysis of bright fringes:





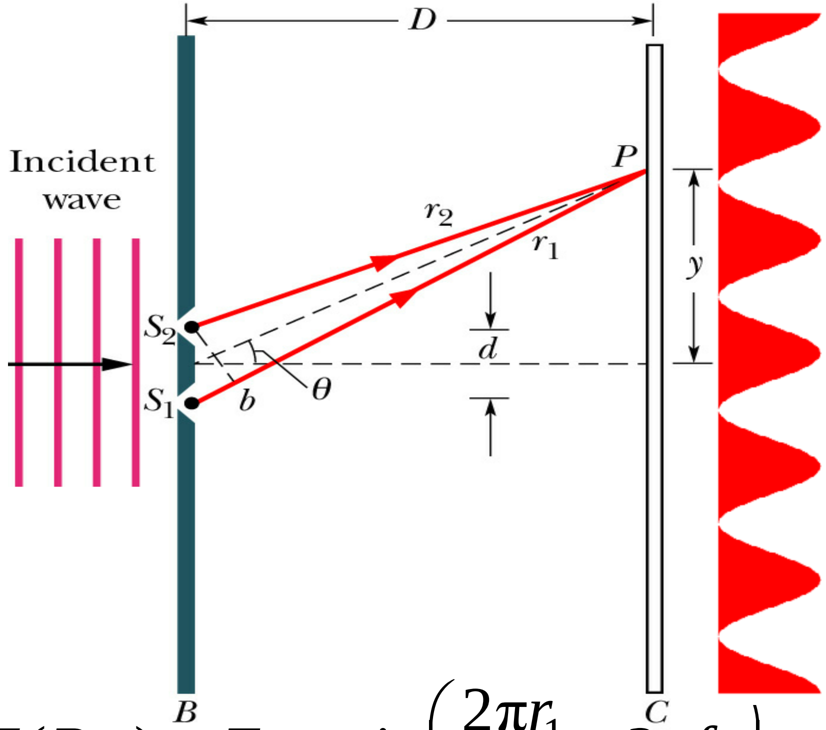
$$r_1 - r_2 = \delta = d \sin\theta$$

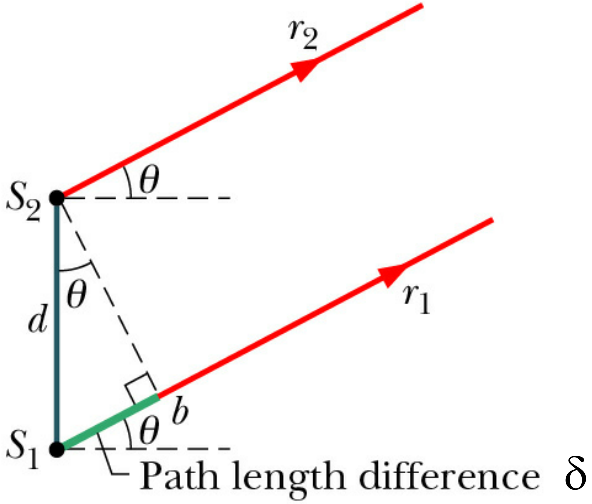
$$E(P,t) = E_{\max}\sin\left(\frac{2\pi r_1}{\lambda} - 2\pi f t\right) + E_{\max}\sin\left(\frac{2\pi r_2}{\lambda} - 2\pi f t\right)$$

$$= 2E_{\max}\sin\left(\frac{\pi(r_1+r_2)}{\lambda} - 2\pi f t\right)\cos\left(\frac{\pi(r_1-r_2)}{\lambda}\right)$$

➔ intensity maxima occur for $\frac{\pi(r_1-r_2)}{\lambda} = m\pi$ $\Rightarrow d\sin\theta = m\lambda$

Diffraction pattern from a plane wave incident on a double slit:



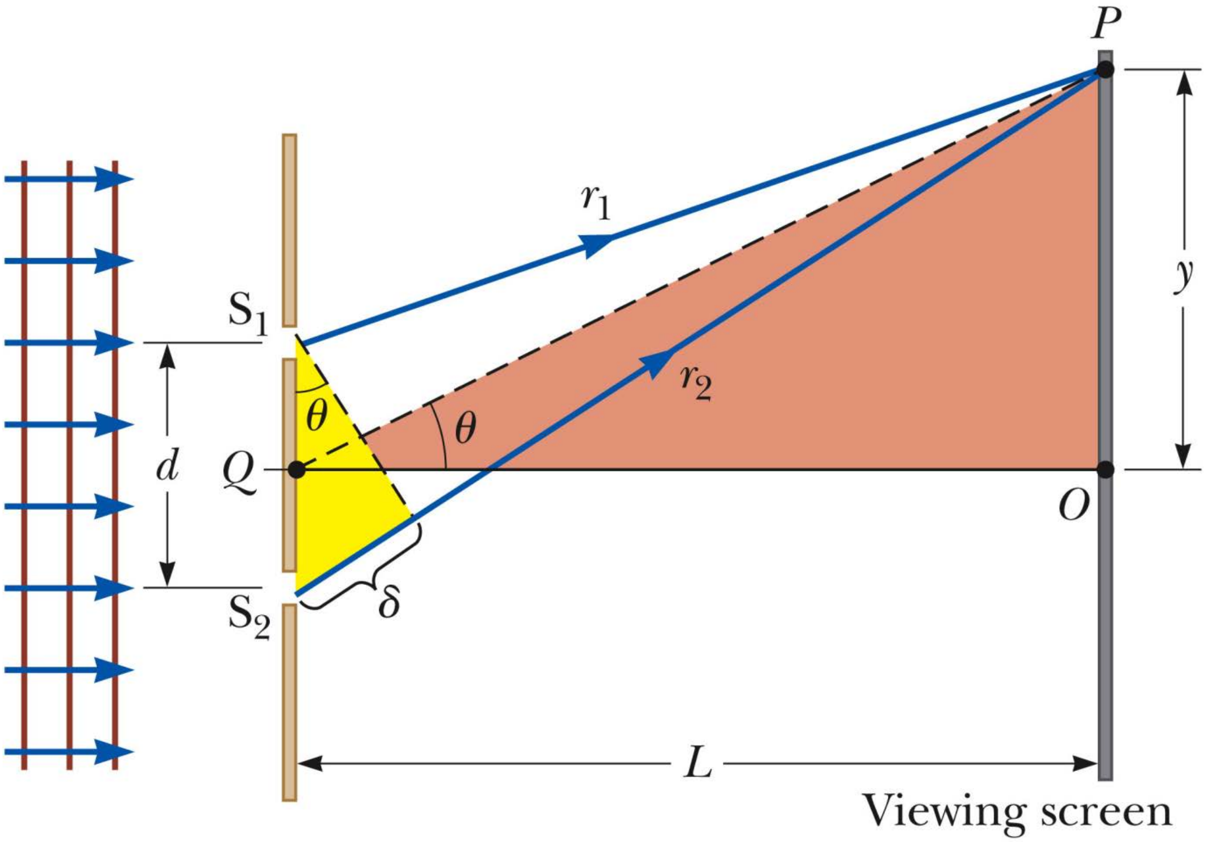


$$r_1 - r_2 = \delta = d \sin\theta$$

$$E(P,t) = E_{\max}\sin\left(\frac{2\pi r_1}{\lambda} - 2\pi f t\right) + E_{\max}\sin\left(\frac{2\pi r_2}{\lambda} - 2\pi f t\right)$$

$$= 2E_{\max}\sin\left(\frac{\pi(r_1 + r_2)}{\lambda} - 2\pi f t\right)\cos\left(\frac{\pi(r_1 - r_2)}{\lambda}\right)$$

➔ intensity maxima occur for $\frac{\pi(r_1 - r_2)}{\lambda} = m\pi$ $\Rightarrow d\sin\theta = m\lambda$



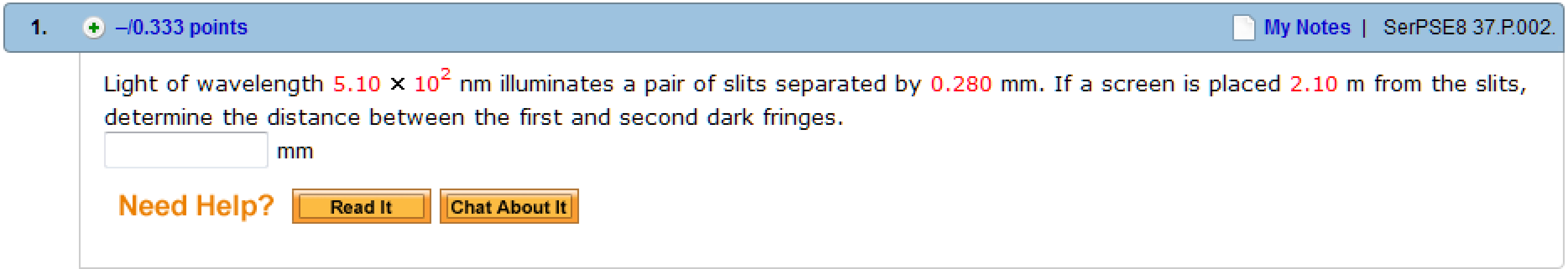
$$I \approx I_{\max} \cos^2\left(\pi \frac{d}{\lambda}\frac{y}{L}\right)$$

$$E(P,t) = E_{\max} \sin\left(\frac{2\pi r_1}{\lambda} - 2\pi f t\right) + E_{\max} \sin\left(\frac{2\pi r_2}{\lambda} - 2\pi f t\right)$$

$$= 2E_{\max} \sin\left(\frac{\pi(r_1 + r_2)}{\lambda} - 2\pi f t\right) \cos\left(\frac{\pi(r_1 - r_2)}{\lambda}\right)$$

$$r_2 - r_1 = \delta = d \sin\theta \qquad y = L \tan\theta \approx L \sin\theta$$

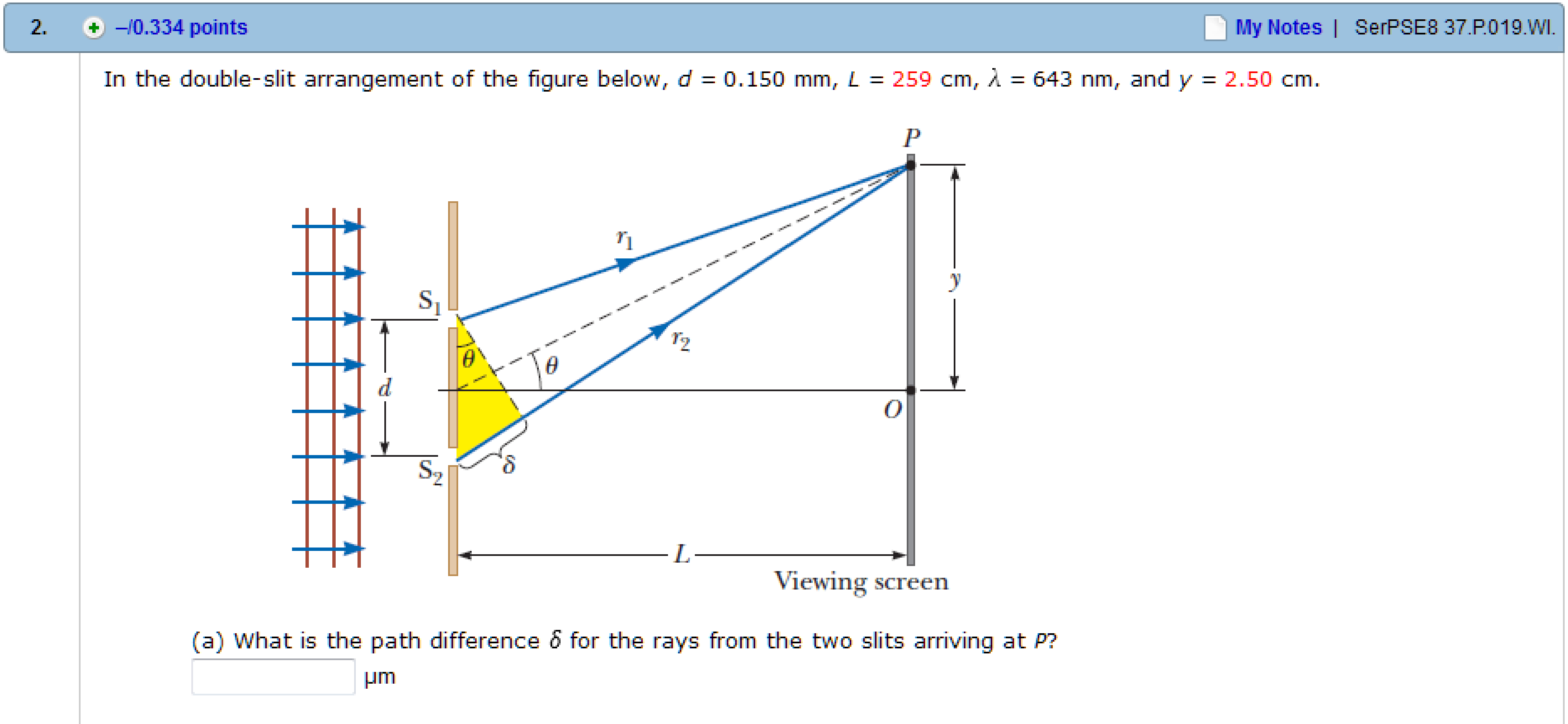
Webassign hints:



1. –/0.333 points SerPSE8 37.P.002.

Light of wavelength 5.10×10^2 nm illuminates a pair of slits separated by 0.280 mm. If a screen is placed 2.10 m from the slits, determine the distance between the first and second dark fringes.

☐ mm



2. –/0.334 points SerPSE8 37.P.019.WI.

In the double-slit arrangement of the figure below, d = 0.150 mm, L = 259 cm, λ = 643 nm, and y = 2.50 cm.

(a) What is the path difference δ for the rays from the two slits arriving at P?

☐ µm

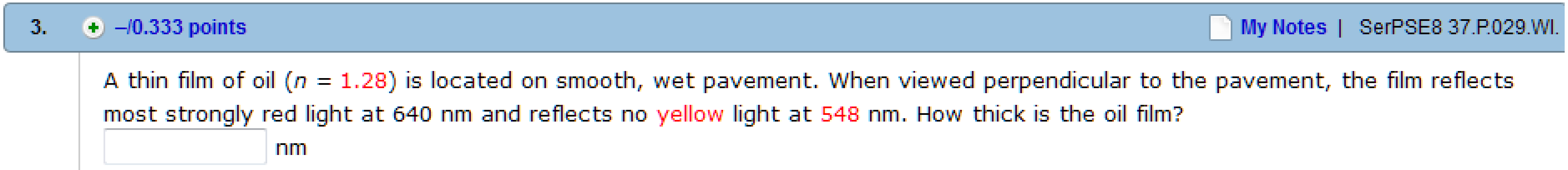
Recall that the 2-slit intensity pattern has the form:

$$I = I_{\max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

$$\Rightarrow \text{bright fringes when } \frac{\pi d \sin\theta}{\lambda} = m\pi \quad \sin\theta = \frac{m\lambda}{d}$$

$$\Rightarrow \text{dark fringes when } \frac{\pi d \sin\theta}{\lambda} = \left(m + \frac{1}{2}\right)\pi \quad \sin\theta = \frac{\left(m + \frac{1}{2}\right)\lambda}{d}$$

Webassign hint:

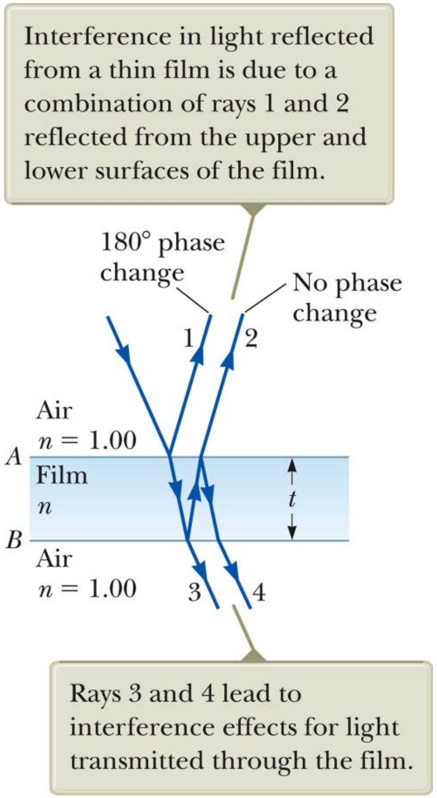


3. –/0.333 points

My Notes | SerPSE8 37.P.029.WI.

A thin film of oil (n = 1.28) is located on smooth, wet pavement. When viewed perpendicular to the pavement, the film reflects most strongly red light at 640 nm and reflects no yellow light at 548 nm. How thick is the oil film?

☐ nm



Interference in light reflected from a thin film is due to a combination of rays 1 and 2 reflected from the upper and lower surfaces of the film.

180° phase change

No phase change

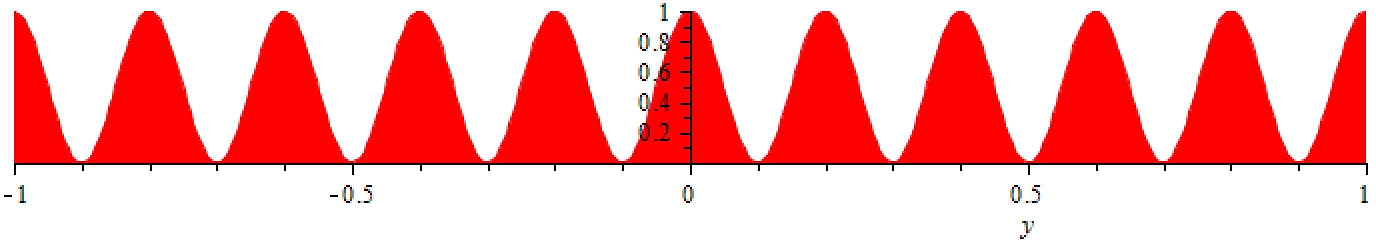
Rays 3 and 4 lead to interference effects for light transmitted through the film.

In illustration case:

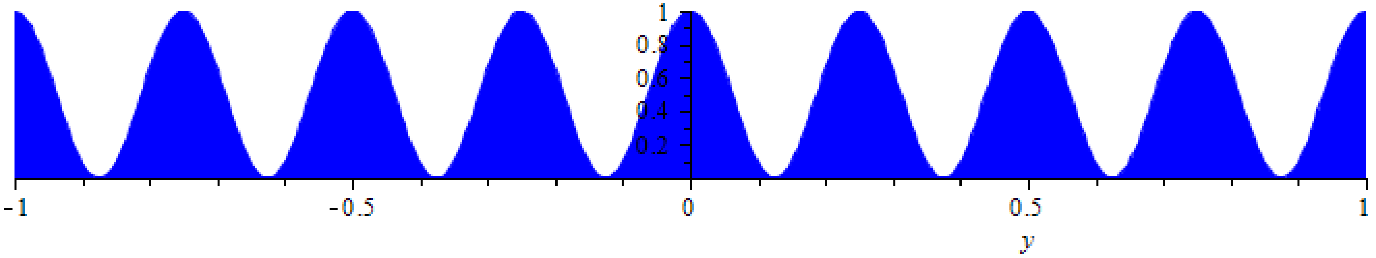
Constructive interference: $$2nt = \left(m + \frac{1}{2}\right)\lambda$$

Destructive interference: $$2nt = m\lambda$$

Light intensity patterns seen on screen for very thin double slit



λ_1

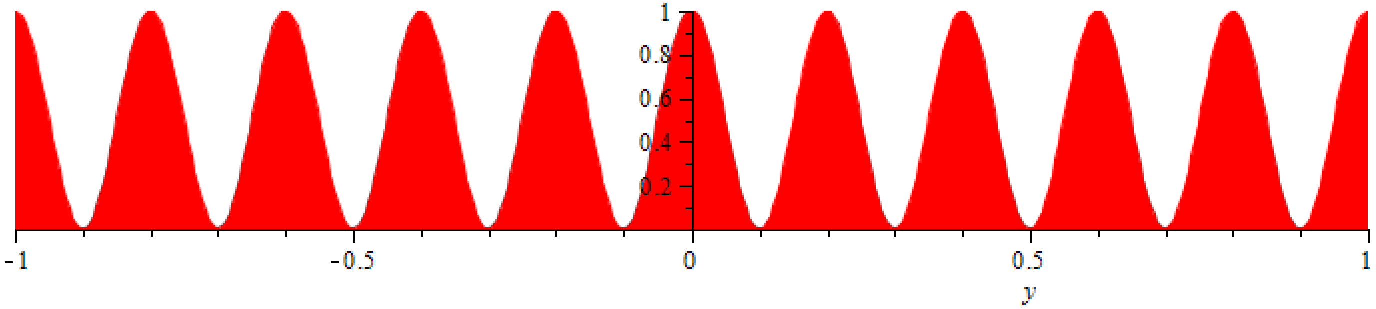


λ_2

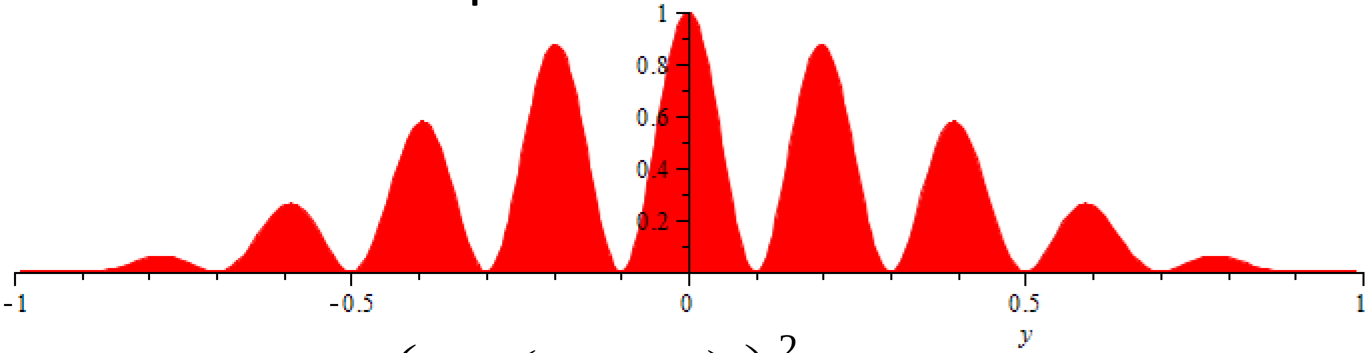
Assuming that d and L are the same which plot corresponds the the greater wavelength?

A. $\lambda_1 > \lambda_2$
B. $\lambda_2 > \lambda_1$

Ideal infinitely thin 2-slit pattern



Finite thickness 2-slit pattern

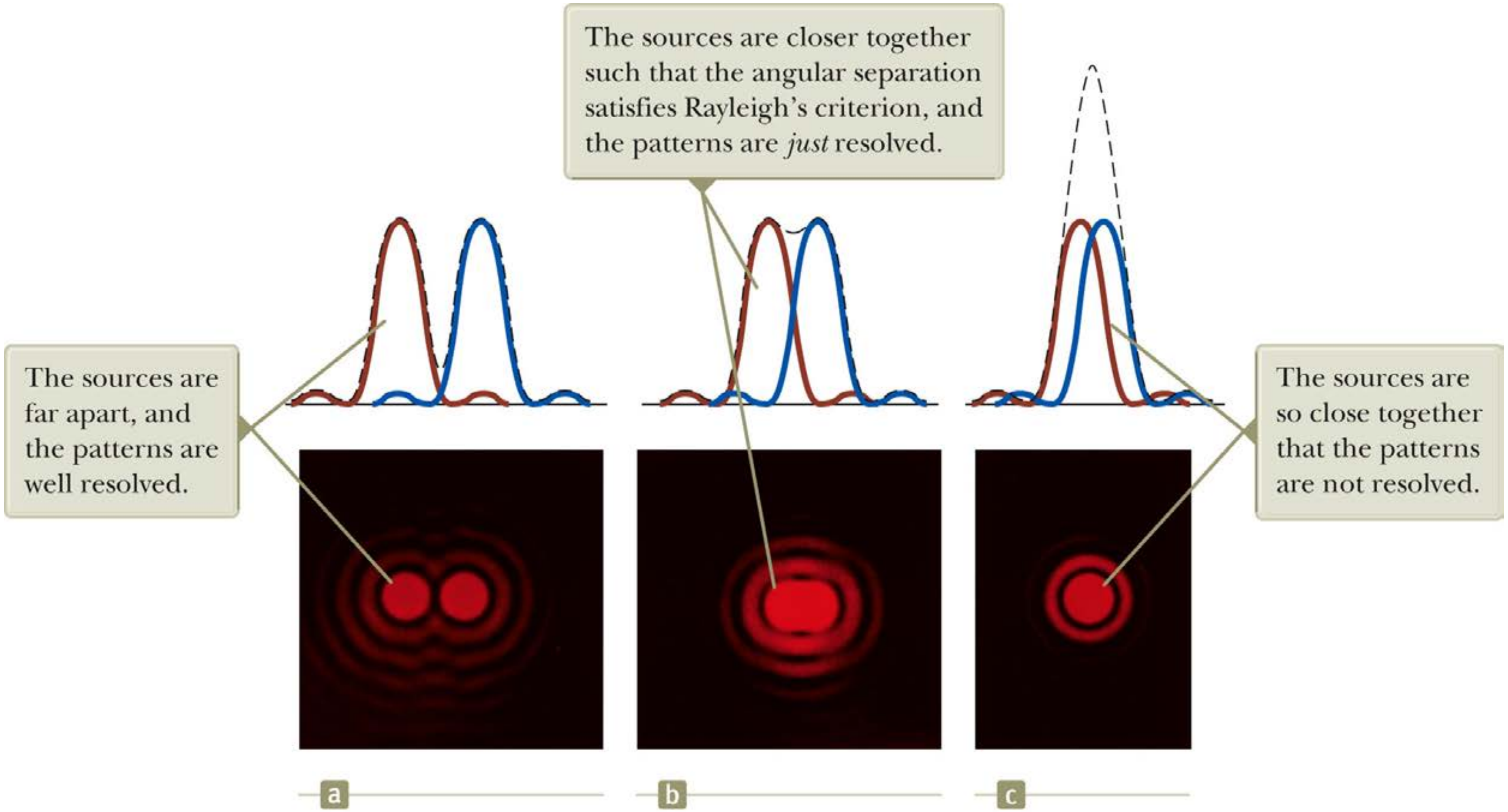


$$I \approx I_{\max} \cos^2\left(\pi \frac{d}{\lambda}\frac{y}{L}\right)\left(\frac{\sin\left(\pi \frac{a}{\lambda}\frac{y}{L}\right)}{\pi \frac{a}{\lambda}\frac{y}{L}}\right)^2$$

$d \equiv$ slit separation

$a \equiv$ slit width

Effects of diffraction when you may not want it – images of small objects near the “diffraction” limit

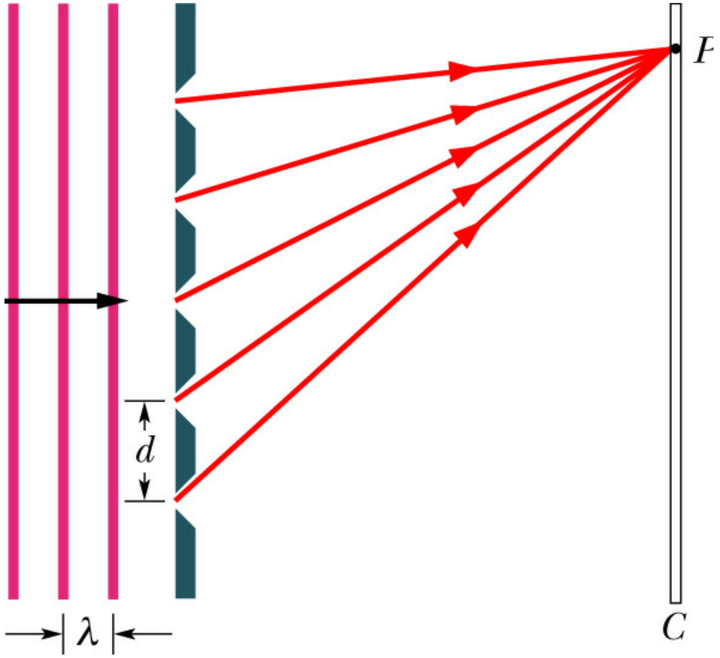


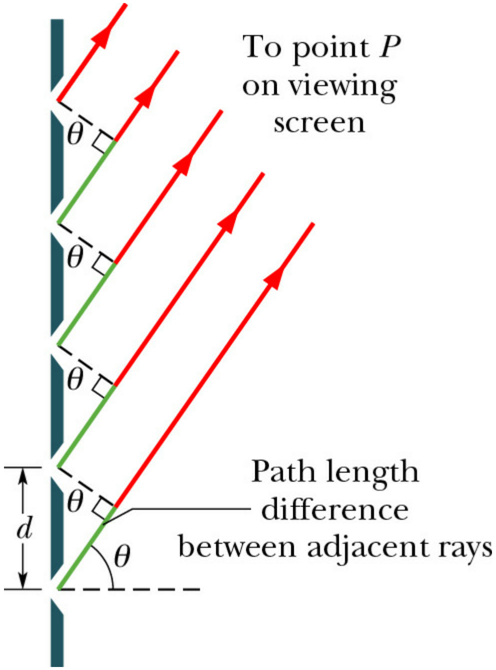
The sources are closer together such that the angular separation satisfies Rayleigh’s criterion, and the patterns are *just* resolved.

The sources are far apart, and the patterns are well resolved.

The sources are so close together that the patterns are not resolved.

Enhancement of diffraction – diffraction gratings

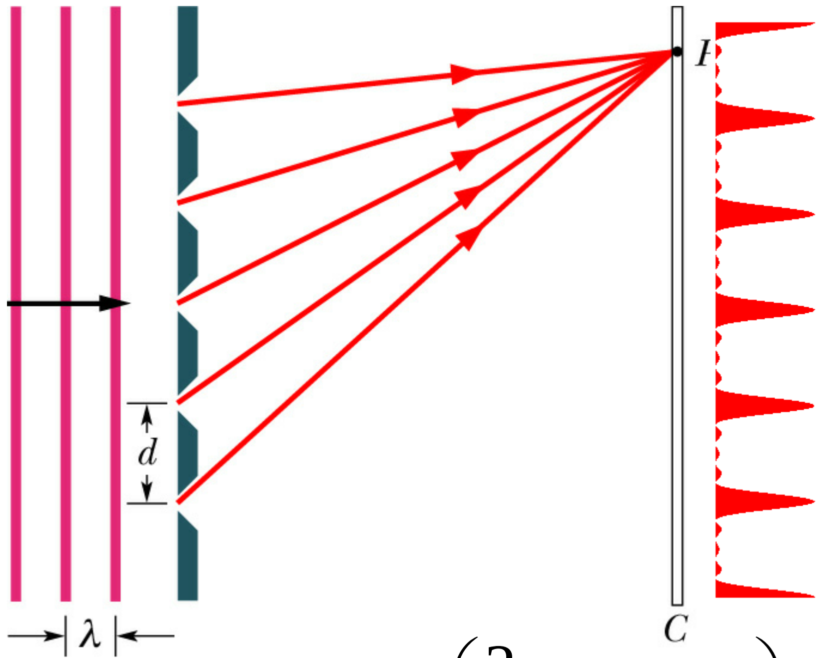


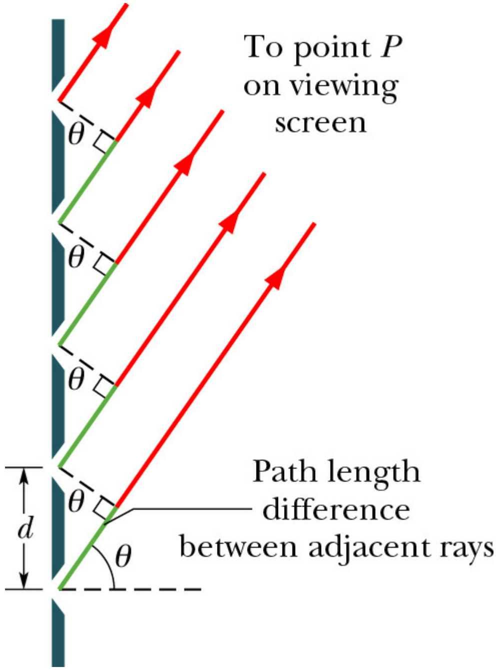


$$E(P,t)=\sum_i E_{\max}\sin\left(\frac{2\pi r_i}{\lambda}-2\pi f t\right)$$

$$=E_{\max}\sin\left(\frac{2\pi r_{av}}{\lambda}-2\pi f t\right)\frac{\sin\left(\frac{N\pi d\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)}$$

Diffraction pattern for *N* slits – diffraction grating





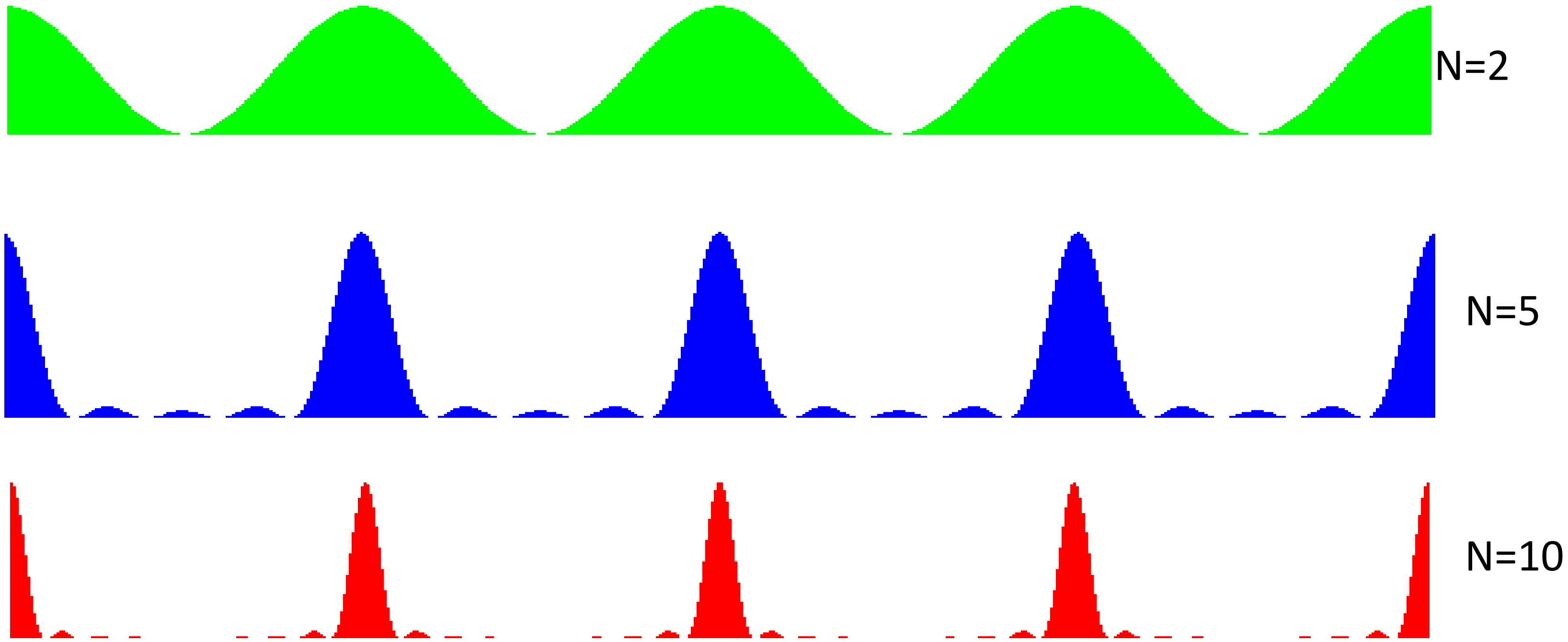
$$E(P,t)=\sum_i E_{\max}\sin\left(\frac{2\pi r_i}{\lambda}-2\pi f t\right)$$

$$=E_{\max}\sin\left(\frac{2\pi r_{av}}{\lambda}-2\pi f t\right)\frac{\sin\left(\frac{N\pi d\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)}$$

Intensity maxima at $d\sin\theta = m\lambda$

Intensity pattern from multiple slit grating:

$$I = I_{\max}\left[\frac{\sin\left(\dfrac{N\pi d\sin\theta}{\lambda}\right)}{\sin\left(\dfrac{\pi d\sin\theta}{\lambda}\right)}\right]^2$$



Sanity check:

If the intensity pattern for N slits is given by

$$I = I_{\max}\left[\frac{\sin\left(\frac{N\pi d \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d \sin\theta}{\lambda}\right)}\right]^2$$

what happened to the formula two-slit intensity pattern?

A. NEVER trust your physics professor
B. More evidence that physics does not make sense
C. The formula look different, but are equivalent for N=2

$$I = I_{\max}\left[\frac{\sin\left(\frac{N\pi d\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)}\right]^2$$

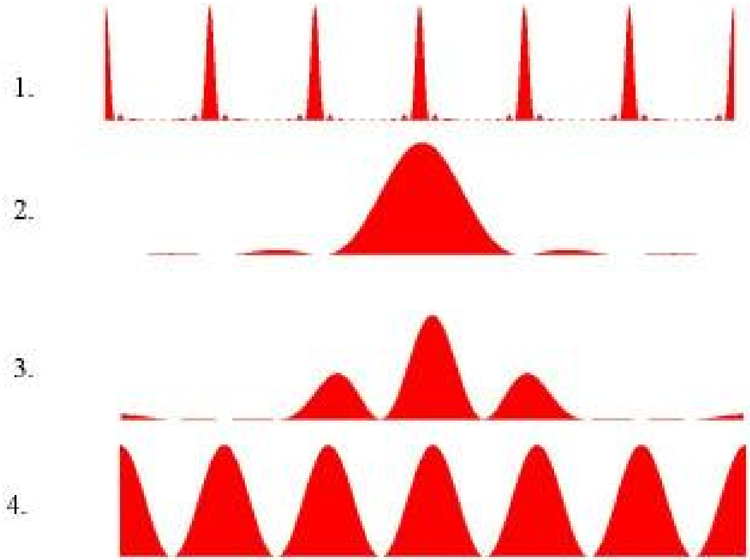
For $N = 2$:

$$I = I_{\max}\left[\frac{\sin\left(\frac{2\pi d\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)}\right]^2 \equiv I_{\max}\left[\frac{\sin(2\varphi)}{\sin(\varphi)}\right]^2$$

$$= I_{\max}\left[\frac{2\sin(\varphi)\cos(\varphi)}{\sin(\varphi)}\right]^2$$

$$= I'_{\max}\cos^2(\varphi)$$

$$= I'_{\max}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)$$



thin multiple slits

fat single slit

fat double slits

thin double slits

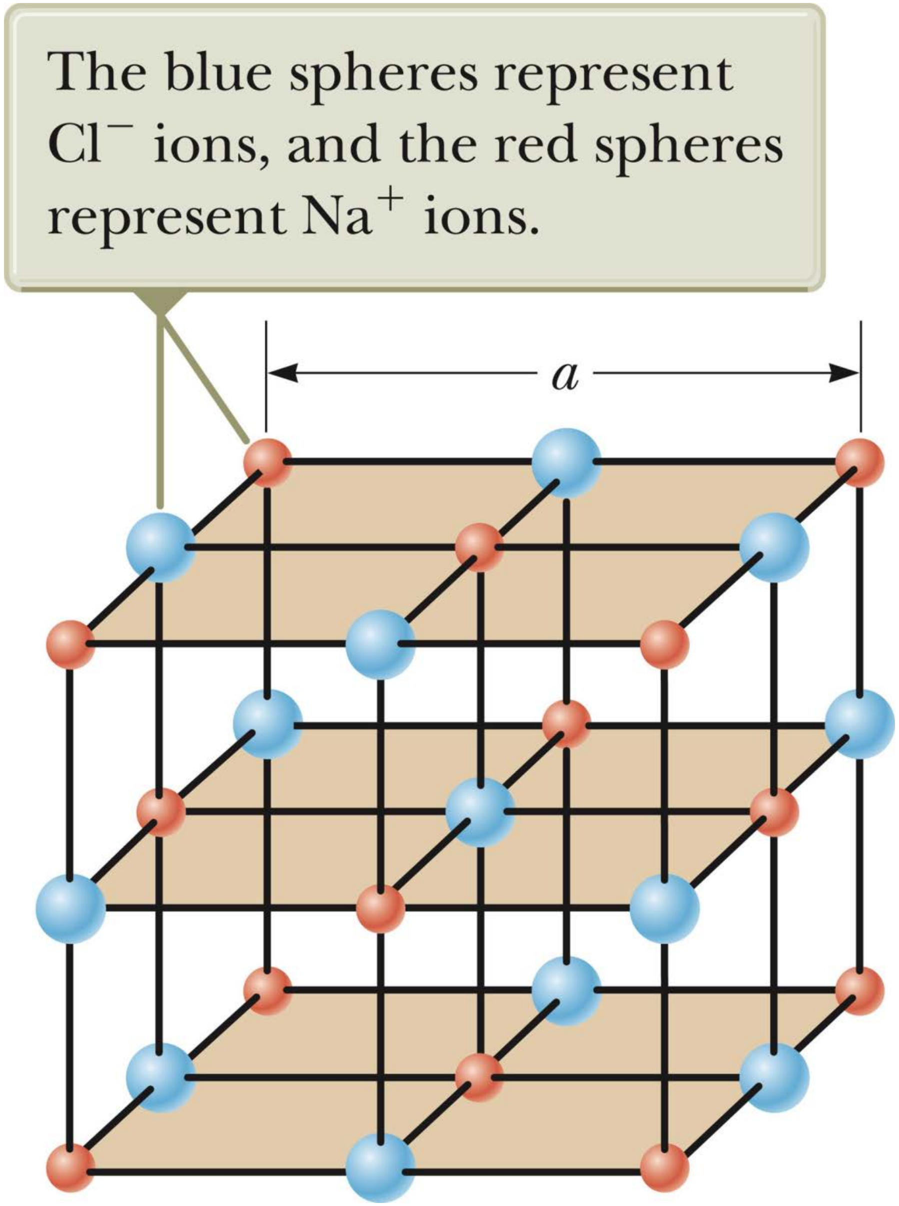
Consider the 4 plots which represent the intensity of monochromatic light on a screen a large distance away from various slit configurations. For each of the plots identify the type of slits -- thin double slits, fat double slits, thin multiple slits, fat multiple slits, 1 thin slit, 1 fat slit, etc.

Diffraction gratings in nature -- crystals

Model of NaCl

Typical atomic distances

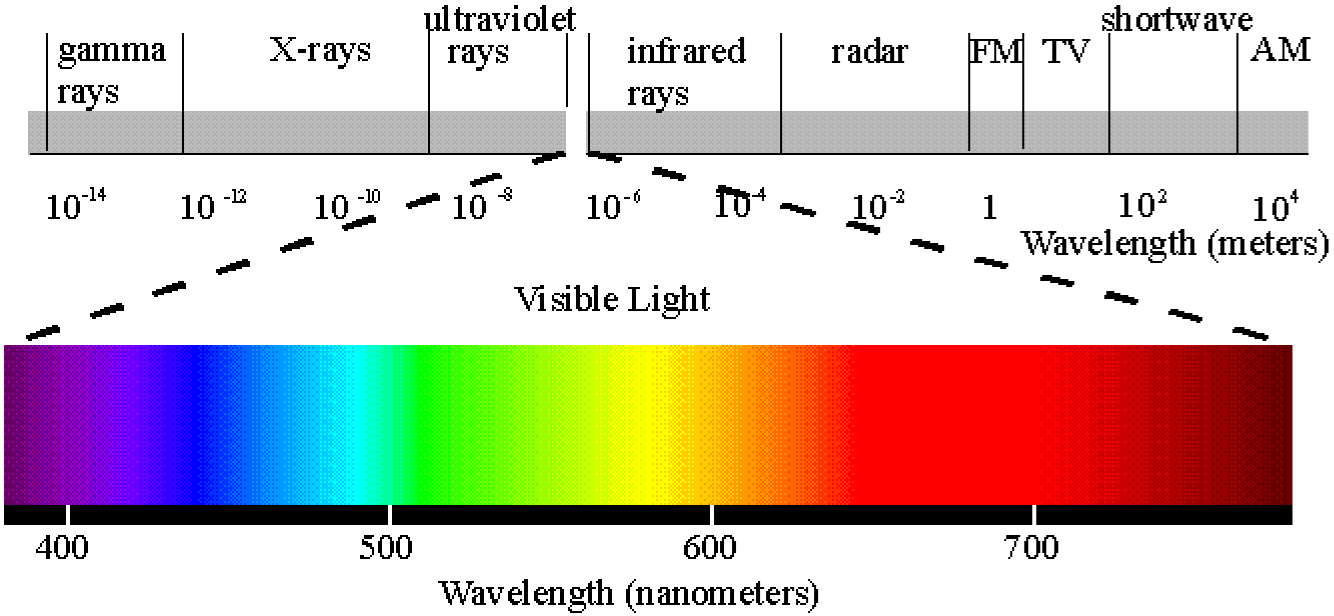
a =0.1-1.0 nm



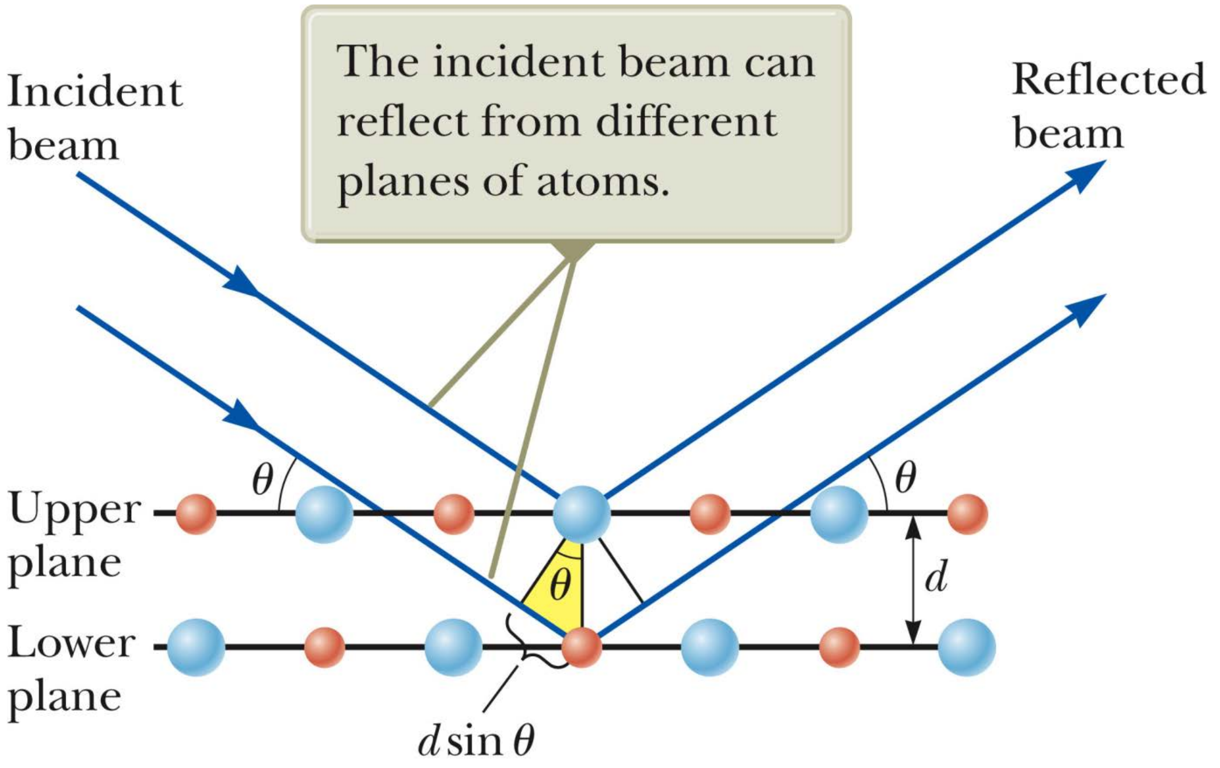
The blue spheres represent Cl^- ions, and the red spheres represent Na^+ ions.

Recall form of diffraction pattern for grating :

$$I = I_{\max}\left[\frac{\sin\left(\frac{N\pi d\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)}\right]^2 \quad \Rightarrow \text{maxima when } d\sin\theta = m\lambda$$



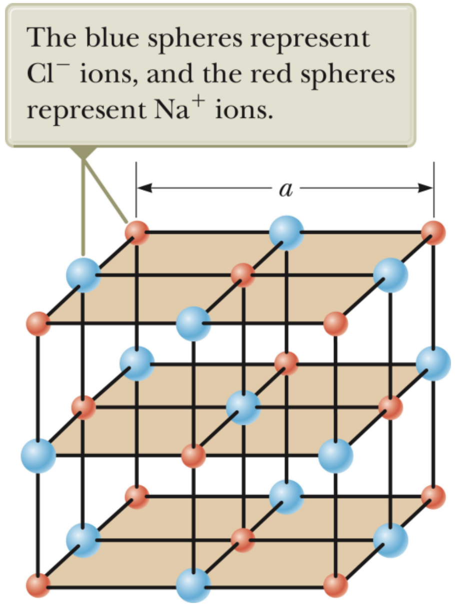
X-ray diffraction geometry:



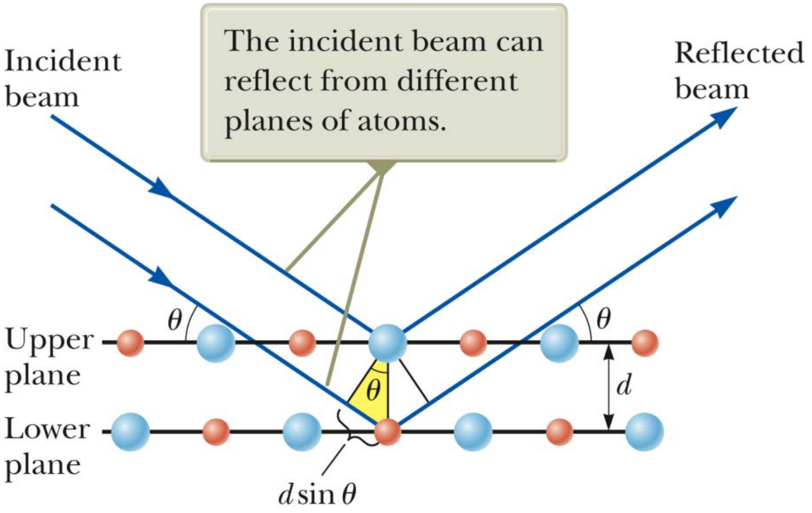
X - ray bright spots -- Bragg condition :

$$2d\sin\theta = m\lambda$$

Example of X-ray scattering for NaCl



The blue spheres represent Cl^- ions, and the red spheres represent Na^+ ions.



The incident beam can reflect from different planes of atoms.

Note that in this case $d=a/2$

Example of X-ray diffraction data at different temperatures

54

K. Homma et al. / Solid State Ionics 182 (2011) 53–58

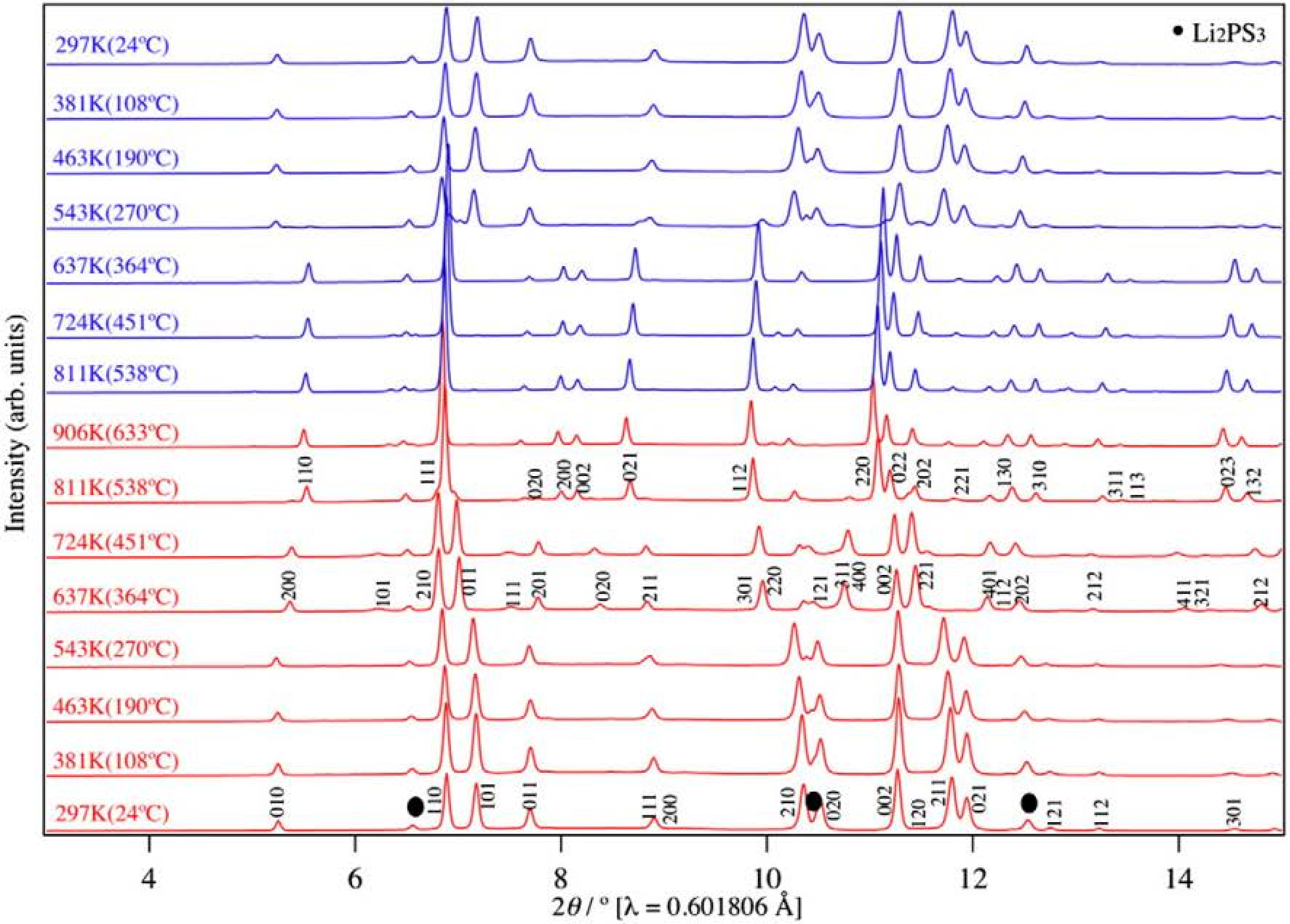
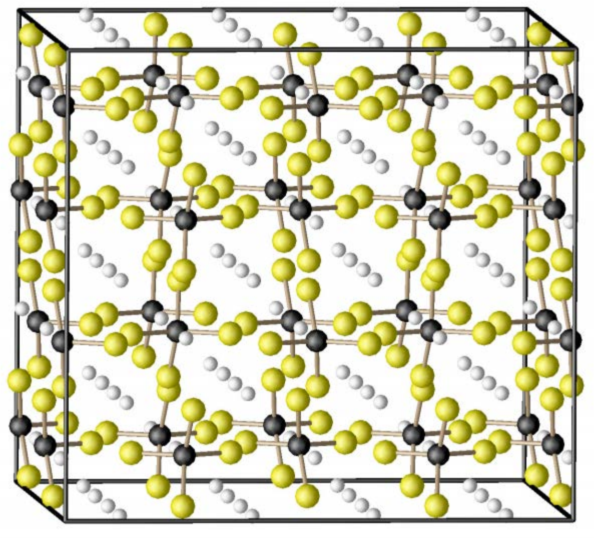


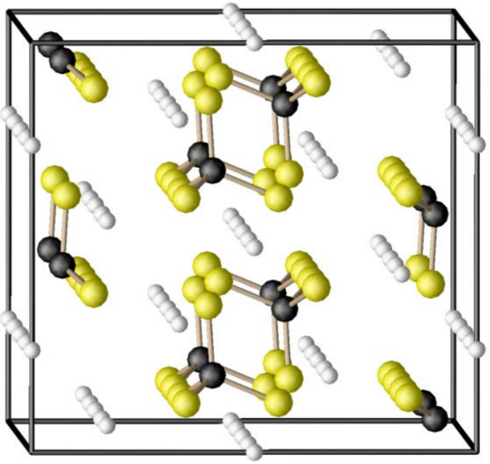
Fig. 1. High-temperature X-ray diffraction patterns of Li_3PS_4 synthesized at 773 K. The diffraction patterns were recorded both for the heating and cooling processes, from 297 K to 906 K and then 906 to 297 K, respectively.

Constituents of $Li_{10}GeP_2S_{12}$:



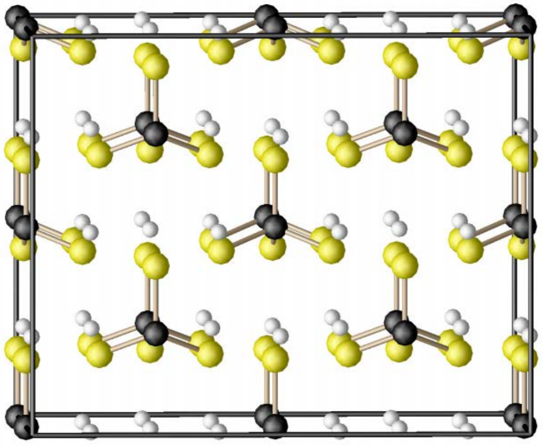
$\alpha^* \text{- } Li_3PS_4 \quad Pbcn$

$\Delta H = -8.12\ \text{eV}$



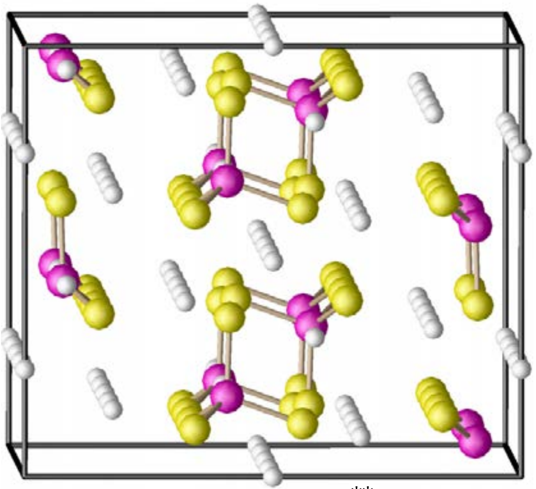
$\beta^* - Li_3PS_4 \quad Pnma$

$\Delta H = -8.28\ \text{eV}$



$\gamma^* - Li_3PS_4 \quad Pmn2_1$

$\Delta H = -8.36\ \text{eV}$



$Li_4GeS_4 \quad Pnma^{**}$

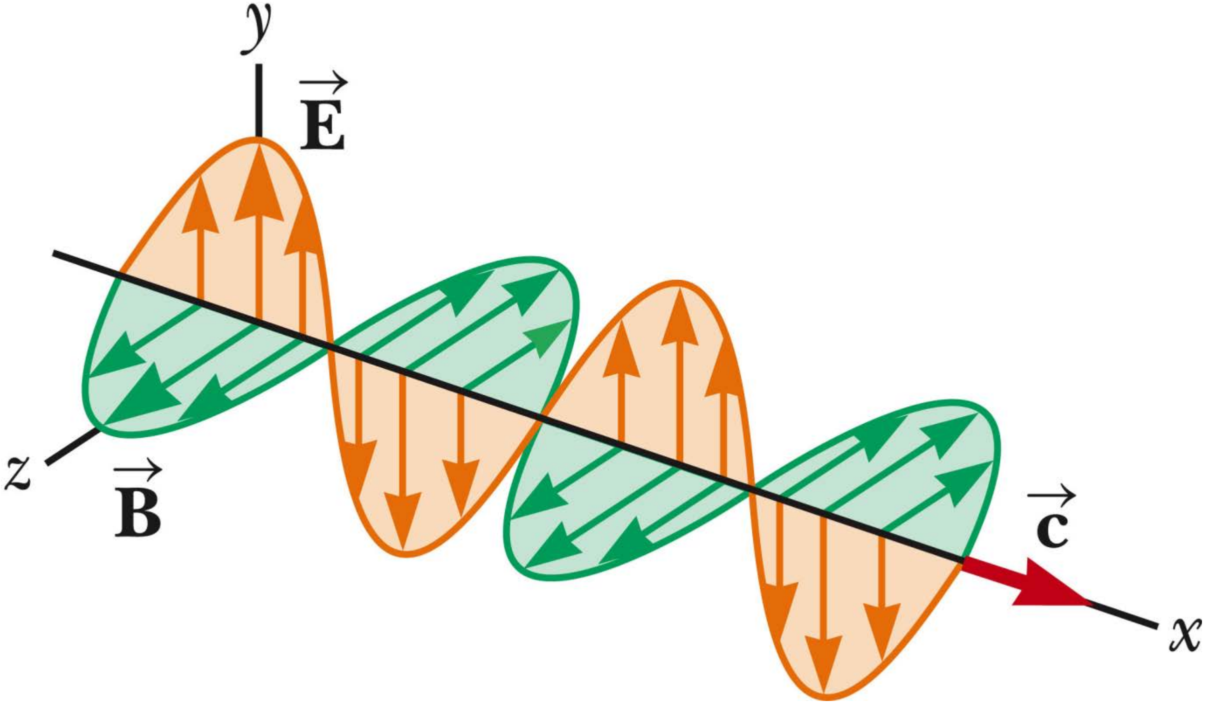
$\Delta H = -10.19\ \text{eV}$

● S
● Li
● P
● Ge

*K. Homma et al, *Solid State Ionics* **182**, 53-58 (2011)

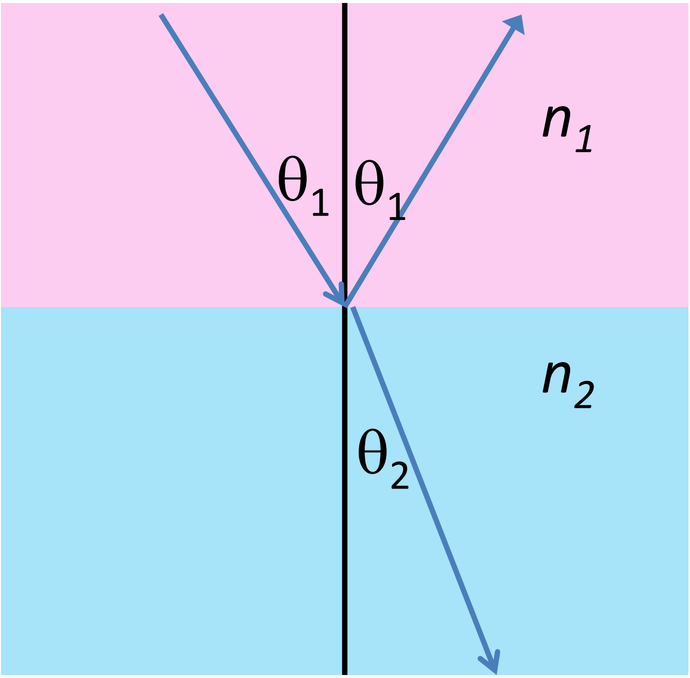
M. Murayama et al, *Solid State Ionics* **154-155, 789-794 (2002)

Polarization of light



In this case, **E** is polarized in y direction.

General case – reflection and refraction



For E polarized in scattering plane

$$\frac{E_2}{E_0} = \frac{2n_1 n_2 \cos\theta_1}{n_2^2 \cos\theta_1 + n_1 n_2 \cos\theta_2}$$

$$\frac{E_{1R}}{E_0} = \frac{n_2^2 \cos\theta_1 - n_1 n_2 \cos\theta_2}{n_2^2 \cos\theta_1 + n_1 n_2 \cos\theta_2}$$

For E polarized out of scattering plane

$$\frac{E_2}{E_0} = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

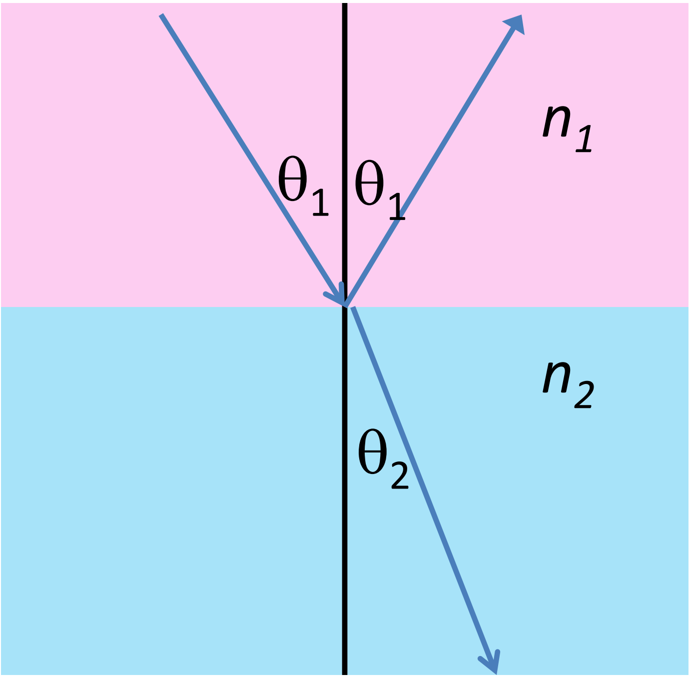
$$\frac{E_{1R}}{E_0} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

For $\theta_1 = 0 = \theta_2$

$$\frac{E_2}{E_0} = \frac{2n_1}{n_2 + n_1}$$

$$\frac{E_{1R}}{E_0} = \frac{n_2 - n_1}{n_2 + n_1}$$

General case – reflection and refraction



For E polarized in scattering plane

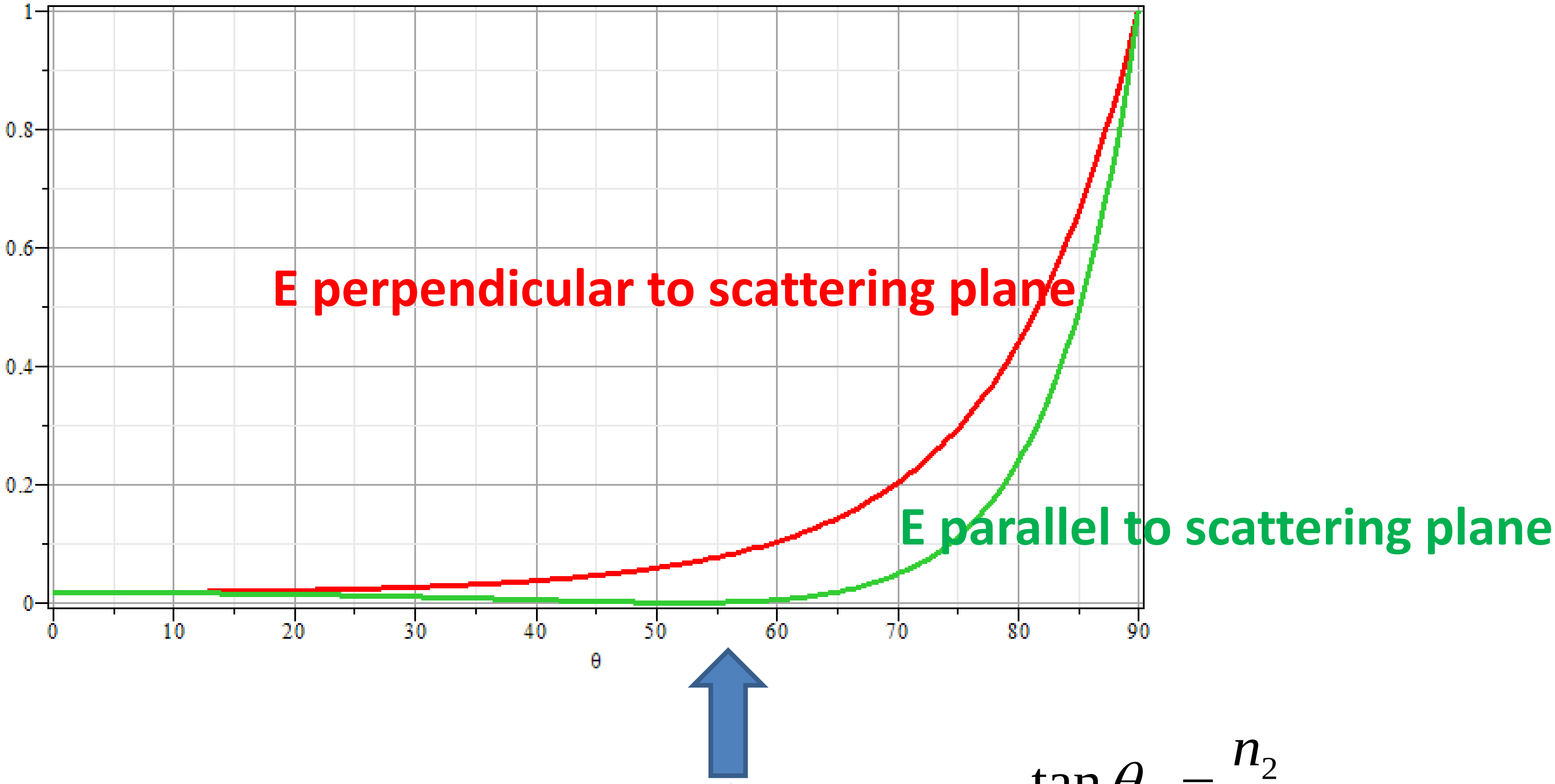
$$R=\left|\frac{E_{1R}}{E_0}\right|^2=\left|\frac{n_2\cos\theta_1-n_1\cos\theta_2}{n_2\cos\theta_1+n_1\cos\theta_2}\right|^2$$

For E polarized out of scattering plane

$$R=\left|\frac{E_{1R}}{E_0}\right|^2=\left|\frac{n_1\cos\theta_1-n_2\cos\theta_2}{n_1\cos\theta_1+n_2\cos\theta_2}\right|^2$$

Snell's law : $n_1\sin\theta_1=n_2\sin\theta_2$

Plot of reflectivity R versus θ



E perpendicular to scattering plane

E parallel to scattering plane

Brewster’s angle

$$\tan\theta_P = \frac{n_2}{n_1}$$