Plan for Lecture 22 (Chapter 38):

**Diffraction of light**

1. Diffraction gratings
2. X-ray diffraction
3. Other properties of light

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**Time, Einstein, and the Coolest Stuff in the Universe**

A free public lecture by Nobel Laureate

Dr. William Phillips
National Institute of Standards and Technology
8:00 PM Friday, April 20
Bren Dix Recital Hall
Wake Forest University

[www.wfu.edu/physics/sps/spszone52012conf/welcome.html](http://www.wfu.edu/physics/sps/spszone52012conf/welcome.html)

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Submit your feedback or report any issues on GitHub at [https://github.com/wfu-physics/phy-lec-notes](https://github.com/wfu-physics/phy-lec-notes)
3rd exam solutions
• Solutions posted on web
• Exam review session??
  Would you like to attend an exam review session?
  (A) yes
  (B) no
If you would like a review session, can you meet
  (A) Today Tuesday at 2 PM (here)
  (B) Today Tuesday at 5 PM Olin 107
  (C) Tomorrow Wed. at 1 PM Olin 107
  (D) Tomorrow Wed. at 2 PM Olin 107
  (E) Other?
• Similar problems may appear on final exam

Fourth exam – scheduled during the week of April 23
(evenings 6-10 PM) Note: Content mostly on material
in Chapters 35-38, optics, diffraction, plus possibly some
ideas of Quantum Mechanics and possibly some topics
from Exam 3
Which of these times are you likely to prefer:
A. Monday 4/23
B. Tuesday 4/24
C. Wednesday 4/25
D. Thursday 4/26
E. Friday 4/27

Wave phenomena associated with light
Plane polarized electromagnetic wave
at an instant of time:
\[ E_x \left( x, t \right) = E_{\text{max}} \sin \left( \frac{2\pi}{\lambda} (x - vt) + \phi \right) \]
Superposition of two electromagnetic waves (electric field portion)
\[ E_{\text{max}} \left( x, t \right) = E_x \left( x, t \right) + E_y \left( x, t \right) \]
\[ E_y \left( x, t \right) = E_{\text{max}} \sin \left( \frac{2\pi}{\lambda} (x - vt) + \frac{1}{2} \phi \right) \]
\[ = 2E_{\text{max}} \sin \left( \frac{2\pi}{\lambda} (x - vt) + \frac{1}{2} \phi \right) \cos \left( \frac{\phi}{2} \right) \]
Note that this result follows from the trigonometric identity:
\[
\sin(A) + \sin(B) = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)
\]
Squared magnitude:
\[
4 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)
\]
In our case:
\[
E_{\text{tot}}(x,t) = E_{\text{max}} \sin\left(\frac{2\pi}{\lambda}(x-\nu t)\right) + E_{\text{max}} \sin\left(\frac{2\pi}{\lambda}(x+\nu t)\right) + \phi
\]
\[
= 2E_{\text{max}} \sin\left(\frac{2\pi}{\lambda}(x-\nu t)\right) \cos\left(\frac{\phi}{2}\right)
\]
Intensity of the EM waves:
\[
I = S_{\text{ext}} = \frac{1}{\varepsilon_0} \left|E_{\text{tot}}^2\right|_{\text{eq}} = \frac{4}{2\varepsilon_0} \left|E_{\text{max}} \cos\left(\frac{\phi}{2}\right)\right| = I_{\text{max}} \cos^2\left(\frac{\phi}{2}\right)
\]

Example: \(\phi=0.5\) rad – plotting snapshot of EM wave

Example: \(\phi=3\) rad – plotting snapshot of EM wave

Intensity as a function of \(\phi\):

\[
E_{\text{tot}}(x,t) = E_{\text{max}} \sin\left[\frac{2\pi}{\lambda}(x-\nu t)\right] + E_{\text{max}} \sin\left[\frac{2\pi}{\lambda}(x+\nu t)\right] + \phi
\]
\[
= 2E_{\text{max}} \sin\left[\frac{2\pi}{\lambda}(x-\nu t)\right] \cos\left(\frac{\phi}{2}\right)
\]

What is the significance of this result?

A. No significance – only an evil physics professor could love such a result.
B. Shows that any two electromagnetic waves can interfere
C. Shows that two electromagnetic wave with the same amplitude, wavelength, and velocity can interfere if they have different phases \(\phi\)
Young’s double slit geometry:
Mathematical analysis of bright fringes:

\[ E(P,t) = E_{\text{max}} \sin \left( \frac{2\pi r_1}{\lambda} - 2\pi ft \right) + E_{\text{max}} \sin \left( \frac{2\pi r_2}{\lambda} - 2\pi ft \right) \]
\[ = 2E_{\text{max}} \sin \left( \frac{\pi (r_1 + r_2)}{\lambda} - 2\pi ft \right) \cos \left( \frac{\pi (r_1 - r_2)}{\lambda} \right) \]

\[ \Rightarrow \text{intensity maxima occur for} \quad \frac{\pi (r_1 - r_2)}{\lambda} = m\pi \quad \Rightarrow d \sin \theta = m\lambda \]

Diffraction pattern from a plane wave incident on a double slit:

\[ E(P,t) = E_{\text{max}} \sin \left( \frac{2\pi r_1}{\lambda} - 2\pi ft \right) + E_{\text{max}} \sin \left( \frac{2\pi r_2}{\lambda} - 2\pi ft \right) \]
\[ = 2E_{\text{max}} \sin \left( \frac{\pi (r_1 + r_2)}{\lambda} - 2\pi ft \right) \cos \left( \frac{\pi (r_1 - r_2)}{\lambda} \right) \]

\[ \Rightarrow \text{intensity maxima occur for} \quad \frac{\pi (r_1 - r_2)}{\lambda} = m\pi \quad \Rightarrow d \sin \theta = m\lambda \]

\[ I = I_{\text{max}} \cos \left( \frac{\pi \frac{d}{L} y}{\lambda} \right) \]

\[ E(P,t) = E_{\text{max}} \sin \left( \frac{2\pi r_1}{\lambda} - 2\pi ft \right) + E_{\text{max}} \sin \left( \frac{2\pi r_2}{\lambda} - 2\pi ft \right) \]
\[ = 2E_{\text{max}} \sin \left( \frac{\pi (r_1 + r_2)}{\lambda} - 2\pi ft \right) \cos \left( \frac{\pi (r_1 - r_2)}{\lambda} \right) \]
\[ r_2 - r_1 = \delta = d \sin \theta \quad y = L \tan \theta \approx L \sin \theta \]
Webassign hints:

Recall that the 2-slit intensity pattern has the form:

\[ I = I_{\text{max}} \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right) \]

- bright fringes when \( \frac{m \lambda}{d} \sin \theta = m \pi \) \( \Rightarrow \sin \theta = \frac{m \lambda}{d} \)
- dark fringes when \( \frac{m \lambda}{d} \sin \theta = (m + \frac{1}{2}) \pi \) \( \Rightarrow \sin \theta = \frac{(m + \frac{1}{2}) \lambda}{d} \)

In illustration case:

Constructive interference: \( 2nt = (m + \frac{1}{2}) \lambda \)

Destructive interference: \( 2nt = m \lambda \)
Light intensity patterns seen on screen for very thin double slit

Assuming that $d$ and $L$ are the same which plot corresponds to the greater wavelength?
A. $\lambda_2 > \lambda_1$
B. $\lambda_2 < \lambda_1$

Ideal infinitely thin 2-slit pattern

Finite thickness 2-slit pattern

$\lambda_1$
$\lambda_2$

$I = I_{\text{max}} \cos^2 \left( \frac{\pi \frac{a \lambda}{d L}}{2 \pi \frac{a \lambda}{d L}} \right)$

$d$ = slit separation
$a$ = slit width

Effects of diffraction when you may not want it – images of small objects near the “diffraction” limit
Enhancement of diffraction – diffraction gratings

\[ E(P,t) = \sum_j E_{\text{max}} \sin \left( \frac{2\pi r_j}{\lambda} - 2\pi g_j t \right) \]

\[ = E_{\text{max}} \sin \left( \frac{2\pi r_{\text{max}}}{\lambda} - 2\pi g t \right) \frac{\sin \left( N_d d \sin \theta \right)}{\sin \left( d \sin \theta \right)} \]

First-order diffraction between adjacent grating

Diffraction pattern for N slits – diffraction grating

\[ E(P,t) = \sum_j E_{\text{max}} \sin \left( \frac{2\pi r_j}{\lambda} - 2\pi g_j t \right) \]

\[ = E_{\text{max}} \sin \left( \frac{2\pi r_{\text{max}}}{\lambda} - 2\pi g t \right) \frac{\sin \left( N_d d \sin \theta \right)}{\sin \left( d \sin \theta \right)} \]

Intensity maxima at \[ d \sin \theta = m\lambda \]

Intensity pattern from multiple slit grating:

\[ I = I_{\text{max}} \left[ \frac{\sin \left( N_d d \sin \theta \right)}{\sin \left( \frac{\pi d \sin \theta}{\lambda} \right)} \right]^2 \]

For \( N = 2, 5, 10 \):
Sanity check:
If the intensity pattern for \( N \) slits is given by
\[
I = I_{\text{max}} \left( \frac{\sin \left( \frac{N \pi d \sin \theta}{\lambda} \right)}{\sin \left( \frac{\pi d \sin \theta}{\lambda} \right)} \right)^2
\]
what happened to the formula two-slit intensity pattern?
A. NEVER trust your physics professor
B. More evidence that physics does not make sense
C. The formula look different, but are equivalent for \( N=2 \)

\[
I = I_{\text{max}} \left( \frac{\sin \left( \frac{N \pi d \sin \theta}{\lambda} \right)}{\sin \left( \frac{\pi d \sin \theta}{\lambda} \right)} \right)^2
\]
For \( N=2 \):
\[
I = I_{\text{max}} \left( \frac{\sin \left( \frac{2 \pi d \sin \theta}{\lambda} \right)}{\sin \left( \frac{\pi d \sin \theta}{\lambda} \right)} \right)^2
= I_{\text{max}} \left( \frac{2 \sin(\varphi) \cos(\varphi)}{\sin(\varphi)} \right)^2
= I_{\text{max}} \cos^2(\varphi)
= I'_{\text{max}} \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right)
\]

Consider the 4 plots which represent the intensity of monochromatic light on a screen a large distance away from various slit configurations. For each of the plots identify the type of slits — thin double slits, fat double slits, thin multiple slits, fat multiple slits, 1 thin slit, 1 fat slit, etc.
Recall form of diffraction pattern for grating:

\[ I = I_{\text{max}} \left( \frac{\sin \left( \frac{\lambda \sin \theta}{d} \right)}{\sin \frac{\lambda \sin \theta}{d}} \right)^2 \Rightarrow \text{maxima when } d \sin \theta = m\lambda \]

X-ray diffraction geometry:

The incident beam can reflect from different planes of atoms.

\[ 2d \sin \theta = m\lambda \]
Note that in this case $d = a/2$
In this case, \( \mathbf{E} \) is polarized in \( y \) direction.

**General case – reflection and refraction**

For \( \mathbf{E} \) polarized in scattering plane

\[
\begin{align*}
E_1 &= \frac{2n_1 n \cos \theta_1}{E_0} \\
E_2 &= \frac{n_1}{n_1 + n_2} E_0 \\
E_4 &= \frac{n_2}{n_1 + n_2} E_0 \\
\end{align*}
\]

For \( \theta_1 = 0 = \theta_2 

\[
\begin{align*}
E_1 &= \frac{n_1 \cos \theta_1}{E_0} \\
E_2 &= \frac{n_1}{n_1 + n_2} E_0 \\
E_4 &= \frac{n_2}{n_1 + n_2} E_0 \\
\end{align*}
\]

For \( E \) polarized out of scattering plane

\[
\begin{align*}
E_1 &= \frac{n_1 \cos \theta_1}{E_0} \\
E_2 &= \frac{n_1}{n_1 + n_2} E_0 \\
E_4 &= \frac{n_2}{n_1 + n_2} E_0 \\
\end{align*}
\]

Snell’s law : \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \)
Plot of reflectivity $R$ versus $\theta$

$E$ perpendicular to scattering plane

$E$ parallel to scattering plane

Brewster’s angle $\tan \theta_p = \frac{n_2}{n_1}$