## PHY 114 A General Physics II
## 11 AM-12:15 PM TR Olin 101

**Plan for Lecture 23 (Chapter 40-42):**

**Some topics in Quantum Theory**

1. **Particle behaviors of electromagnetic waves**
2. **Wave behaviors of particles**
3. **Quantized energies**



| | | | | | |
|---|---|---|---|---|---|
| 13 | 03/08/2012 | Faraday's law | 31.1-31.5 | 31.12,31.23,31.40 | 03/20/2012 |
| | 03/13/2012 | *No class (Spring Break)* | | | |
| | 03/15/2012 | *No class (Spring Break)* | | | |
| 14 | 03/20/2012 | Induction and AC circuits | 32.1-32.6 | 32.4,32.20,32.43 | 03/22/2012 |
| 15 | 03/22/2012 | AC circuits | 33.1-33.9 | 33.8,33.24,33.71 | 03/27/2012 |
| 16 | 03/27/2012 | Electromagnetic waves | 34.1-34.3 | 34.3,34.10,34.13 | 03/29/2012 |
| 17 | 03/29/2012 | Electromagnetic waves | 34.4-34.7 | 34.22,34.46,34.57 | 04/03/2012 |
| 18 | 04/03/2012 | Ray optics Evening exam | 35.1-35.8 | 35.20,35.27,35.35 | 04/10/2012 |
| 19 | 04/05/2012 | Image formation Evening exam | 36.1-36.4 | 36.8,36.31,36.42 | 04/10/2012 |
| 20 | 04/10/2012 | Image formation | 36.5-36.10 | 36.52,36.54,36.64 | 04/12/2012 |
| 21 | 04/12/2012 | Wave interference | 37.1-37.6 | 37.2,37.19,37.29 | 04/17/2012 |
| 22 | 04/17/2012 | Diffraction | 38.1-38.6 | 38.24,38.30,38.37 | 04/19/2012 |
| 23 | 04/19/2012 | Quantum Physics | 40.1-42.10 | 40.41,41.12,42.10 | 04/24/2012 |
| 24 | 04/24/2012 | Molecules and solids Evening exam | 43.1-43.8 | 43.2,43.40,43.43 | 05/01/2012 |
| 25 | 04/26/2012 | Nuclear reactions Evening exam | 45.1-45.4 | 45.6,45.20,45.30 | 05/01/2012 |
| 26 | 05/01/2012 | Nuclear radiation | 45.5-45.7 | | |
| | 05/08/2012 | Final exam 9 AM | | | |



**Time, Einstein, and the Coolest Stuff in the Universe**

A free public lecture by Nobel Laureate

**Dr. William Phillips**

National Institute of Standards and Technology

**8:00 PM Friday, April 20**

**Brendle Recital Hall**
**Wake Forest University**

Part of SPS zone 5 conference
April 20-21, 2012

Offer 1 point extra credit for attendance*

*After the lecture, email me that you attended. In the following email exchange you will be asked to answer one question about the lecture.

**www.wfu.edu/physics/sps/spszone52012conf/welcome.html**

Webassign hint:

–/0.333 points | My Notes | SerPSE8 38.P.030.M

The hydrogen spectrum includes a red line at 656 nm and a blue-violet line at 434 nm. What are the angular separations between these two spectral lines for all visible orders obtained with a diffraction grating that has 4 160 grooves/cm? (In this problem assume that the light is incident normally on the gratings.)

first order separation ____ °

second order separation ____ °

third order separation ____ °

$$d \sin\theta = m\lambda$$

$$\text{For } N = 4160 \text{ grooves/cm, } d = \frac{1}{N}$$

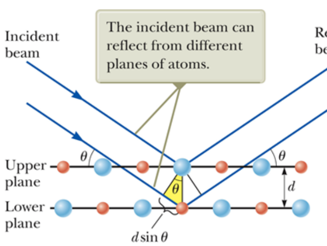


Webassign hint:

4. –/0.333 points | My Notes | SerPSE8 38.P.037.

Potassium iodide (KI) has the same crystalline structure as NaCl, with atomic planes separated by 0.353 nm. A monochromatic x-ray beam shows a first-order diffraction maximum when the grazing angle is 7.50°. Calculate the x-ray wavelength. (Assume first order.)

____ nm




Condition for bright spot :

$$2d \sin\theta = m\lambda = \lambda$$



**If you have not already done so – please reply to my email concerning your intentions regarding Exam 4.**

The material you have learned up to now in PHY 113 & 114 was known in 1900 and is basically **still true**. Some details (such as at high energy, short times, etc. ) have been modified with Einstein's theory of relativity, and with the development of quantum theory.

Which of the following technologies do **not** need quantum mechanics.

A. X-ray diffraction
B. Neutron diffraction
C. Electron microscope
D. MRI (Magnetic Resonance Imaging)
E. Lasers

Which of the following technologies do **not** need quantum mechanics.

A. Scanning tunneling microscopy
B. Atomic force microscopy
C. Data storage devices
D. Microwave ovens
E. LED lighting



Image of Si atoms on a nearly perfect surface at T=7 K.

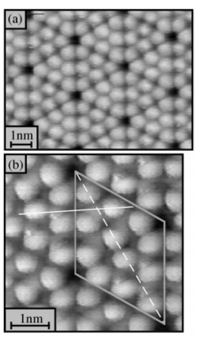



Image made using atomic force microscopy.

From ***Physical Review Letters*** March 20, 2000 -- Volume 84, Issue 12, pp. 2642-2645



Quantum physics –

- Electromagnetic waves sometimes behave like particles
  - → one "photon" has a quantum of energy $E=hf$
    momentum $p=h/\lambda=hf/c$
- Particles sometimes behave like waves
  - → "wavelength" of particle related to momentum:
    $\lambda=h/p$
  - → quantum particles can "tunnel" to places classically "forbidden"
  - → Stationary quantum states have quantized energies

Classical physics

Wave equation for electric field in Maxwell's equations (plane wave boundary conditions):

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} = c^2 \frac{\partial^2 \mathbf{E}}{\partial x^2} \quad \text{for example}: \mathbf{E}(x,t) = E_{\max}\hat{\mathbf{j}}\sin\left(k\left(x-ct\right)\right)$$

Equation for particle trajectory ***r(t)*** in conservative potential *U(**r**)* and total energy *E*

$$\frac{1}{2}m\left(\frac{d\mathbf{r}}{dt}\right)^2 + U\left(\mathbf{r}\right) = E$$

$$\text{for example}: \quad \mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}_0 t - \frac{1}{2}g\hat{\mathbf{k}}t^2$$



Particle ←→wave properties in classical physics

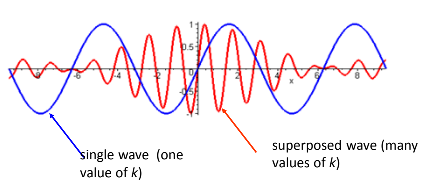
| Particle properties | Wave properties |
|---|---|
| Position as a function of time is known -- **r**(t) | Phonomenon is spread out over many positions at an instant of time. |
| Particle is spatially confined when E≤U(r). | Notion of spatial confinement non-trivial. |
| Particles are independent. | Interference effects. |



Mathematical representation of particle and wave behaviors.

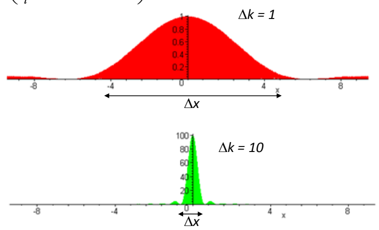
Consider a superposition of periodic waves at *t=0:*

$$E(x,t) = \sum_i E_{\max}\sin\left(k_i x\right)$$



$$[E(x,0)]^2 = \left(\sum_i E_{\max} \sin(k_i x)\right)^2$$

$\Delta x \, \Delta k \approx 2\pi$




$\Delta$x smaller ➔ more particle like

$\Delta$k smaller ➔ more wave like



$\Delta x \, \Delta k \approx 2\pi$ ➔ Heisenberg's uncertainty principle

De Broglie's particle moment – wavelength relation:

$$p = \frac{h}{\lambda} = \frac{h/2\pi}{\lambda/2\pi} = \hbar k$$

Heisenberg's hypotheses: $\Delta x \Delta p \geq \frac{\hbar}{2}$

$$\Delta t \Delta E \geq \frac{\hbar}{2}$$

$h = 6.6\times10^{-34}$ Js = $4.14\times10^{-15}$ eVs



Wave equations

Electromagnetic waves:

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} = c^2 \frac{\partial^2 \mathbf{E}}{\partial x^2}$$

Matter waves: (Schrödinger equation)

$$-i\hbar \frac{\partial}{\partial t} \Psi(x,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)\right]\Psi(x,t)$$

Comparison of different wave equations

| Electromagnetic waves | Matter waves |
|---|---|
| Vector – **E** or **B** fields<br>Second order *t* dependence<br>Examples:<br>$E_y(x,t) = E_{\max}\sin(kx-\omega t)$<br>$B_z(x,t) = \frac{E_{\max}}{c}\sin(kx-\omega t)$ | Scalar – probability amplitude<br>First order *t* dependence<br>Examples:<br>$\Psi(x,t) = \Psi_0\sin(kx)e^{-iEt/\hbar}$<br>$\Psi(r,t) = \frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}e^{-iE_0 t/\hbar}$<br>$E_0 = -\frac{e^2}{8\pi\varepsilon_0 a_0}$ |

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What is the meaning of the matter wave function $\Psi(x,t)$?

- $\Psi(x,t)$ is not directly measurable
- $|\Psi(x,t)|^2$ is measurable – represents the density of particles at position *x* at time *t*.
- For a single particle system – represents the probability of measuring particle at position *x* at time *t*.
- For many systems of interest, the wave function can be written in the form $\Psi(x,t) = \psi(x)e^{-iEt/}$

$$|\Psi(x,t)|^2 = |\psi(x)|^2$$

$$\int_{-\infty}^{\infty}|\Psi(x,t)|^2 dx = 1$$

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Wave-like properties of particles

Louis de Broglie suggested that a wavelength could be associated with a particle's momentum

$$p = \frac{h}{\lambda} \Rightarrow -i\frac{h}{2\pi}\frac{\partial}{\partial x} \equiv -i\hbar\frac{\partial}{\partial x}$$

"Wave" equation for particles – Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)\right]\Psi(x,t) = -i\frac{h}{2\pi}\frac{\partial}{\partial t}\Psi(x,t)$$

Stationary-state wavefunctions: $\Psi(\mathbf{r},t) = \psi(\mathbf{r})e^{-iEt/\hbar}$

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)\right]\Psi(x,t) = E\Psi(x,t)$$

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Example -- free particle -- $U(r)=0$: $\Psi(\mathbf{r},t)=\psi(\mathbf{r})e^{-iEt/\hbar}$

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right]\Psi(x,t)=E\Psi(x,t)$$

$$\Psi(x,t)=\Psi_0 \sin(kx)e^{-iEt/\hbar}$$

$$E=\frac{\hbar^2 k^2}{2m}$$

$$k=\frac{2\pi}{\lambda} \Rightarrow \lambda=\frac{h}{\sqrt{2mE}} \quad \text{or} \quad E=\frac{h^2}{2m\lambda^2}$$

Example: Suppose we want to create a beam of electrons ($m$=9.1x10$^{-31}$kg) for diffraction with $\lambda$=1x10$^{-10}$m. What is the energy $E$ of the beam?

$$E=\frac{h^2}{2m\lambda^2}=\frac{\left(6.6\times10^{-34}J\right)^2}{2\cdot9.1\times10^{-31}kg\cdot\left(10^{-10}m\right)^2}=2.4\times10^{-17}J=150eV$$

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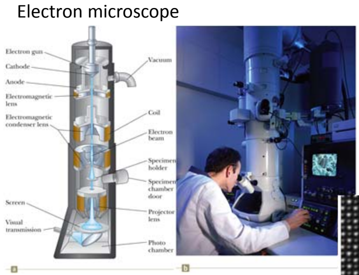
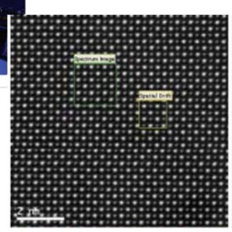
Electron microscope




Typically E=120,000-200,000 eV for high resolution EM



From Microscopy Today article May 2009

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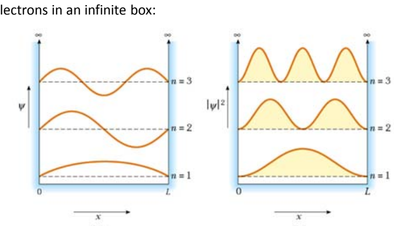
Electrons in an infinite box:




$$E\,\psi(x)=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right]\psi(x) \quad \text{for } 0\le x\le L$$

$$\psi(x)=\psi_0\sin\left(\frac{n\pi x}{L}\right) \qquad n=1,2,3\cdots \qquad E_n=\frac{\hbar^2\pi^2 n^2}{2m}$$

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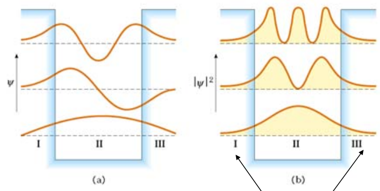
Electrons in a finite box:




finite probability of electron existing outside of classical region

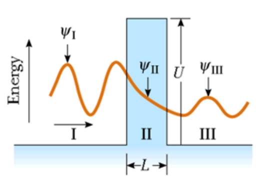


Why would it be interesting to study electrons in a finite box?

A. It isn't
B. It is the mathematically most simple example of quantum system
C. Quantum well systems can be manufactured to design new devices



Tunneling of electrons through a barrier

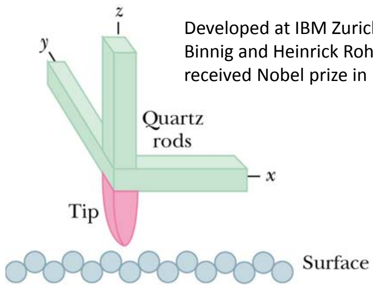



surface region

tip

vacuum

How a scanning tunneling microscope works:

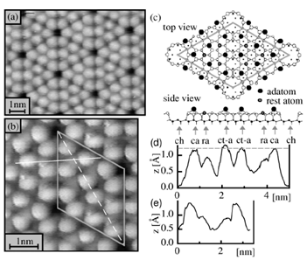



Developed at IBM Zurich by Gerd Binnig and Heinrick Rohrer who received Nobel prize in 1986.



Visualization of $|\psi(x)|^2$

A surface if a nearly perfect Si crystal




Physical Review Letters -- March 20, 2000 -- Volume 84, Issue 12, pp. 2642-2645



The physics of atoms –

Features are described by solutions to the matter wave equation – Schrödinger equation:

$$-i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\mathbf{r}^2}+U(\mathbf{r})\right]\Psi(\mathbf{r},t)$$

"reduced" mass of electron and proton

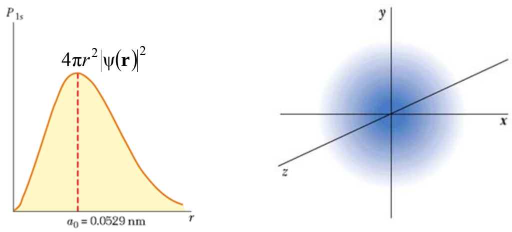
$$-\frac{Ze^2}{4\pi\varepsilon_0 r}$$

Stationary-state wavefunctions: $\Psi(\mathbf{r},t)=\psi(\mathbf{r})e^{-iEt/\hbar}$

Solutions: $E_n=-\frac{Z^2e^2}{8\pi\varepsilon_0 a_0}\frac{1}{n^2}=-13.6\frac{Z^2}{n^2}\,\text{eV}$

$$a_0=\frac{4\pi\varepsilon_0\hbar^2}{me^2}=0.0529\,\text{nm}$$

Form of probability density for ground state ($n$ = 1)





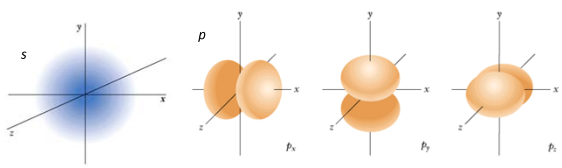

Angular degrees of freedom

-- since the force between the electron and nucleus depends only on distance and not on angle, angular momentum **L ≡ r x p** is conserved. Quantum numbers associated with angular momentum:

$$\mathbf{L}^2 = \hbar^2 \ell(\ell+1) \qquad \ell = 0,1,2,....(n-1)$$

$$L_z = \hbar m \qquad -\ell \le m \le \ell \quad \text{total of } 2\ell+1 \text{ states}$$

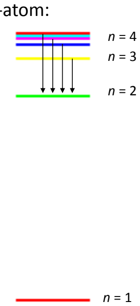
Notation: $\ell = 0 \Rightarrow s, \quad 1 \Rightarrow p, \quad 2 \Rightarrow d$






Summary of results for H-atom:

$$E_n = -13.6\frac{Z^2}{n^2}\text{eV}$$




Balmer series spectra

degeneracy associated with each $n$: $2n^2$

Atomic states of atoms throughout periodic table:

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 H | | | | | | | | | | | | | | | | | 2 He |
| 3 Li | 4 Be | | | | | | | | | | | 5 B | 6 C | 7 N | 8 O | 9 F | 10 Ne |
| 11 Na | 12 Mg | | | | | | | | | | | 13 Al | 14 Si | 15 P | 16 S | 17 Cl | 18 Ar |
| 19 K | 20 Ca | 21 Sc | 22 Ti | 23 V | 24 Cr | 25 Mn | 26 Fe | 27 Co | 28 Ni | 29 Cu | 30 Zn | 31 Ga | 32 Ge | 33 As | 34 Se | 35 As | 36 Kr |
| 37 Rb | 38 Sr | 39 Y | 40 Zr | 41 Nb | 42 Mo | 43 Tc | 44 Ru | 45 Rh | 46 Pd | 47 Ag | 48 Cd | 49 In | 50 Sn | 51 Sb | 52 Te | 53 I | 54 Xe |
| 55 Cs | 56 Ba | 57 La | 72 Hf | 73 Ta | 74 W | 75 Re | 76 Os | 77 Ir | 78 Pt | 79 Au | 80 Hg | 81 Tl | 82 Pb | 83 Bi | 84 Po | 85 At | 86 Rn |
| 87 Fr | 88 Ra | 89 Ac | | | | | | | | | | | | | | | |

$$E\psi(\mathbf{r}) = \left[ -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} + U(\mathbf{r}) \right]\psi(\mathbf{r})$$

effective potential for an electron in atom



Example: Cu (Z=29) $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^1$