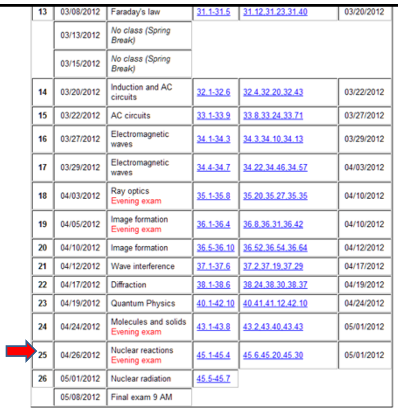
PHY 114 A General Physics II
11 AM-12:15 PM TR Olin 101

Plan for Lecture 25 (Chapters 44-45):

Some topics in nuclear physics

1. **Nuclear binding energies**
2. **Radioactivity**
3. **Nuclear reactions**

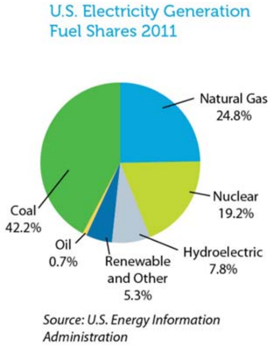


13	03/08/2012	Faraday's law	31.1-31.5	31.12,31.23,31.40	03/20/2012
	03/13/2012	*No class (Spring Break)*			
	03/15/2012	*No class (Spring Break)*			
14	03/20/2012	Induction and AC circuits	32.1-32.6	32.4,32.20,32.43	03/22/2012
15	03/22/2012	AC circuits	33.1-33.9	33.8,33.24,33.71	03/27/2012
16	03/27/2012	Electromagnetic waves	34.1-34.3	34.3,34.10,34.13	03/29/2012
17	03/29/2012	Electromagnetic waves	34.4-34.7	34.22,34.46,34.57	04/03/2012
18	04/03/2012	Ray optics Evening exam	35.1-35.8	35.20,35.27,35.35	04/10/2012
19	04/05/2012	Image formation Evening exam	36.1-36.4	36.8,36.31,36.42	04/10/2012
20	04/10/2012	Image formation	36.5-36.10	36.52,36.54,36.64	04/12/2012
21	04/12/2012	Wave interference	37.1-37.6	37.2,37.19,37.29	04/17/2012
22	04/17/2012	Diffraction	38.1-38.6	38.24,38.30,38.37	04/19/2012
23	04/19/2012	Quantum Physics	40.1-42.10	40.41,41.12,42.10	04/24/2012
24	04/24/2012	Molecules and solids Evening exam	43.1-43.8	43.2,43.40,43.43	05/01/2012
25	04/26/2012	Nuclear reactions Evening exam	45.1-45.4	45.6,45.20,45.30	05/01/2012
26	05/01/2012	Nuclear radiation	45.5-45.7		
	05/08/2012	Final exam 9 AM			

What do you think of when you hear the phrase nuclear reaction?

A. Clean energy source
B. Radiation danger
C. Nuclear weapons
D. No opinion

http://www.nei.org

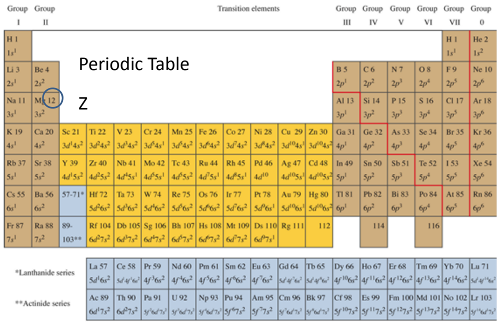


Source: U.S. Energy Information Administration

Properties of Nuclei

Z Atomic number (number of protons) N Neutron number

A = Z+N (number of nucleons)



Constituents of the nucleus

TABLE 44.1 *Masses of Selected Particles in Various Units*

Particle	Mass kg	u	MeV/c^2
Proton	$1.672\,62 \times 10^{-27}$	1.007 276	938.27
Neutron	$1.674\,93 \times 10^{-27}$	1.008 665	939.57
Electron	$9.109\,38 \times 10^{-31}$	$5.485\,79 \times 10^{-4}$	0.510 999
$^1_1$H atom	$1.673\,53 \times 10^{-27}$	1.007 825	938.783
$^4_2$He nucleus	$6.644\,66 \times 10^{-27}$	4.001 506	3 727.38
$^{12}_{6}$C atom	$1.992\,65 \times 10^{-27}$	12.000 000	11 177.9

Example for proton :

$$mc^2 = 1.67263\times10^{-27}(299792458)^2 = 1.50\times10^{-10}J$$
$$= 938.27MeV$$

Nuclear notation:

$^{A}_{Z}\mathrm{X}$

Z Atomic number (number of protons)A =
A Nucleon number

Some examples:

$^{56}_{26}\mathrm{Fe}$ 91.754%

$^{238}_{92}\mathrm{U}$ 99.275%

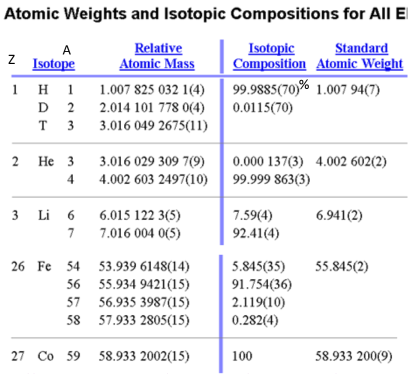
Natural abundance

Some isotopes of C:

$^{12}_{6}\mathrm{C}$ 98.93%

$^{13}_{6}\mathrm{C}$ 1.07%

Natural abundance



Atomic Weights and Isotopic Compositions for All E

Z	Isotope	A	Relative Atomic Mass	Isotopic Composition	Standard Atomic Weight
1	H	1	1.007 825 032 1(4)	99.9885(70)%	1.007 94(7)
	D	2	2.014 101 778 0(4)	0.0115(70)	
	T	3	3.016 049 2675(11)		
2	He	3	3.016 029 309 7(9)	0.000 137(3)	4.002 602(2)
		4	4.002 603 2497(10)	99.999 863(3)	
3	Li	6	6.015 122 3(5)	7.59(4)	6.941(2)
		7	7.016 004 0(5)	92.41(4)	
26	Fe	54	53.939 6148(14)	5.845(35)	55.845(2)
		56	55.934 9421(15)	91.754(36)	
		57	56.935 3987(15)	2.119(10)	
		58	57.933 2805(15)	0.282(4)	
27	Co	59	58.933 2002(15)	100	58.933 200(9)

Ref: http://www.physics.nist.gov/PhysRefData/Compositions/index.html

mass of neutral atom ➔ includes Z electrons

Z		A			
27	Co	59	58.933 2002(15)	100	58.933 200(9)

Mass of nucleus:

$M_{sum} = Zm_p + (A\text{-}Z)m_n + Zm_e = 59.474281$ u

$\Delta M = 0.5410806$ u

What should we do with this mass deficit?

(A) Chalk it up to inaccuracy of my calculator.

(B) Figure that NIST made a mistake.

(C) Give up on physics as a quantitative science.

(D) Find some meaning associated with ΔM.

mass of neutral atom ➔ includes Z electrons

Z		A			
27	Co	59	58.933 2002(15)	100	58.933 200(9)

Mass of nucleus:

$M_{sum} = Zm_p + (A-Z)m_n + Zm_e = 59.474281$ u

$\Delta M = 0.5410806$ u $= 504.0135$ MeV $/c^2$

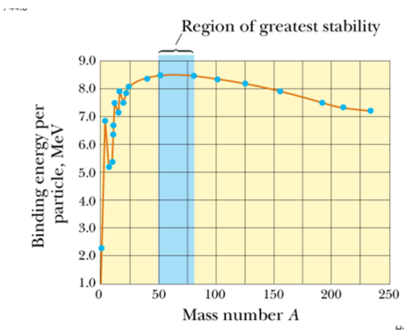
$\Delta M/A = 8.5426$ MeV $/c^2$ /nucleon ➔ energy associated with nuclear "binding"

Another example:

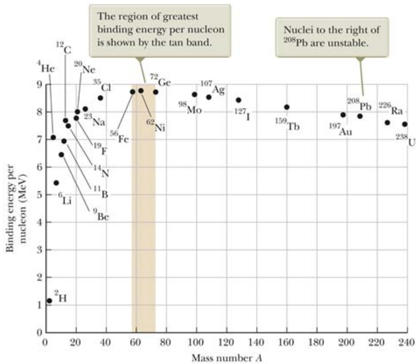
2	He	3	3.016 029 309 7(9)	0.000 137(3)	4.002 602(2)
		4	4.002 603 2497(10)	99.999 863(3)	

$\Delta M/A(^4_2He) = 7.1$ MeV/c^2/nucleon

$\Delta M/A(^3_2He) = 2.6$ MeV/c^2/nucleon



Nuclear binding energies per nucleon


The region of greatest binding energy per nucleon is shown by the tan band.

Nuclei to the right of ^{208}Pb are unstable.

Nuclear binding energy :

$$^{A}_{Z}X \rightarrow Z(H) + (A-Z)(n)$$

Neutral atom with Z electrons and Z protons

Neutral H atom

Neutrons

Nuclear forces are so strong that within the nucleus, mass can be converted to energy and visa versa: E = mc^2 .

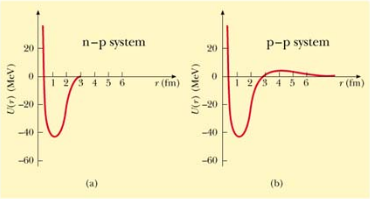
Nuclear binding energy :

$$E_b = -\left[M(^{A}_{Z}X) - Z \cdot M(^{1}_{1}H) - (A-Z)m_n\right]c^2$$

Example $^{2}_{1}\mathrm{H}$:

$$\begin{aligned} E_b &= -\left[M(^{2}_{1}X) - Z \cdot M(^{1}_{1}H) - (A-Z)m_n\right]c^2 \\ &= -[2.014102 - 1.007825 - 1.008665]u \cdot 931.494 MeV/u \\ &= 2.2\ MeV \end{aligned}$$

➔There must be a strong attraction between nuclear particles



1 fm = 10^{-15}m

Not all nuclei are stable:

Some types of nuclear decay

$\alpha \equiv {}^{4}_{2}\mathrm{He}^{++}$ Ex: ${}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + \alpha$

$\beta \equiv e^{-}$ or e^{+} Ex: ${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + e^{-}$

$\gamma \equiv$ (photon) Ex: ${}^{12}_{5}\mathrm{B} \rightarrow {}^{12}_{6}\mathrm{C}^{*} + e^{-} + \bar{\nu}$

Measure of radioactive decays:

$$\text{Decay rate}: \quad \frac{dN}{dt} = -\lambda N$$

$$\text{Solution}: \quad N(t) = N_0 e^{-\lambda t}$$

$$\left|\frac{dN}{dt}(t)\right| = N_0 \lambda e^{-\lambda t}$$

Half-life:

When N(t) = ½ N_0:

$$N(T_{1/2}) = N_0 e^{-\lambda T_{1/2}} \equiv \tfrac{1}{2} N_0$$

$$\Rightarrow T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.6931472}{\lambda}$$

$$\text{Note that}: \; N(t) = N_0 \left(\frac{1}{2}\right)^{\left(t/T_{1/2}\right)}$$

Some units of nuclear decay:

1 Ci (Curie) ≡ 3.7 x 10^{10} decays/s

1 Bq (Becquerel) ≡ 1 decay/s

Example:

Suppose that you have a sample of 10^{23} ${}^{14}_{6}\mathrm{C}$ nuclei each of which has a half-life of 5730 years. How many Curies of radiation is this?

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.6931472}{\lambda}$$

$$\lambda = \frac{0.6931472}{T_{1/2}} = 3.8 \times 10^{-12}\,\text{decays/s}$$

$$\left|\frac{dN}{dt}\right| = \lambda N = 3.8 \times 10^{-12} \times 10^{23}\,\text{decays/s} = 10.4\,\text{Ci}$$

Suppose you have a sample of 6.02 x 10^{23} atoms of ^{97}Tc which has a half-life of 2.6 x 10^6 years.

1. How many Curies does this sample have?

$$\frac{dN}{dt} = \frac{\ln 2}{T_{1/2}} N_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}} = \frac{\ln 2}{T_{1/2}} N$$

$$= \frac{\ln 2 \cdot 6.02 \times 10^{23}}{2.6 \times 10^6 \times 3.16 \times 10^7 s} \frac{1Ci}{3.7 \times 10^{10} / s}$$

$$= 0.137 Ci$$

2. How long will it take before the sample has only 3.01 x 10^{23} ^{97}Tc atoms left?
   - (a) 1.3 x 10^6 years
   - (b) 2.6 x 10^6 years
   - (c) 5.2 x 10^6 years
   - (d) 26 x 10^6 years
   - (e) None of these.

Effects of radiation in matter

Release of energetic particles –

α, n, p → move atoms in materials

e⁻, e⁺, γ → remove or excite electrons

→ cause chemical reactions

Quantitative measures of radiation dose

rad (“radiation absorbed dose”): amount of radiation that increases the energy of 1kg of absorbed materials by 0.01 J.

gray (Gy): 1 Gy = 100 rad

RBE (“relative biological effectiveness” factor): ratio of biological damage of radiation type to that of γ rays

rem (“radiation equivalent in man”): rad x RBE

sievert (Sv): 1 Sv = 100 rem

Some RBE factors

Radiation type	RBE
γ rays	1
β particles	1-1.7
n (slow)	4-5
n & p (fast)	10
α particles	10-20
heavy ions	20

Some typical values of dose

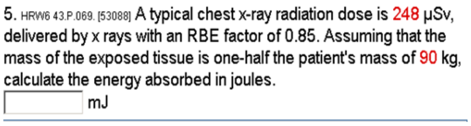
Source	Dose
Background radiation	0.13 rem/year
Recommended limit	0.5 rem/year
Diagnostic chest X-ray	0.01 rem
Mammogram	0.1 rem
Fatal dose	400-500 rem

Summary of radiation units:

Radiation dose: Amount of radiation absorbed/unit mass

1 Gy = 1 J/kg of absorbed radiation = 100 rad

$$1\text{ Sv} = \underbrace{\text{RBE}}_{\text{relative biological effectiveness}} \text{ x (dose in Gy units)} = 100\text{ rem}$$



5. HRW6 43.P.069. [53088] A typical chest x-ray radiation dose is 248 μSv, delivered by x rays with an RBE factor of 0.85. Assuming that the mass of the exposed tissue is one-half the patient's mass of 90 kg, calculate the energy absorbed in joules.

☐ mJ

248×10^{-6} x 0.85 x 45 = 9.5×10^{-3}J

Example:

A sealed capsule containing the radiopharmaceutical ${}^{32}_{15}P$ with an initial radioactivity of 5.22×10^6 Bq, is implanted into a 0.1 kg tumor. Each decay produces e^- particles at an energy of 7×10^5 eV. Determine the absorbed dose in a 10 day period. $T_{1/2}$ =14.26 days. Assume all emitted particles are absorbed.

$$\text{dose} = \underbrace{\text{RBE}}_{1} \text{ x } \underbrace{\text{(number of decays)}}_{N_0\left(1-\left(\frac{1}{2}\right)^{10/14.26}\right)} \text{ x } \underbrace{\text{(energy released)}}_{7\times10^5 \text{ x } 1.6\times10^{-19}\text{J}}/\overset{0.1\text{ kg}}{\text{mass}}$$

$$N_0 = \left|\frac{dN}{dt}\right|_0 \bigg/ \frac{\ln 2}{T_{1/2}} = 5.22\times10^6 \bigg/ \frac{\ln 2}{14.26\times86400} = 9.2785\times10^{12}$$

dose = 4 J/kg = 400 rem = 4 Sv

Other facts about nuclei

- Nuclei are confined to a very small region of space – typical nuclear radii are 10^{-15}m = fm (compare with atomic radius of 10^{-10}m)
- For some nuclei, there are stable forms with the same Z, but different A (isotopes)
- Some nuclei are meta-stable; they transform (decay) into other forms

Decay processes:	Examples:
α decay	
${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}\rightarrow{}^{\mathrm{A-4}}_{\mathrm{Z-2}}\mathrm{Y}+{}^{4}_{2}\mathrm{He}$	${}^{238}_{92}\mathrm{U}\rightarrow{}^{234}_{90}\mathrm{Th}+{}^{4}_{2}\mathrm{He}$
β decay	
${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}\rightarrow{}^{\mathrm{A}}_{\mathrm{Z+1}}\mathrm{Y}+\mathrm{e}^{-}+\overline{\upsilon}$	${}^{14}_{6}\mathrm{C}\rightarrow{}^{14}_{7}\mathrm{N}+\mathrm{e}^{-}+\overline{\upsilon}$
${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}\rightarrow{}^{\mathrm{A}}_{\mathrm{Z-1}}\mathrm{Y}+\mathrm{e}^{+}+\upsilon$	
γ decay	
${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}^{*}\rightarrow{}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}+\gamma$	${}^{12}_{6}\mathrm{C}^{*}\rightarrow{}^{12}_{6}\mathrm{C}+\gamma$

Summary of some ideas about nuclear physics

•The basic forces that describe nuclei are stronger, shorter range, and more complicated than electromagnetic forces. Nuclear binding energies are typically 8 MeV/nucleon.

•Most nuclear decay processes are described by a simple rate equation

$$\frac{dN}{dt}=-\lambda N \quad \Rightarrow N(t)=N_0 e^{-\lambda t}=N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$$

For example: ^{238}U→ ^{234}Th + ^{4}He $T_{1/2}$=4.5×10^9 yr.

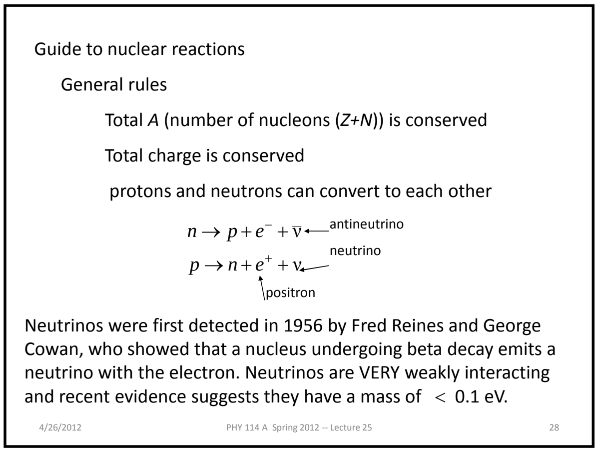
•Radiation effects on humans and other sensitive forms of life and devices are due to the light decay products – α,β,γ particles

Radiation dose: Amount of radiation absorbed/unit mass

1 Gy = 1 J/kg of absorbed radiation = 100 rad

1 Sv = RBE x (dose in Gy units) = 100 rem

relative biological effectiveness



Guide to nuclear reactions

General rules

Total *A* (number of nucleons (*Z+N*)) is conserved

Total charge is conserved

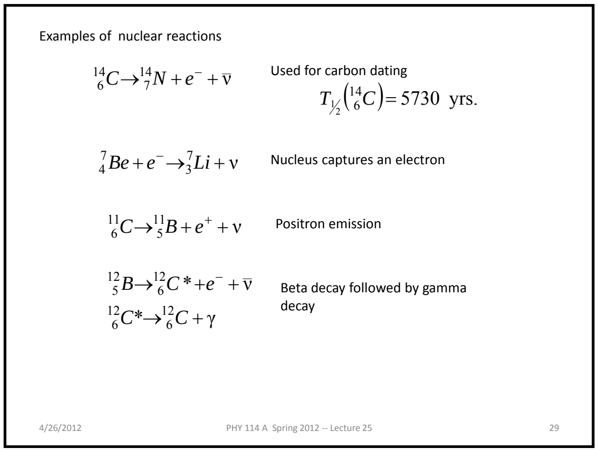
protons and neutrons can convert to each other

$$n \rightarrow p + e^- + \bar{\nu} \leftarrow \text{antineutrino}$$

$$p \rightarrow n + e^+ + \nu \leftarrow \text{neutrino}$$

positron

Neutrinos were first detected in 1956 by Fred Reines and George Cowan, who showed that a nucleus undergoing beta decay emits a neutrino with the electron. Neutrinos are VERY weakly interacting and recent evidence suggests they have a mass of $<$ 0.1 eV.



Examples of nuclear reactions

$${}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^- + \bar{\nu}$$

Used for carbon dating

$$T_{1/2}\left({}^{14}_{6}C\right) = 5730 \text{ yrs.}$$

$${}^{7}_{4}Be + e^- \rightarrow {}^{7}_{3}Li + \nu$$

Nucleus captures an electron

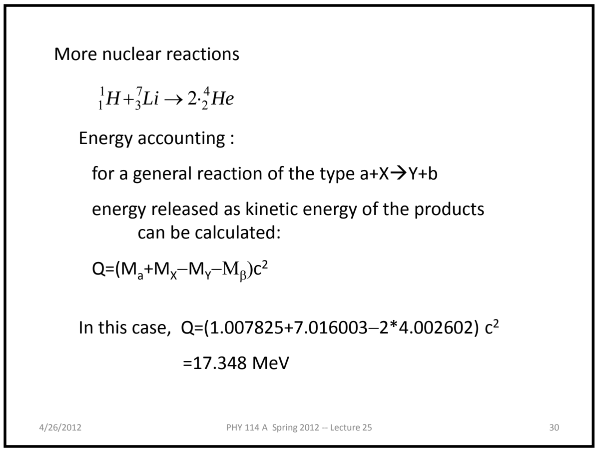
$${}^{11}_{6}C \rightarrow {}^{11}_{5}B + e^+ + \nu$$

Positron emission

$${}^{12}_{5}B \rightarrow {}^{12}_{6}C^* + e^- + \bar{\nu}$$

$${}^{12}_{6}C^* \rightarrow {}^{12}_{6}C + \gamma$$

Beta decay followed by gamma decay



More nuclear reactions

$${}^{1}_{1}H + {}^{7}_{3}Li \rightarrow 2 \cdot {}^{4}_{2}He$$

Energy accounting :

for a general reaction of the type a+X→Y+b

energy released as kinetic energy of the products can be calculated:

$Q=(M_a+M_X-M_Y-M_\beta)c^2$

In this case, $Q=(1.007825+7.016003-2*4.002602)\ c^2$

=17.348 MeV

Fission: Some history –

1932 James Chadwick (England) discovered neutron

Enrico Fermi (Italy) discovered that neutrons could be absorbed by nuclei to form new elements

Lise Meitner, Otto Hahn, Fritz Strassmann, Otto Frisch (Germany) discovered fission of U

Example:

$$n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2n$$

$$\begin{array}{ccccccccc} {}^{140}_{54}Xe & \rightarrow & {}^{140}_{55}Cs & \rightarrow & {}^{140}_{56}Ba & \rightarrow & {}^{140}_{57}La & \rightarrow & {}^{140}_{58}Ce \\ (14\mathrm{s}) & & (64\mathrm{s}) & & (13\mathrm{d}) & & (40\mathrm{h}) & & (\mathrm{stable}) \end{array}$$

$$\begin{array}{ccccc} {}^{94}_{38}Sr & \rightarrow & {}^{94}_{39}Y & \rightarrow & {}^{94}_{40}Zr \\ (75\mathrm{s}) & & (19\mathrm{min}) & & (\mathrm{stable}) \end{array}$$

Q≈200 MeV

Other decay products for $n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U$