PHY 114 A General Physics II
11 AM-12:15 PM TR Olin 101

Plan for Lecture 3:

1. Introduction to Gauss's law
2. Relationship between Coulomb's law and Gauss's law

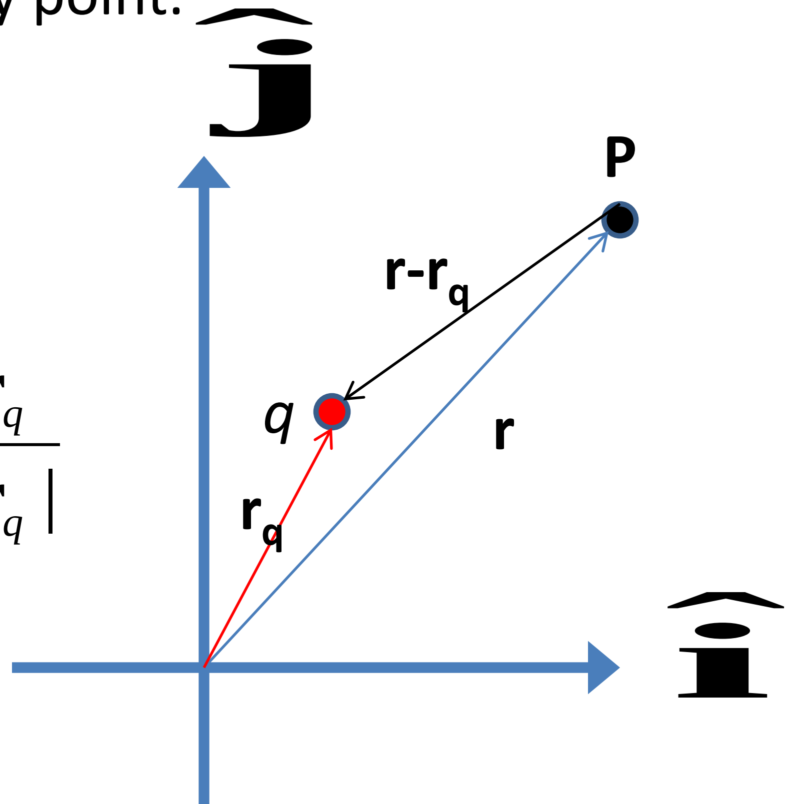
Announcements:

No.	Lecture Date	Topic	Text Sections	Problem Assignments	Assignment Due Date
1	01/19/2012	Coulomb's law	23.1-23.4	23.6,23.8a,23.13	01/24/2012
2	01/24/2012	Electric field	23.4-23.7	23.22,23.20,23.61a	01/26/2012
3	01/26/2012	Gauss's Law	24.1-24.3	24.22a,24.23,24.40	01/31/2012
4	01/31/2012	Electric potential	25.1-25.4	25.12,25.23,25.34,25.01	02/02/2012
5	02/02/2012	Electric potential	25.5-25.8	(Review for exam)	
	02/07/2012	Exam			

i-clicker registration problems:

- Campbell, Thane
- Chung, Jae
- Dearmon, Jake
- Desaly Gonzalez
- Klebous, Sandy
- Samsel, David
- Tulowiecki, Alex
- Valderrey, Melina

Comment about finding the electric field at an arbitrary point.



$$\mathbf{E}_P = \frac{k_e q}{|\mathbf{r}-\mathbf{r}_q|^2}\frac{\mathbf{r}-\mathbf{r}_q}{|\mathbf{r}-\mathbf{r}_q|}$$

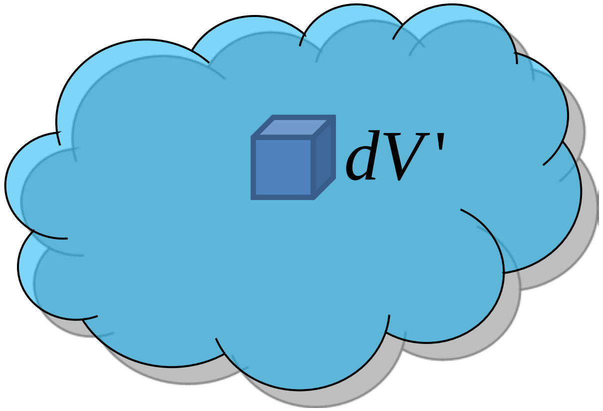
$$= \frac{k_e q\left(\mathbf{r}-\mathbf{r}_q\right)}{\left(\left|\mathbf{r}-\mathbf{r}_q\right|\right)^3}$$

$$= \frac{k_e q\left(\left(x-x_q\right)\hat{\mathbf{i}}+\left(y-y_q\right)\hat{\mathbf{j}}+\left(z-z_q\right)\hat{\mathbf{k}}\right)}{\left(\left(x-x_q\right)^2+\left(y-y_q\right)^2+\left(z-z_q\right)^2\right)^{3/2}}$$

First consider the notion of charge density:

$$\rho(\mathbf{r}) \equiv \frac{dq(\mathbf{r})}{dV} \qquad \text{or} \qquad \rho = \frac{\sum_{i \in dV} dq_i}{dV}$$

Corresponding electric field:



$$\mathbf{E}(\mathbf{r}) = \sum_q \frac{k_e q}{|\mathbf{r}-\mathbf{r}_q|^2} \frac{\mathbf{r}-\mathbf{r}_q}{|\mathbf{r}-\mathbf{r}_q|}$$

$$= k_e \int \frac{\rho(\mathbf{r}')(\mathbf{r}-\mathbf{r}')}{\left(|\mathbf{r}-\mathbf{r}'|\right)^{3/2}} dV'$$

Gauss's Law States:

$$\oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = 4\pi k_e \int \rho(\mathbf{r}) dV = \frac{1}{\varepsilon_0} \int \rho(\mathbf{r}) dV$$

Gauss's Law States:

$$\oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = 4\pi k_e \int_V \rho(\mathbf{r}) dV = \frac{1}{\varepsilon_0} \int_V \rho(\mathbf{r}) dV$$

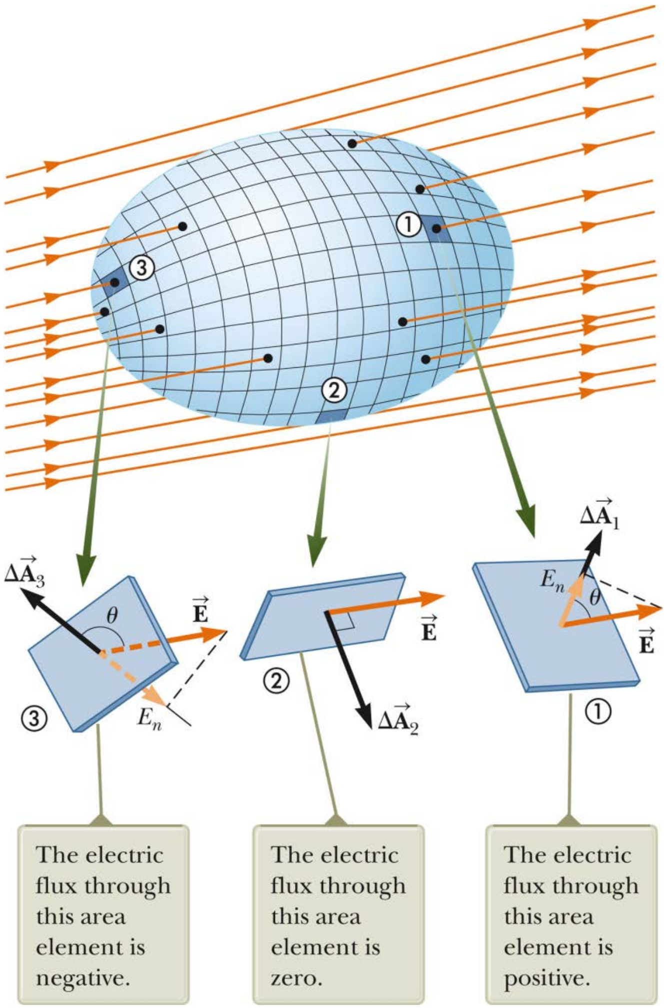
Surface integral enclosing volume V

Volume integral of charge within volume V

Differential version of Gauss's Law:

$$\nabla \cdot \mathbf{E}(\mathbf{r}) = \frac{\rho(\mathbf{r})}{\varepsilon_0}$$

How to evaluate Gauss's law:

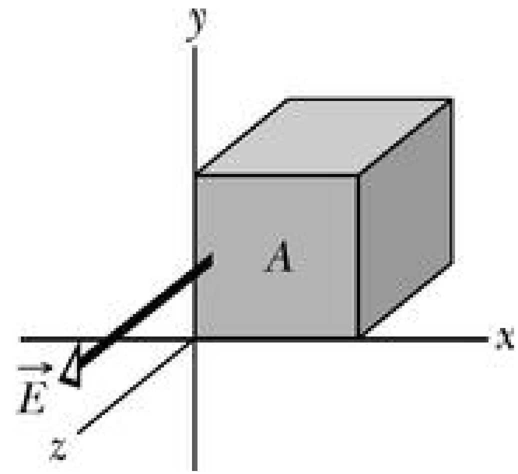


$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\varepsilon_0} \int \rho(\mathbf{r}) dV$$

i-clicker questions

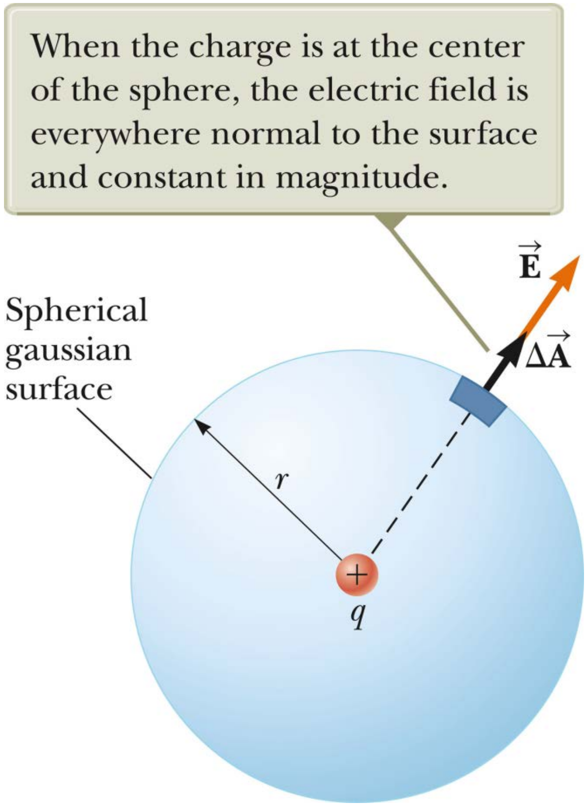
Consider the cube with each face having an area A in the presence of a uniform electric field **E** along the z-axis. Possible answers are

A. EA B. –EA C. 2EA
D. – 2EA E. 0



1. What is the flux through the front surface in the x-y plane?
2. What is the flux through the back surface in the x-y plane?
3. What is the flux through the side surface in left y-z plane?
4. What is the total flux through complete surface of the cube?

Example of Gauss's Law for familiar case:



$$\mathbf{E} = \frac{k_e q}{|\mathbf{r}|^2}\hat{\mathbf{r}}$$

$$d\mathbf{A} = \hat{\mathbf{r}}\,|\mathbf{r}|^2\, d\Omega$$

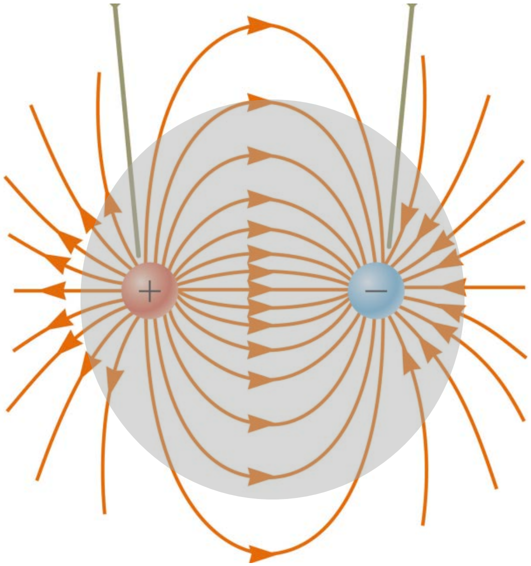
$$\mathbf{E}\cdot d\mathbf{A} = k_e q\, d\Omega$$

$$\oint \mathbf{E}\cdot d\mathbf{A} = 4\pi k_e q$$

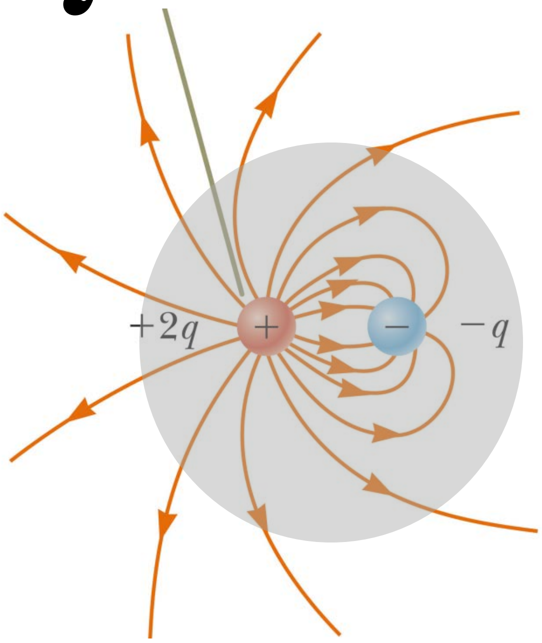
$$\mathbf{E} = \frac{k_e q}{|\mathbf{r}|^2}\hat{\mathbf{r}}$$

$$\oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} \neq 0$$



dipole



dipole plus monopole

i-clicker question

Consider the following statements

1. We can define a gravitational field (N/kg) $\mathbf{E}_g = -\frac{Gm}{|\mathbf{r}|^2}\hat{\mathbf{r}}$

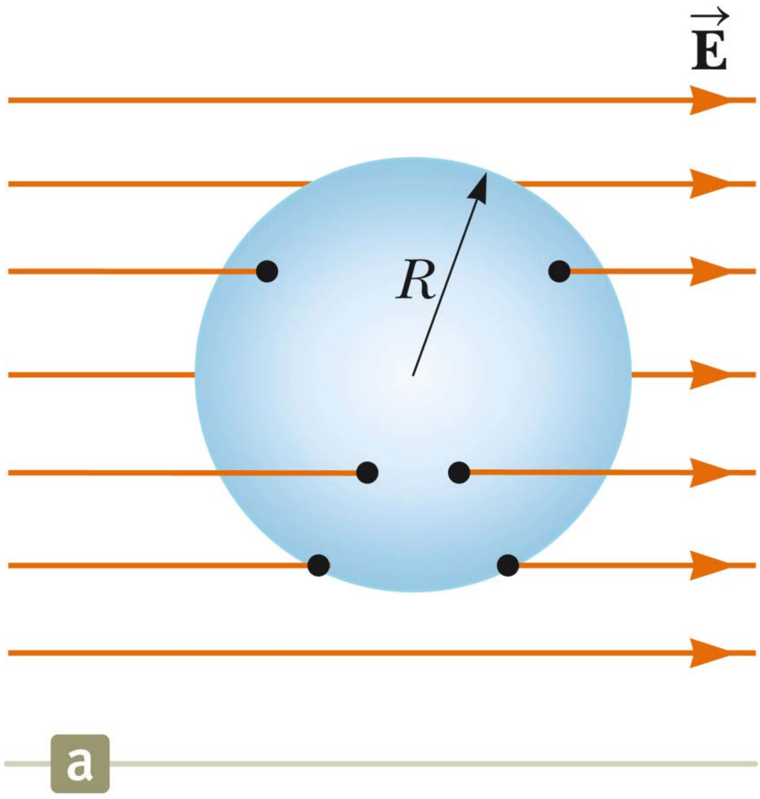
2. The Gauss's law for gravity is $\oint \mathbf{E}_g \cdot d\mathbf{A} = 4\pi Gm$

3. The Gauss's law for gravity is $\oint \mathbf{E}_g \cdot d\mathbf{A} = -4\pi Gm$

A. Only 1 is true
B. 1 and 2 are true
C. 1 and 3 are true
D. Gauss's law cannot be applied to gravitational fields

i-clicker question

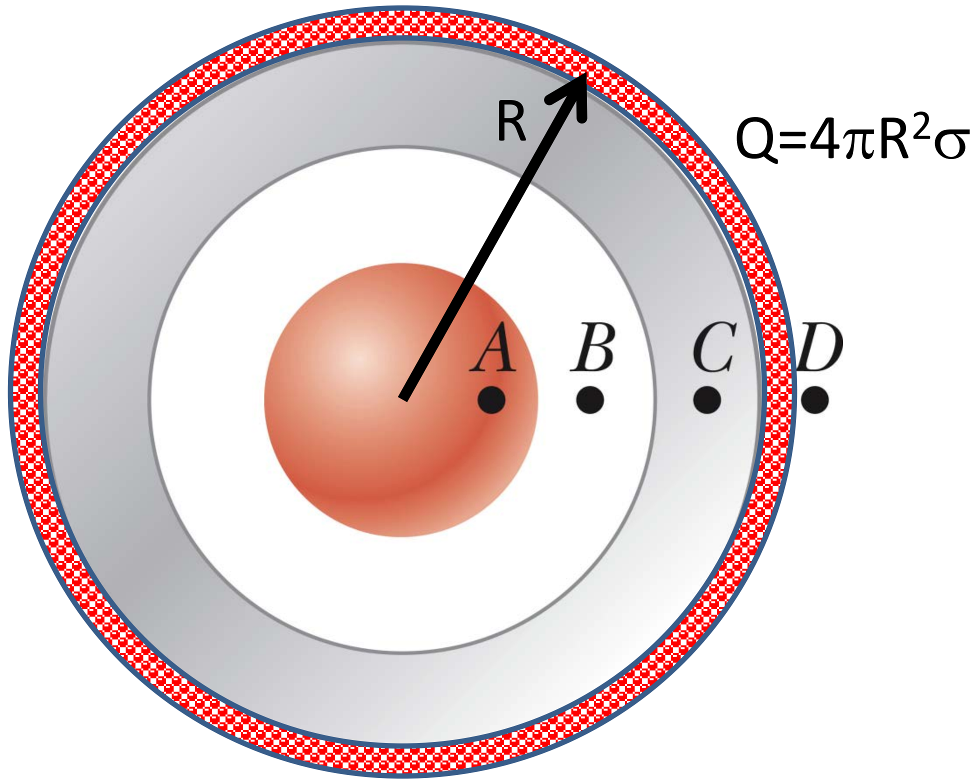
Consider the following diagram showing a Gaussian sphere in a region of a uniform electric field. What can you conclude about the total charge Q within the sphere?



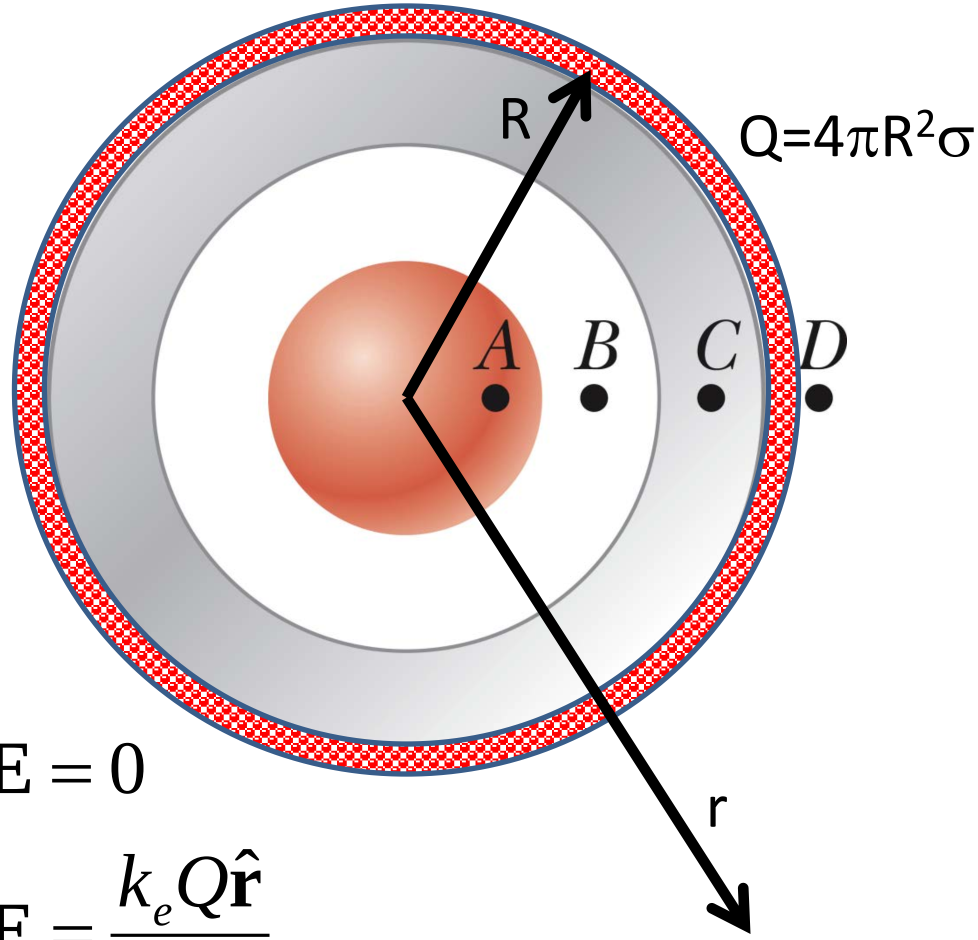
A. $Q > 0$
B. $Q < 0$
C. $Q = 0$
D. Not enough information

i-clicker question

The grey shell represents a metal having a uniform surface charge (indicated in red).



Where is the electric field non-zero?



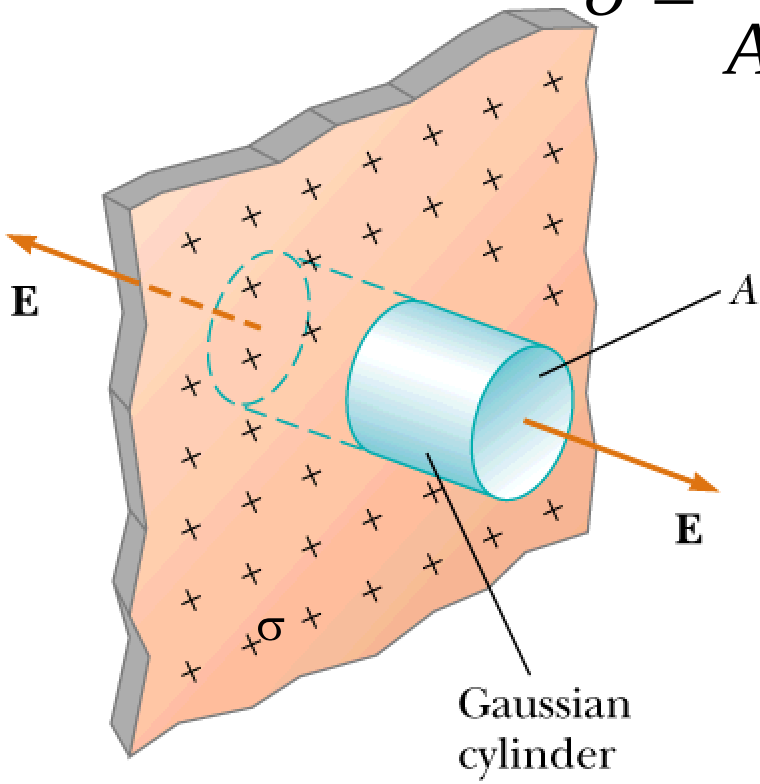
For $r < R$ $\qquad \mathbf{E} = 0$

For $r > R$ $\qquad \mathbf{E} = \frac{k_e Q \hat{\mathbf{r}}}{r^2}$

When is Gauss's law convenient to use?

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\varepsilon_0} \int \rho(\mathbf{r}) dV$$

Electrostatic field from charged sheet



$$\sigma = \frac{Q_{tot}}{A_{tot}}$$

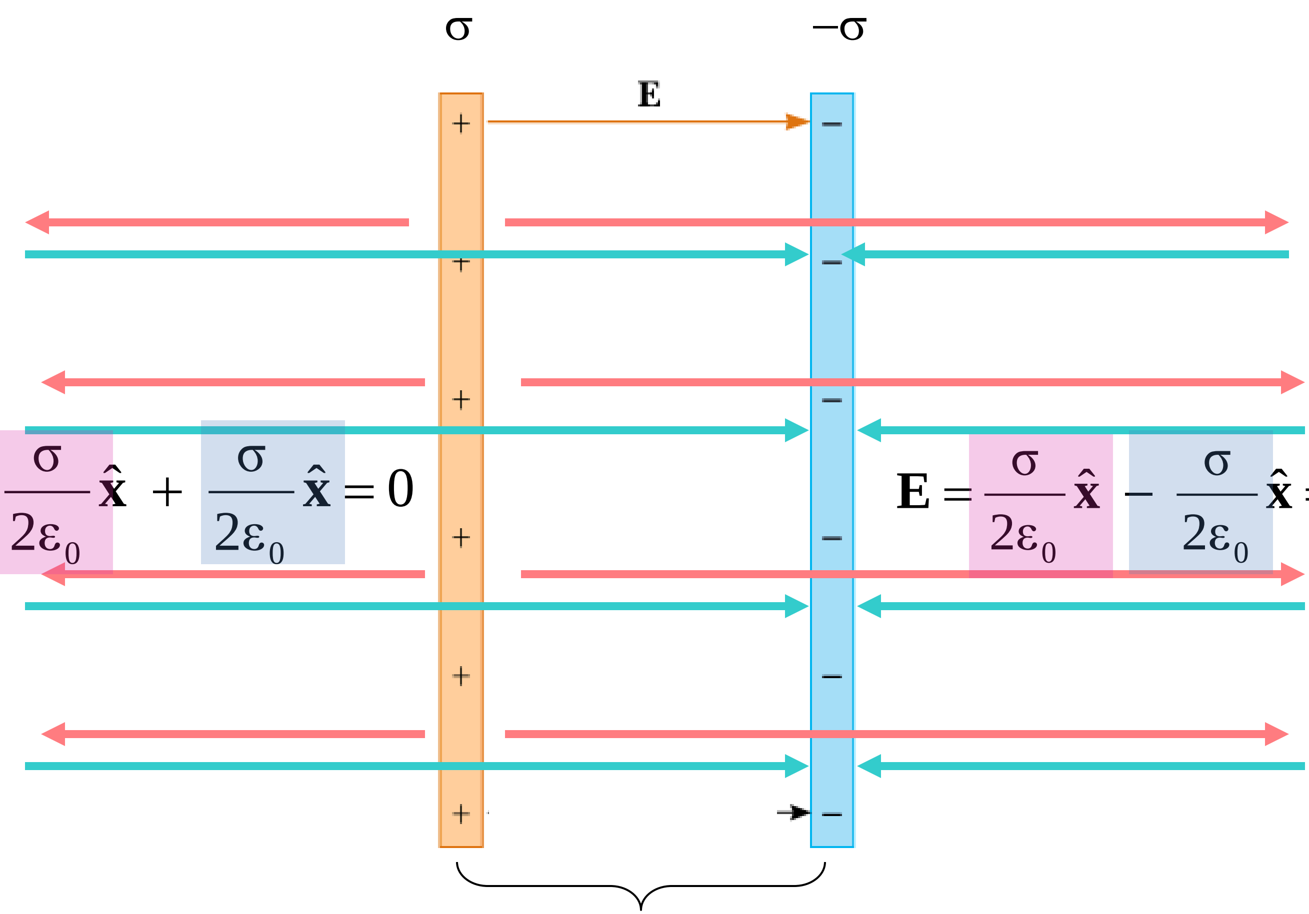
2 ends of cylinder

$$2EA = \frac{1}{\varepsilon_0}\int \rho(\mathbf{r})dV = \frac{1}{\varepsilon_0}\sigma A$$

$$\Rightarrow E = \frac{\sigma A}{2A\varepsilon_0} = \frac{\sigma}{2\varepsilon_0}$$

$$\Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

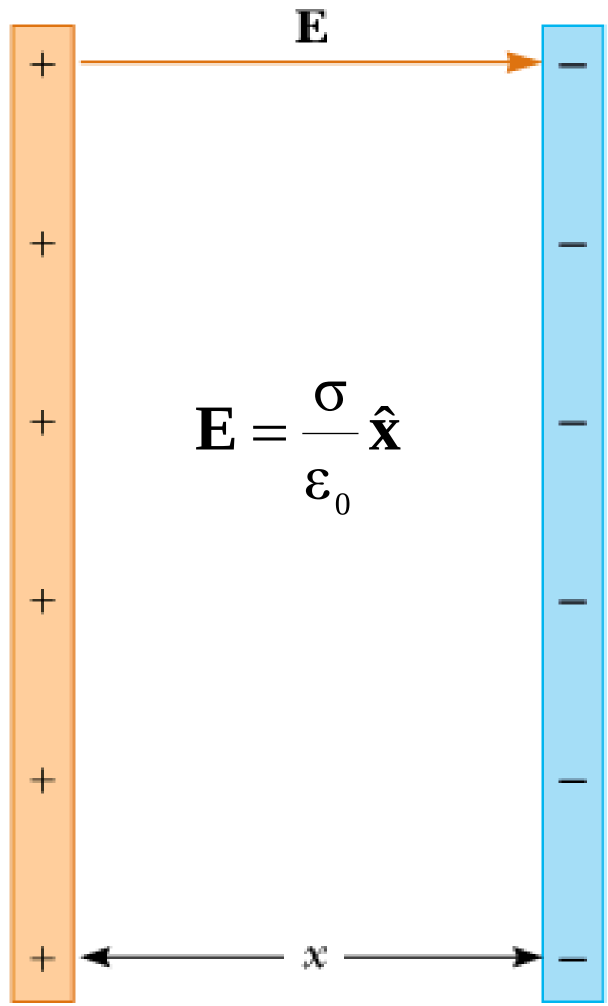
permittivity constant $=8.854\text{x}10^{-12}\ C^2/(Nm^2)$



$$\mathbf{E} = -\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{x}} + \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{x}} = 0$$

$$\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{x}} - \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{x}} = 0$$

$$\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{x}} + \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{x}} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{x}}$$

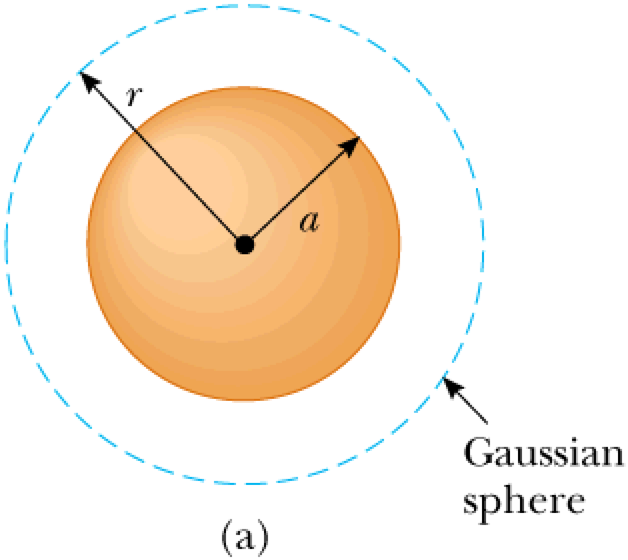


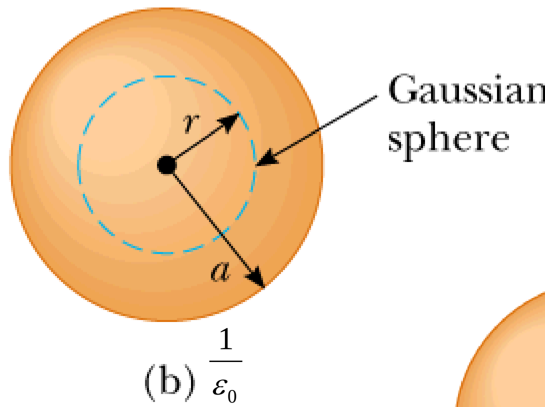
$$\mathbf{E} = 0$$

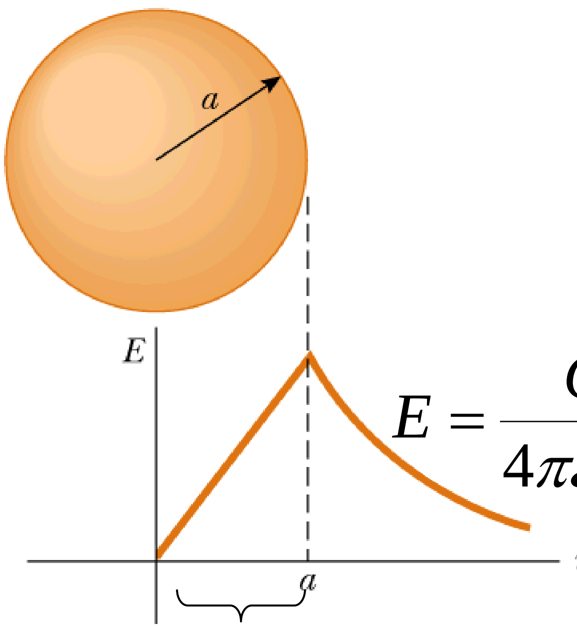
$$\mathbf{E} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{x}}$$

$$\mathbf{E} = 0$$

Electric field inside and outside uniformly charged sphere:







$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

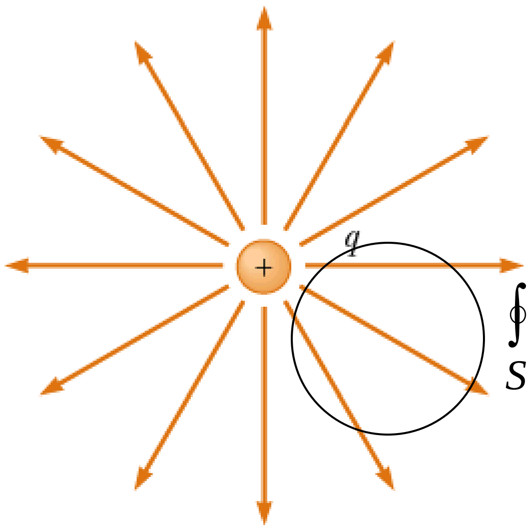
$$E = \frac{Qr}{4\pi\varepsilon_0 a^3}$$

Summary – Gauss's law

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\varepsilon_0} \int_V dV \rho$$

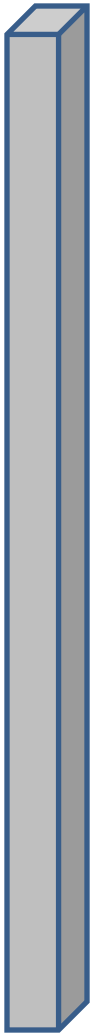
closed surface

volume inside surface

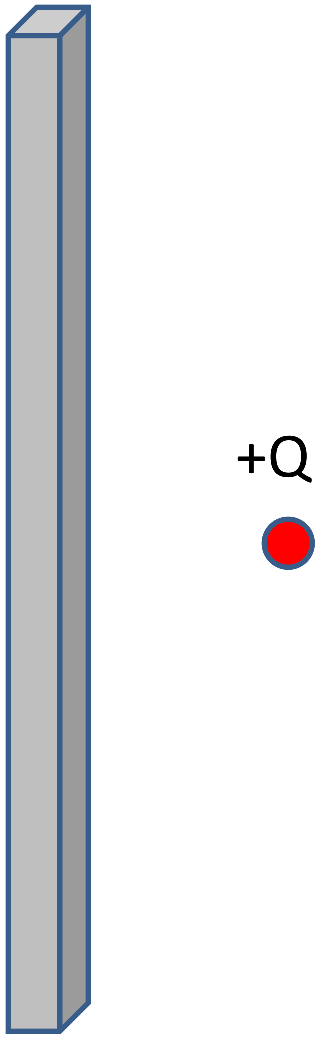


$$\oint_S \mathbf{E} \cdot d\mathbf{A} = 0$$

Consider a neutral (electrically isolated) metal sheet:



Consider a neutral (electrically isolated) metal sheet:



What happens when you bring a point charge +Q close to the sheet?

A. There is no force on the charge.
B. The charge is attracted to the sheet.
C. The charge is repelled from the sheet.