Plan for Lecture 3:

1. Introduction to Gauss’s law
2. Relationship between Coulomb’s law and Gauss’s law

Announcements:

- i-clicker registration problems:
  - Campbell, Thane
  - Chung, Jae
  - Dearmon, Jake
  - Desaly Gonzalez
  - Klebous, Sandy
  - Samsel, David
  - Tulowiecki, Alex
  - Valderrey, Melina
Comment about finding the electric field at an arbitrary point.

\[ E_r = \frac{kq}{|r-r_q|} \]

First consider the notion of charge density:

\[ \rho(r) = \frac{dq(r)}{dV} \quad \text{or} \quad \rho = \frac{\sum dq_i}{dV} \]

Corresponding electric field:

\[ E(r) = \sum \frac{kq}{|r-r_q|} \]

\[ = k \int \frac{\rho(r') \cdot (r-r')}{|r-r'|^{3/2}} dV' \]

Gauss's Law States:

\[ \int_\Sigma E \cdot dA = 4\pi k \int_V \rho(r) dV = \frac{1}{\varepsilon_0} \int_V \rho(r) dV \]

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Surface integral enclosing volume \( V \)

Volume integral of charge within volume \( V \)

Differentiation of Gauss's Law:

\[ V \cdot E(r) = \frac{\rho(r)}{\varepsilon_0} \]
How to evaluate Gauss’s law:

$$\iiint E \cdot dA = \frac{1}{\varepsilon_0} \iiint \rho(r) dV$$

1. **i-clicker questions**

   Consider the cube with each face having an area $A$ in the presence of a uniform electric field $E$ along the $z$-axis. Possible answers are

   A. $EA$
   B. $-EA$
   C. $2EA$
   D. $-2EA$
   E. $0$

   1. What is the flux through the front surface in the $x$-$y$ plane?
   2. What is the flux through the back surface in the $x$-$y$ plane?
   3. What is the flux through the side surface in the $y$-$z$ plane?
   4. What is the total flux through complete surface of the cube?

Example of Gauss’s Law for familiar case:

$$E = \frac{kq}{|r|^2} \hat{r}$$

$$dA = \hat{r} |r|^2 d\Omega$$

$$E \cdot dA = kq d\Omega$$

$$\iiint E \cdot dA = 4\pi kq$$
\[ E = \frac{kq}{|r|^2} \hat{r} \]

\[ \int E(r) \cdot dA = 0 \]
\[ \int E(r) \cdot dA \neq 0 \]

i-clicker question
Consider the following statements
1. We can define a gravitational field (N/kg) \( E_g = -\frac{Gm}{|r|^2} \hat{r} \)
2. The Gauss’s law for gravity is \( \int E_g \cdot dA = 4\pi Gm \)
3. The Gauss’s law for gravity is \( \int E_g \cdot dA = -4\pi Gm \)

A. Only 1 is true
B. 1 and 2 are true
C. 1 and 3 are true
D. Gauss’s law cannot be applied to gravitational fields
i-clicker question
Consider the following diagram showing a Gaussian sphere in a region of a uniform electric field. What can you conclude about the total charge $Q$ within the sphere?

A. $Q > 0$
B. $Q < 0$
C. $Q = 0$
D. Not enough information

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The grey shell represents a metal having a uniform surface charge (indicated in red).

Where is the electric field non-zero?

For $r < R$ \[ E = 0 \]
For $r > R$ \[ E = \frac{kQr}{r^2} \]
When is Gauss’s law convenient to use?

\[ \int \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\varepsilon_0} \int \rho(r) dV \]

Electrostatic field from charged sheet

\[ \sigma = \frac{Q_{\text{tot}}}{A_{\text{tot}}} \]

\[ 2E = \frac{1}{\varepsilon_0} \int \rho(r) dV = \frac{1}{\varepsilon_0} \sigma dV \]

\[ E = \frac{\sigma A}{2A\varepsilon_0} = \frac{\sigma}{2\varepsilon_0} \]

permittivity constant = 8.854x10^-12 C^2/(Nm^2)

\[ \sigma = \sigma - \sigma \]

\[ \hat{E} \]
Electric field inside and outside uniformly charged sphere:

\[ E = \frac{Q}{4\pi \varepsilon_0 r^2} \]

Summary – Gauss’s law

\[ \oint_{\text{closed surface}} \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\varepsilon_0} \int_{\text{volume inside surface}} d\mathbf{p} \]
Consider a neutral (electrically isolated) metal sheet:

What happens when you bring a point charge +Q close to the sheet?

A. There is no force on the charge.
B. The charge is attracted to the sheet.
C. The charge is repelled from the sheet.