

PHY 114 A General Physics II
11 AM-12:15 PM TR Olin 101

Plan for Lecture 3:

1. Introduction to Gauss's law

2. Relationship between Coulomb's law and Gauss's law

1/26/2012 PHY 114 A Spring 2012 -- Lecture 3 1

Announcements:

No.	Lecture Date	Topic	Text Sections	Problem Assignments	Assignment Due Date
1	01/19/2012	Coulomb's law	23.1-23.4	23.6 , 23.8a , 23.13	01/24/2012
2	01/24/2012	Electric field	23.4-23.7	23.22 , 23.20 , 23.61a	01/26/2012
3	01/26/2012	Gauss's Law	24.1-24.3	24.22a , 24.23 , 24.40	01/31/2012
4	01/31/2012	Electric potential	25.1-25.4	25.12 , 25.23 , 25.34 , 25.01	02/02/2012
5	02/02/2012	Electric potential	25.5-25.8	(Review for exam)	
	02/07/2012	Exam			

1/26/2012 PHY 114 A Spring 2012 -- Lecture 3 2

i-clicker registration problems:

- Campbell, Thane
- Chung, Jae
- Dearmon, Jake
- Desaly Gonzalez
- Klebous, Sandy
- Samsel, David
- Tulowiecki, Alex
- Valderrey, Melina

1/26/2012 PHY 114 A Spring 2012 -- Lecture 3 3

Comment about finding the electric field at an arbitrary point.

$$\mathbf{E}_p = \frac{k_e q}{|\mathbf{r} - \mathbf{r}_q|^2} \frac{\mathbf{r} - \mathbf{r}_q}{|\mathbf{r} - \mathbf{r}_q|}$$

$$= \frac{k_e q (\mathbf{r} - \mathbf{r}_q)}{(|\mathbf{r} - \mathbf{r}_q|)^3}$$

$$= \frac{k_e q ((x - x_q)\hat{\mathbf{i}} + (y - y_q)\hat{\mathbf{j}} + (z - z_q)\hat{\mathbf{k}})}{((x - x_q)^2 + (y - y_q)^2 + (z - z_q)^2)^{3/2}}$$

1/26/2012 PHY 114 A Spring 2012 -- Lecture 3 4

First consider the notion of charge density :

$$\rho(\mathbf{r}) \equiv \frac{dq(\mathbf{r})}{dV} \quad \text{or} \quad \rho = \frac{\sum_i dq_i}{dV}$$

Corresponding electric field :

$$\mathbf{E}(\mathbf{r}) = \sum_q \frac{k_e q}{|\mathbf{r} - \mathbf{r}_q|^2} \frac{\mathbf{r} - \mathbf{r}_q}{|\mathbf{r} - \mathbf{r}_q|}$$

$$= k_e \int \frac{\rho(\mathbf{r}')(\mathbf{r} - \mathbf{r}')}{(|\mathbf{r} - \mathbf{r}'|)^3} dV'$$

Gauss' s Law States :

$$\oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = 4\pi k_e \int \rho(\mathbf{r}) dV = \frac{1}{\epsilon_0} \int \rho(\mathbf{r}) dV$$

1/26/2012 PHY 114 A Spring 2012 -- Lecture 3 5

Gauss' s Law States :

$$\oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = 4\pi k_e \int_V \rho(\mathbf{r}) dV = \frac{1}{\epsilon_0} \int_V \rho(\mathbf{r}) dV$$

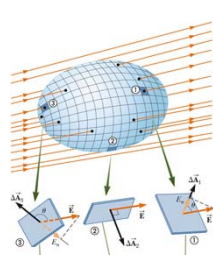
↑ Surface integral enclosing volume V ↑ Volume integral of charge within volume V

Differenti al version of Gauss' s Law :


$$\nabla \cdot \mathbf{E}(\mathbf{r}) = \frac{\rho(\mathbf{r})}{\epsilon_0}$$

1/26/2012 PHY 114 A Spring 2012 -- Lecture 3 6


How to evaluate Gauss's law:




$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\epsilon_0} \int \rho(\mathbf{r}) dV$$



The electric flux through this area element is negative.



The electric flux through this area element is zero.



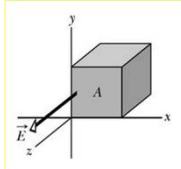
The electric flux through this area element is positive.

1/26/2012 PHY 114 A Spring 2012 – Lecture 3 7

i-clicker questions

Consider the cube with each face having an area A in the presence of a uniform electric field \mathbf{E} along the z -axis. Possible answers are

A. EA B. $-EA$ C. $2EA$
 D. $-2EA$ E. 0

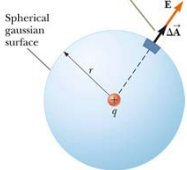


1. What is the flux through the front surface in the x - y plane?
2. What is the flux through the back surface in the x - y plane?
3. What is the flux through the side surface in left y - z plane?
4. What is the total flux through complete surface of the cube?

1/26/2012 PHY 114 Lecture 3 8

Example of Gauss's Law for familiar case:

When the charge is at the center of the sphere, the electric field is everywhere normal to the surface and constant in magnitude.



$$\mathbf{E} = \frac{k_e q}{|\mathbf{r}|^2} \hat{\mathbf{r}}$$

$$d\mathbf{A} = \hat{\mathbf{r}} |\mathbf{r}|^2 d\Omega$$

$$\mathbf{E} \cdot d\mathbf{A} = k_e q d\Omega$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = 4\pi k_e q$$

1/26/2012 PHY 114 A Spring 2012 – Lecture 3 9

$$\mathbf{E} = \frac{k_e q}{|\mathbf{r}|^2} \hat{\mathbf{r}}$$

1/26/2012 PHY 114 A Spring 2012 -- Lecture 3 10

$$\oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = 0 \quad \oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} \neq 0$$

dipole dipole plus monopole

1/26/2012 PHY 114 A Spring 2012 -- Lecture 3 11

i-clicker question
Consider the following statements

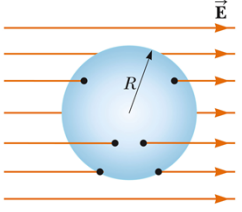
1. We can define a gravitational field (N/kg) $\mathbf{E}_g = -\frac{Gm}{|\mathbf{r}|^2} \hat{\mathbf{r}}$
2. The Gauss's law for gravity is $\oint \mathbf{E}_g \cdot d\mathbf{A} = 4\pi Gm$
3. The Gauss's law for gravity is $\oint \mathbf{E}_g \cdot d\mathbf{A} = -4\pi Gm$

A. Only 1 is true
B. 1 and 2 are true
C. 1 and 3 are true
D. Gauss's law cannot be applied to gravitational fields

1/26/2012 PHY 114 A Spring 2012 -- Lecture 3 12

i-clicker question

Consider the following diagram showing a Gaussian sphere in a region of a uniform electric field. What can you conclude about the total charge Q within the sphere?

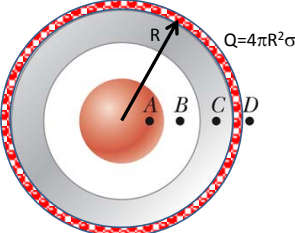


A. $Q > 0$
 B. $Q < 0$
 C. $Q = 0$
 D. Not enough information

1/26/2012 PHY 114 A Spring 2012 -- Lecture 3 13

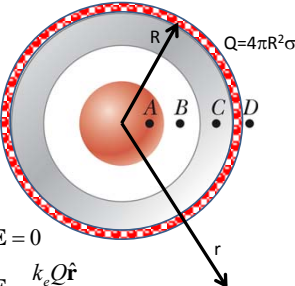
i-clicker question

The grey shell represents a metal having a uniform surface charge (indicated in red).



Where is the electric field non-zero?

1/26/2012 PHY 114 A Spring 2012 -- Lecture 3 14



For $r < R$ $\mathbf{E} = 0$
 For $r > R$ $\mathbf{E} = \frac{k_e Q \hat{r}}{r^2}$

1/26/2012 PHY 114 A Spring 2012 -- Lecture 3 15

When is Gauss's law convenient to use?

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\epsilon_0} \int \rho(\mathbf{r}) dV$$

1/26/2012

PHY 114 A Spring 2012 -- Lecture 3

16

Electrostatic field from charged sheet

$\sigma = \frac{Q_{tot}}{A_{tot}}$
 2 ends of cylinder
 $2EA = \frac{1}{\epsilon_0} \int \rho(\mathbf{r}) dV = \frac{1}{\epsilon_0} \sigma A$
 $\Rightarrow E = \frac{\sigma A}{2A\epsilon_0} = \frac{\sigma}{2\epsilon_0}$
 $\Rightarrow E = \frac{\sigma}{2\epsilon_0}$
 Gaussian cylinder
 permittivity constant $= 8.854 \times 10^{-12} \text{ C}^2/(\text{Nm}^2)$

1/26/2012

PHY 114 Lecture 3

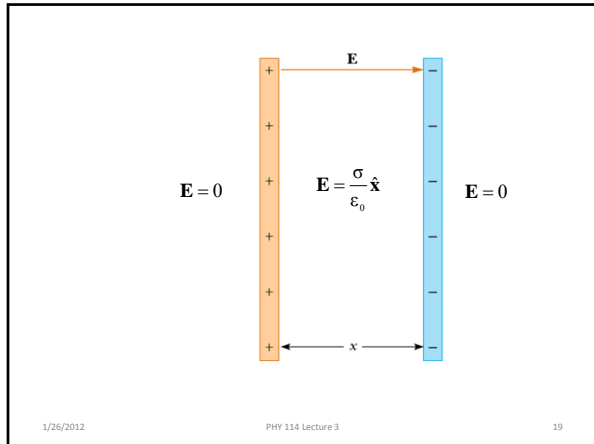
17

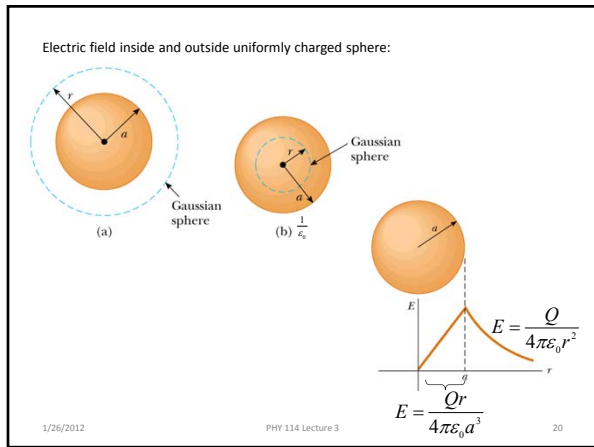
$\mathbf{E} = \frac{\sigma}{2\epsilon_0} \hat{x} + \frac{\sigma}{2\epsilon_0} \hat{x} = 0$
 $\mathbf{E} = \frac{\sigma}{2\epsilon_0} \hat{x} + \frac{\sigma}{2\epsilon_0} \hat{x} = \frac{\sigma}{\epsilon_0} \hat{x}$

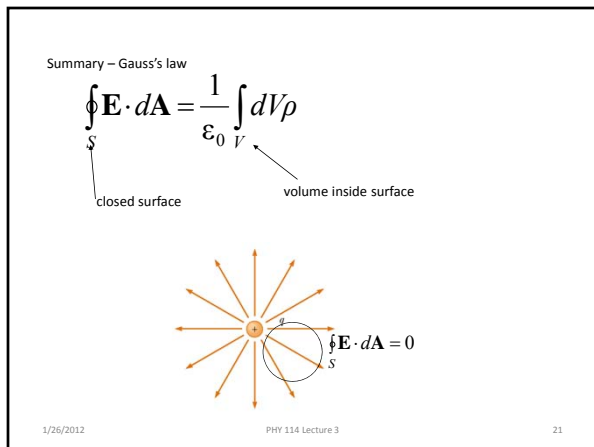
1/26/2012

PHY 114 Lecture 3


18





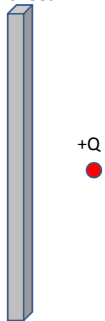


Consider a neutral (electrically isolated) metal sheet:



1/26/2012 PHY 114 A Spring 2012 -- Lecture 3 22

Consider a neutral (electrically isolated) metal sheet:



What happens when you bring a point charge $+Q$ close to the sheet?

- A. There is no force on the charge.
- B. The charge is attracted to the sheet.
- C. The charge is repelled from the sheet.

1/26/2012 PHY 114 A Spring 2012 -- Lecture 3 23
