PHY 341/641
Thermodynamics and Statistical Physics

Lecture 21

Many particle systems (Chapter 6 in STP)

- Ideal gas systems
- Distinguishable/indistinguishable particles
- Quantum/classical dynamics

	2/29/2012	APS -- no class; take-home exam			
	3/02/2012	APS -- no class; take-home exam			
18	3/05/2012	Exam due -- Ising model	5.5	HW 17	03/07/2012
19	3/07/2012	Ising model	5.6-5.7	HW 18	03/09/2012
20	3/09/2012	Phase transformation	5.8-5.10		
	3/12/2012	*Spring Break*			
	3/14/2012	*Spring Break*			
	3/16/2012	*Spring Break*			
21	3/19/2012	Many particle systems	6.1-6.2	HW 19	03/23/2012
22	3/21/2012	Fermi and Bose particles	6.3-6.4		
23	3/23/2012	Example systems	6.5-6.11		
24	3/26/2012	Chemical potential	7.1-7.2		
25	3/28/2012	Phase equilibria	7.3		
	3/30/2012				
	4/02/2012				
	4/04/2012				
	4/06/2012	*Good Friday Holiday*			

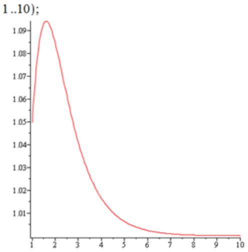
Reminder – second exam in April
-- student presentations 4/30, 5/2 (need to pick topics)

Comment on HW 19:
Maple syntax for numerical evaluation of sum --

> $f := x \rightarrow add\left(\exp\left(-x\cdot n - \frac{2\cdot n^2}{x^2}\right), n = 0..1000\right);$

$$f := x \rightarrow add\left(e^{-x\,n - \frac{2\,n^2}{x^2}}, n = 0..1000\right)$$

> $plot(f(x), x = 1..10);$



Example: Canonical distribution for free particles

Classical canonical distribution for N free particles of mass m moving in d dimensions in box of length L

$$Z(T,V,N) = \frac{1}{N!h^{dN}} \int_{0 \le r_i \le L} d^{dN} r \int d^{dN} p \, e^{-\frac{\beta}{2m}\left(\sum_i p_i^2\right)}$$

$$= \frac{L^{dN}}{N!h^{dN}} (2\pi mkT)^{dN/2}$$

$$= \frac{1}{N!} (L)^{dN} \left(\frac{2\pi mkT}{h^2}\right)^{dN/2}$$

For $d = 3$, $L^3 \equiv V$

$$Z(T,V,N) = \frac{V^N}{N!} \left(\frac{2\pi mkT}{h^2}\right)^{3N/2}$$

Compare with microcanonical ensemble:

$$\Gamma(E,V,N) = \frac{V^N}{N!\Gamma\left(\frac{3N}{2}+1\right)} \left(\frac{2\pi mE}{h^2}\right)^{3N/2}$$

Note: $Z(T,V,N) = \frac{V^N}{N!} \left(\frac{2\pi mkT}{h^2}\right)^{3N/2} = \frac{1}{N!} (Z(T,V,1))^N$?

Justification of $N!$ (within accuracy of Stirling approximation):

$$(N-1)! = \Gamma(N) = e^{-N} N^N \left(\frac{2\pi}{N}\right)^{1/2} \left(1 + O\left(\frac{1}{N}\right) + \cdots\right)$$

Claim: $\frac{\text{Distinguishable}}{\text{Indistinguishable}} = N!$

Example:

microstate s	red	blue	E_s	
~~1~~	~~ϵ_a~~	~~ϵ_a~~	~~$2\epsilon_a$~~	
~~2~~	~~ϵ_b~~	~~ϵ_b~~	~~$2\epsilon_b$~~	
~~3~~	~~ϵ_c~~	~~ϵ_c~~	~~$2\epsilon_c$~~	
4	ϵ_a	ϵ_b	$\epsilon_a + \epsilon_b$	1
5	ϵ_b	ϵ_a	$\epsilon_a + \epsilon_b$	
6	ϵ_a	ϵ_c	$\epsilon_a + \epsilon_c$	2
7	ϵ_c	ϵ_a	$\epsilon_a + \epsilon_c$	
8	ϵ_b	ϵ_c	$\epsilon_b + \epsilon_c$	3
9	ϵ_c	ϵ_b	$\epsilon_b + \epsilon_c$	

Table 6.1: The nine microstates of a system of two noninteracting distinguishable particles (red and blue). Each particle can be in one of three microstates with energy ϵ_a, ϵ_b, or ϵ_c.

$$\frac{\text{Distinguishable}}{\text{Indistinguishable}} = \frac{6}{3} = 2!$$

Classical ←→ quantum systems

Consider a system of non-interacting particles in a box of volume L^3

Classical treatment:

$$Z(T,V,N) = \frac{1}{N!h^{3N}} \int_{0 \le r_i \le L} d^{3N} r \int d^{3N} p \, e^{-\frac{\beta}{2m}\left(\sum_i p_i^2\right)}$$

$$= \frac{V^N}{N!}\left(\frac{2\pi mkT}{h^2}\right)^{3N/2}$$

Quantum treatment:

$$Z_Q(T,V,N) = \frac{1}{N!}\left(Z_Q(T,V,1)\right)^N$$

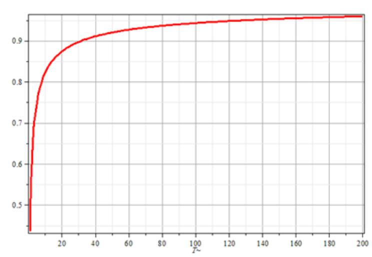
$$Z_Q(T,V,1) = \sum_{n_x,n_y,n_z} e^{-\frac{\beta h^2}{8mL^2}\left(n_x^2+n_y^2+n_z^2\right)}$$

$$\approx \left[L\left(\frac{2\pi kTm}{h^2}\right)^{1/2}\right]^3$$

Thermal de Broglie wavelength:

$$\lambda = \left(\frac{h^2}{2\pi mkT}\right)^{1/2}$$

Comparison of quantum and classical partition functions in 1-d:
$Z_Q(T)/Z(T)$



Interacting systems
Classical treatment

$$Z(T,V,N) = \frac{1}{N!h^{3N}} \int_{0 \le r_i \le L} d^{3N} r \int d^{3N} p \, e^{-\frac{\beta}{2m}\left(\sum_i p_i^2\right) - \beta V(\{\mathbf{r}_i\})}$$

$$= \frac{1}{N!}\left(\frac{2\pi mkT}{h^2}\right)^{3N/2} \int_{0 \le r_i \le L} d^{3N} r e^{-\beta V(\{\mathbf{r}_i\})}$$