PHY 341/641
Thermodynamics and Statistical Physics

Lecture 4

1. First law of thermodynamics
   a. Some examples for ideal gas systems
   b. Some cyclic processes
   c. Efficiency of process
2. Carnot cycle
   a. Efficiency
   b. Entropy
3. Second law of thermodynamics

Summary

$$\text{First law:} \qquad dU = dQ + dW$$

$$dW = -PdV$$

Ideal gas relationships:

$$PV = NkT \quad (k \equiv k_B \quad \text{Boltzmann constant})$$

$$U = \frac{k}{\gamma - 1} NT = \frac{PV}{\gamma - 1} \quad (\gamma = C_P / C_V)$$

Carnot cycle:

Analysis of an ideal heat engine

Nicholas Carnot (French Engineer) 1834

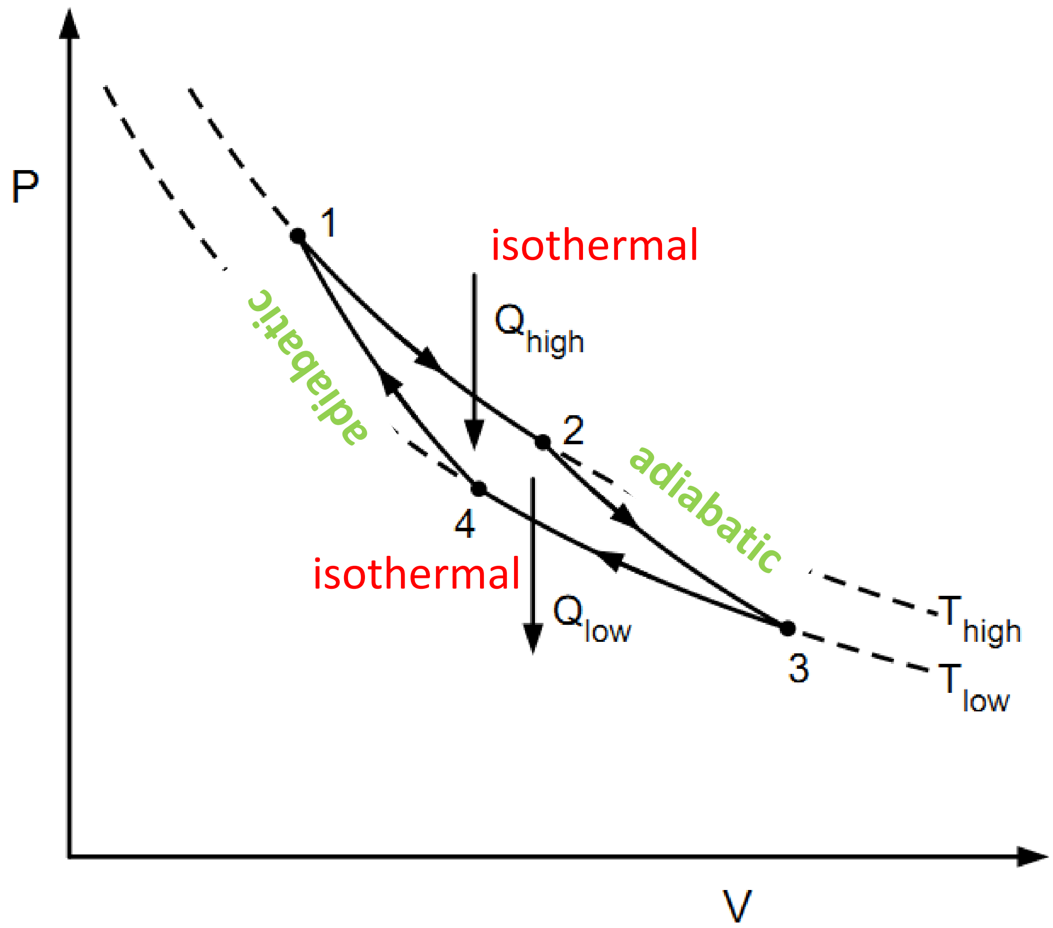
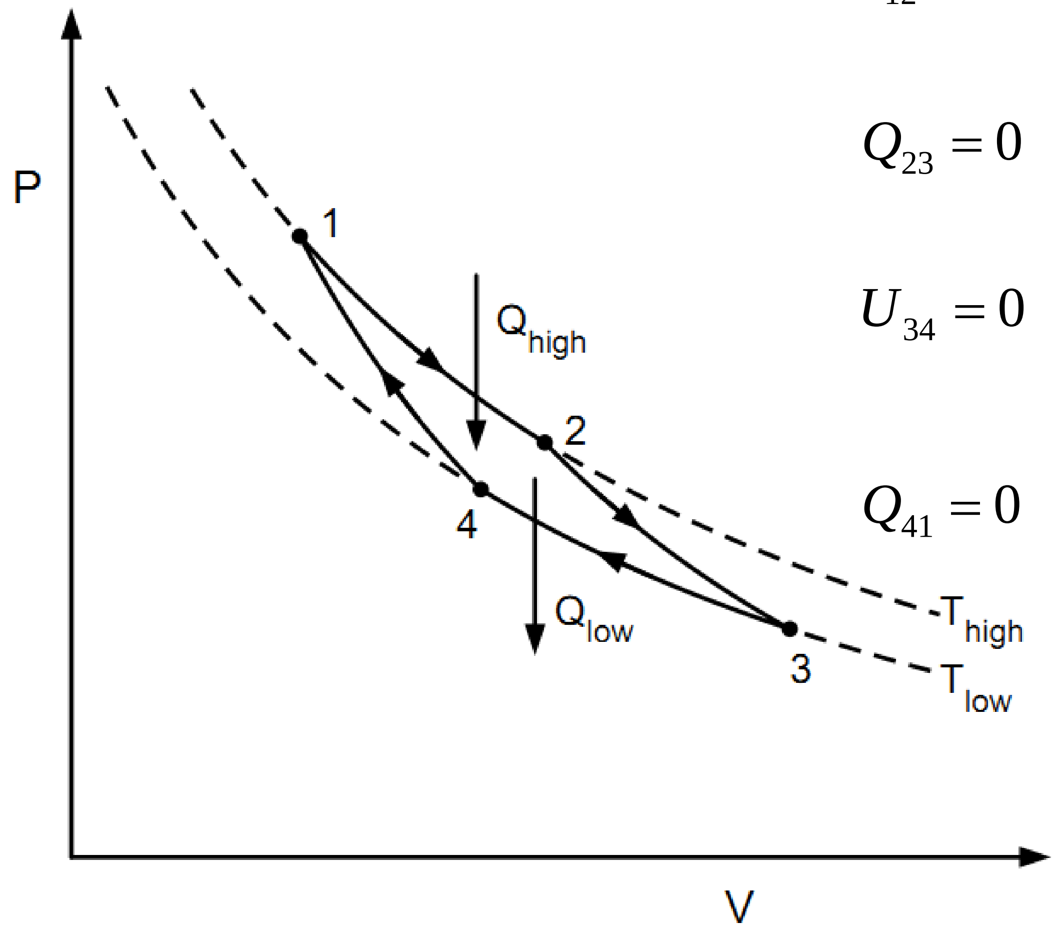


Figure 2.9: The four steps of the Carnot cycle.

Analysis of Carnot cycle for ideal gas system



$$U_{12}=0 \qquad \Rightarrow Q_{12}=-W_{12}=NkT_{high}\ln\frac{V_2}{V_1}$$

$$Q_{23}=0 \qquad \Rightarrow U_{23}=W_{23}=\frac{Nk\left(T_{low}-T_{high}\right)}{\gamma-1}$$

$$U_{34}=0 \qquad \Rightarrow Q_{34}=-W_{34}=NkT_{low}\ln\frac{V_4}{V_3}$$

$$Q_{41}=0 \qquad \Rightarrow U_{41}=W_{41}=\frac{Nk\left(T_{high}-T_{low}\right)}{\gamma-1}$$

$$\varepsilon=\frac{-W_{total}}{Q_{input}}=\frac{-W_{12}-W_{34}}{Q_{12}}$$

$$=1+\frac{NkT_{low}\ln\left(V_4/V_3\right)}{NkT_{high}\ln\left(V_2/V_1\right)}$$

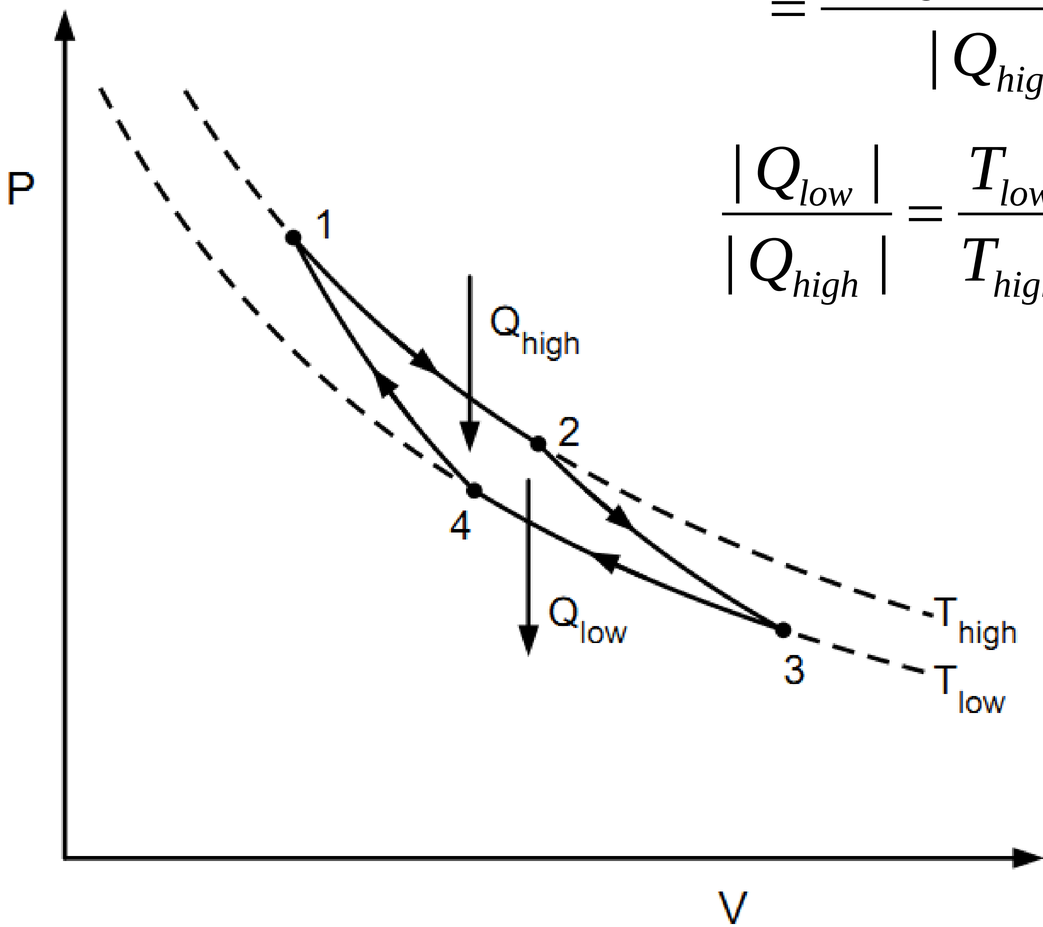
$$=1-\frac{T_{low}}{T_{high}}$$

More analysis of Carnot cycle for ideal gas system

$$\varepsilon = \frac{-W_{total}}{Q_{input}} = \frac{-W_{12} - W_{34}}{Q_{12}} = \frac{Q_{12} + Q_{34}}{Q_{12}}$$

$$= \frac{|Q_{high}| - |Q_{low}|}{|Q_{high}|} = 1 - \frac{T_{low}}{T_{high}}$$

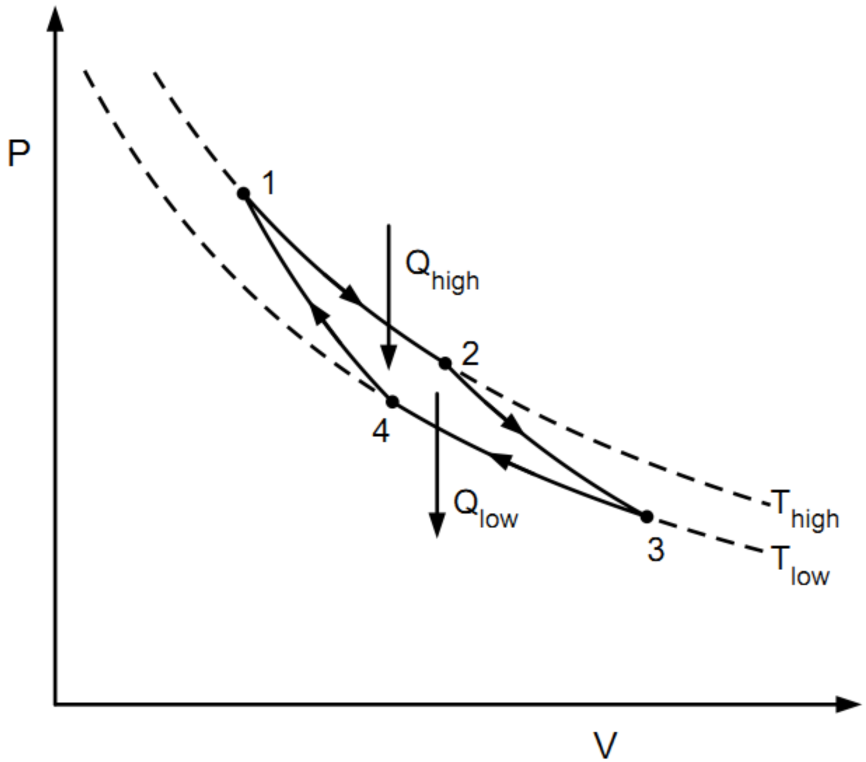
$$\frac{|Q_{low}|}{|Q_{high}|} = \frac{T_{low}}{T_{high}} \qquad \Rightarrow \frac{|Q_{low}|}{T_{low}} = \frac{|Q_{high}|}{T_{high}}$$



Definition of entropy for a reversible process:

$$dS = \frac{dQ}{T}$$

Example for Carnot cycle:

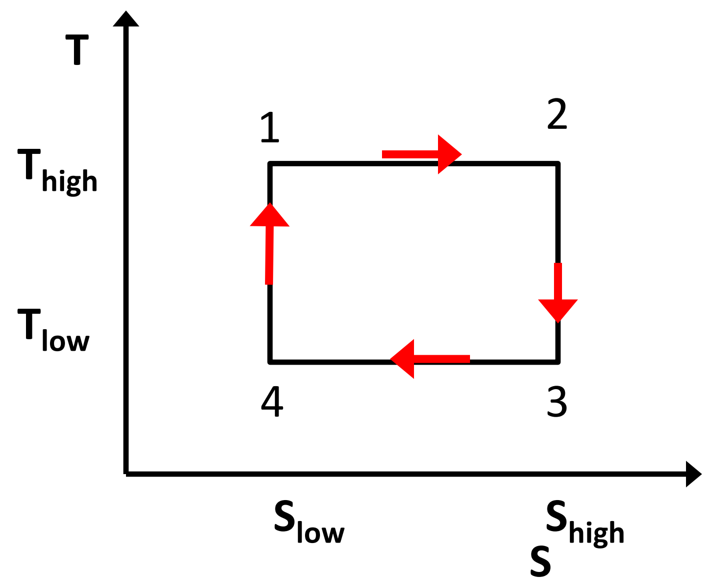


$$S_{12} = \frac{Q_{12}}{T_{high}} = \frac{Q_{high}}{T_{high}} = Nk \ln \frac{V_2}{V_1}$$

$$Q_{23} = 0 \qquad \Rightarrow S_{23} = 0$$

$$S_{34} = \frac{Q_{34}}{T_{low}} = \frac{-|Q_{low}|}{T_{low}} = Nk \ln \frac{V_4}{V_3}$$

$$Q_{41} = 0 \qquad \Rightarrow S_{41} = 0$$



Some practical (idealized) thermodynamic cycles

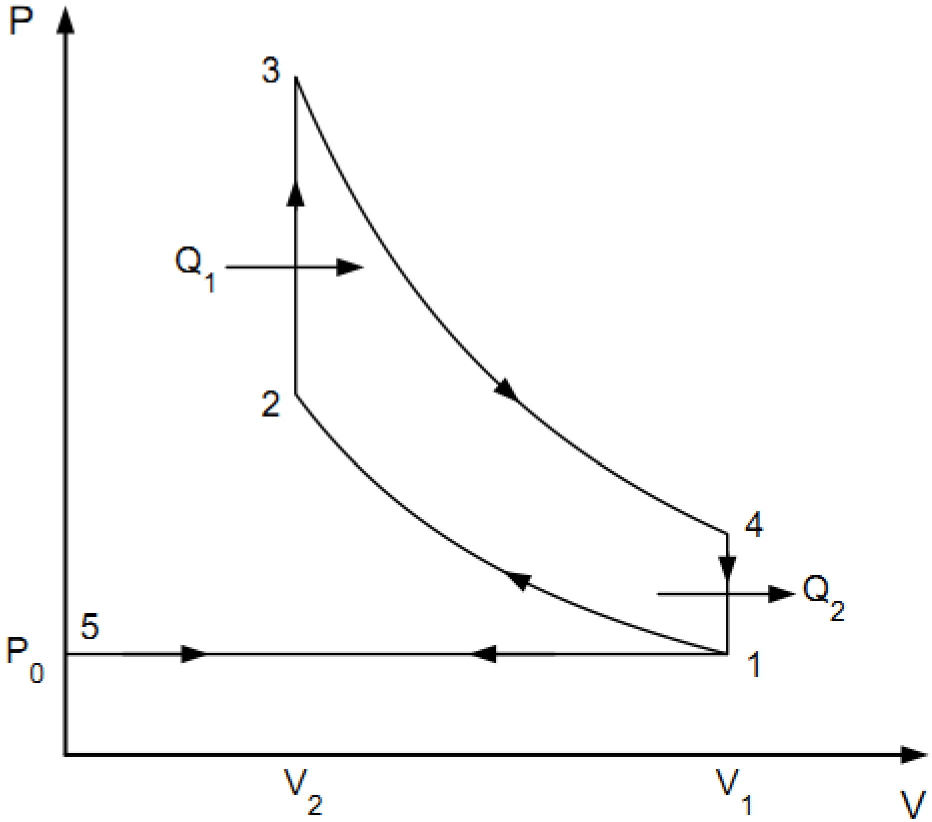


Figure 2.18: The air standard Otto cycle.

- 5→1:Intake stroke: constant pressure (P_0) intake of air and gas
- 1→2:Adiabatic compression of air/gas mixture.
- 2→3:Constant volume pressurization of air/gas misture due to spark.
- 3→4:Power stroke due to adiabatic expansion of air/gas mixture.
- 4→1: Exhaust at constant volume.

Otto cycle continued:

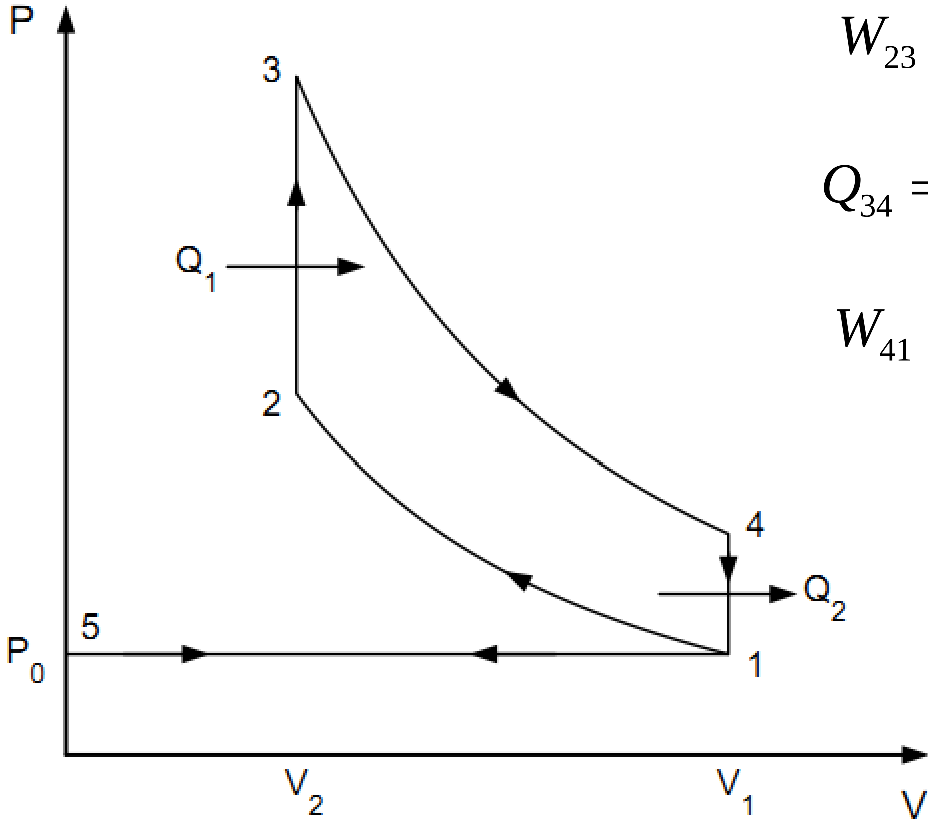


Figure 2.18: The air standard Otto cycle.

$$Q_{12}=0 \quad \Rightarrow U_{12}=W_{12}=\frac{\left(P_2V_2-P_1V_1\right)}{\gamma-1}$$

$$W_{23}=0 \quad \Rightarrow U_{23}=Q_{23}=\frac{\left(P_3V_2-P_2V_2\right)}{\gamma-1}$$

$$Q_{34}=0 \quad \Rightarrow U_{34}=W_{34}=\frac{\left(P_4V_1-P_3V_2\right)}{\gamma-1}$$

$$W_{41}=0 \quad \Rightarrow U_{41}=Q_{41}=\frac{\left(P_4V_1-P_1V_1\right)}{\gamma-1}$$

$$\varepsilon=\frac{-W_{total}}{Q_{input}}=\frac{-W_{12}-W_{34}}{Q_{23}}$$

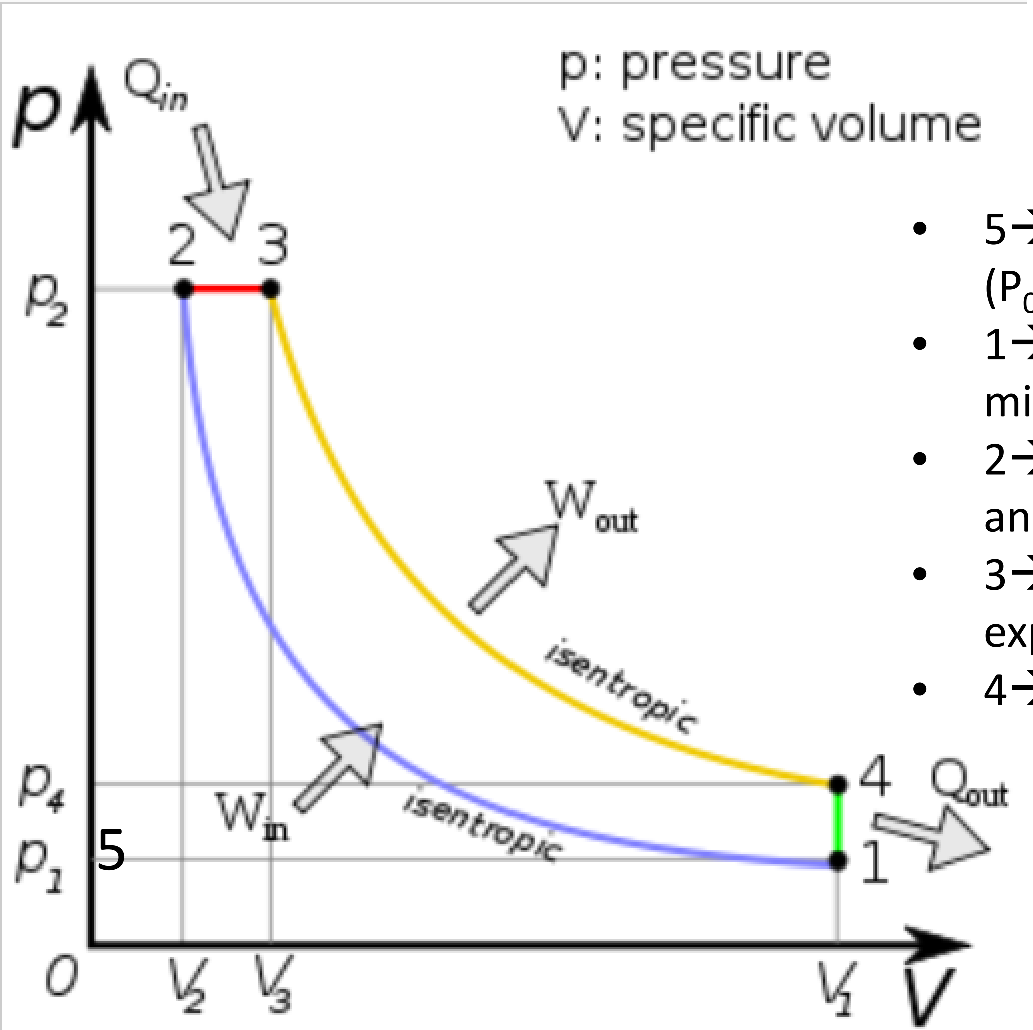
$$=1-\left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\text{Using}: \quad P_3V_2^{\gamma}=P_4V_1^{\gamma}$$

$$P_2V_2^{\gamma}=P_1V_1^{\gamma}$$

Diesel cycle diagram

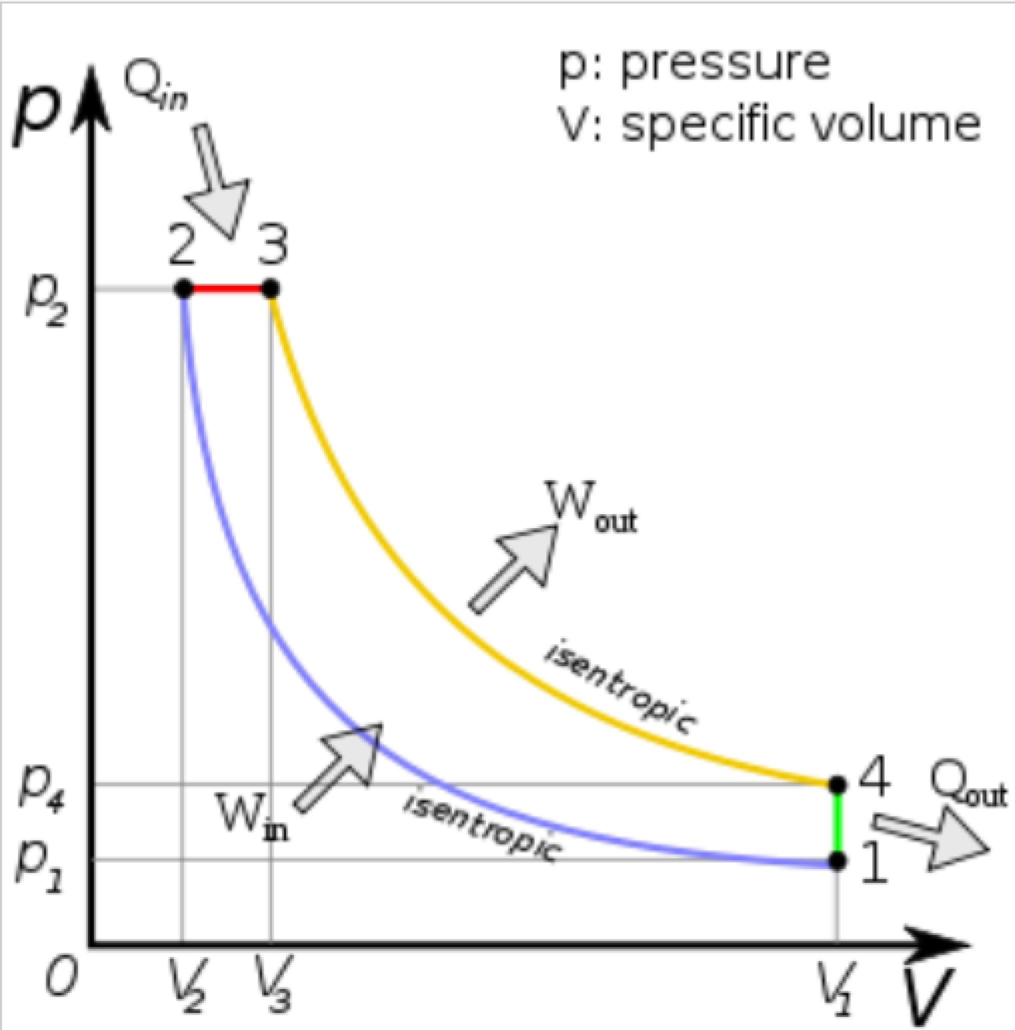
(http://en.wikipedia.org/wiki/Diesel_cycle)



p-V Diagram for the Ideal Diesel cycle. The cycle follows the numbers 1-4 in clockwise direction.

- 5→1:Intake stroke: constant pressure (P_0) intake of air and gas
- 1→2:Adiabatic compression of air/gas mixture.
- 2→3:Constant pressure combustion and expansion.
- 3→4:Power stroke due to adiabatic expansion of air/gas mixture.
- 4→1: Exhaust at constant volume.

Diesel cycle continued:



p-V Diagram for the Ideal Diesel cycle. The cycle follows the numbers 1-4 in clockwise direction.

$$Q_{12}=0 \qquad \Rightarrow U_{12}=W_{12}=\frac{(P_2V_2-P_1V_1)}{\gamma-1}$$

$$U_{23}=\frac{P_2(V_3-V_2)}{\gamma-1} \qquad W_{23}=-P_2(V_3-V_2)$$

$$Q_{23}=\frac{\gamma P_2(V_3-V_2)}{\gamma-1}$$

$$Q_{34}=0 \qquad \Rightarrow U_{34}=W_{34}=\frac{(P_4V_1-P_2V_3)}{\gamma-1}$$

$$W_{41}=0 \qquad \Rightarrow U_{41}=Q_{41}=\frac{V_1(P_1-P_4)}{\gamma-1}$$

$$\varepsilon=\frac{-W_{12}-W_{23}-W_{34}}{Q_{23}}$$

$$\varepsilon=1-\frac{\left(\left(V_3/V_2\right)^{\gamma}-1\right)}{\gamma\left(V_1/V_2\right)^{\gamma-1}\left(\left(V_3/V_2\right)-1\right)}$$

Comparison of Otto and Diesel cycle efficiencies:

Otto cycle:

$$\varepsilon = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma - 1}$$

Diesel cycle:

$$\varepsilon = 1 - \frac{\left(\left(V_3/V_2\right)^{\gamma} - 1\right)}{\gamma \left(V_1/V_2\right)^{\gamma - 1}\left(\left(V_3/V_2\right) - 1\right)}$$

Suppose

$$\gamma \approx 1.4 \quad \text{(air)}$$

$$\frac{V_1}{V_2} \approx 5 \quad \text{(Otto)}$$

$$\frac{V_1}{V_3} \approx 5 \quad \frac{V_1}{V_2} \approx 15 \quad \text{(Diesel)}$$

$$\varepsilon_{Otto} = 0.47$$

$$\varepsilon_{Diesel} = 0.56$$

Summary --

Definition of entropy for a reversible process:

$$dS = \frac{dQ}{T}$$

First law of thermodynamics expressed in terms of entropy:

$$dU = dQ + dW = TdS - PdV$$

$$\text{For an ideal gas:}$$

$$U = \frac{NkT}{\gamma - 1} \qquad PV = NkT$$

$$S = ??$$