PHY 341/641
Thermodynamics and Statistical Physics

Lecture 8

1. Complete discussion of example 2.21
2. Begin introduction to probability concepts (Chapter 3 in STP)

Example from your text on functional dependences in black body radiation following analysis by Boltzmann in 1884.

Assume that we have a system characterized by:

Internal energy: $U(T,V) \equiv u(T)V$ Entropy: $S(T,V)$

Pressure: $P(T) \equiv \frac{1}{3}u(T)$

$$dS = \frac{V}{T}\frac{du}{dT}dT + \left(\frac{\partial S}{\partial V}\right)_T dV$$

$$\left(\frac{\partial S}{\partial T}\right)_V \qquad \left(\frac{\partial S}{\partial V}\right)_T$$

$$\left(\frac{\partial}{\partial V}\right)_T \left(\frac{V}{T}\frac{du}{dT}\right) = \left(\frac{\partial}{\partial T}\right)_V \left(\left(\frac{\partial S}{\partial V}\right)_T \right) \Rightarrow u(T) = KT^4, \; S(T,V) = \frac{4}{3}KT^3V$$

Example from your text on functional dependences in black body radiation following analysis by Boltzmann in 1884 – continued --

Some details:

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV = \frac{d(uV)}{T} + \frac{P}{T} dV$$

$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{V}{T}\frac{du}{dT} \qquad \left(\frac{\partial S}{\partial V}\right)_T = \frac{u}{T} + \frac{u}{3T} = \frac{4u}{3T}$$

$$\left(\frac{\partial}{\partial V}\right)_T \left(\frac{V}{T}\frac{du}{dT}\right) = \left(\frac{\partial}{\partial T}\right)_V \left(\frac{4u}{3T}\right) \Rightarrow \frac{du}{dT} = 4\frac{u}{T}$$

$$\Rightarrow u(T) = KT^4 \qquad S(T,V) = \frac{4}{3}KT^3V$$

Concepts of Probability – Chapter 3 of STP

Probability analysis is helpful for correlating microscopic and macroscopic physics.

Example – 2-sided coin toss

$P(\text{heads}) = ½$

$P(\text{tails}) = ½$

Suppose we have 4 coins tossed at once. On any given toss, what is the probability of having n occurrences of heads?

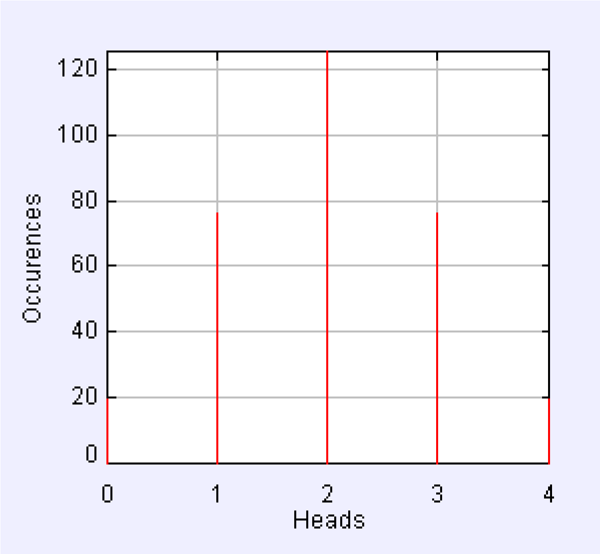
Outcomes for tossing 4 coins

Configurations	n	P(n)
TTTT	0	1/16
HTTT,THTT,TTHT,TTTH	1	4/16
TTHH,THHT,HHTT,THTH,HTHT,HTTH	2	6/16
THHH,HTHH,HHTH,HHHT	3	4/16
HHHH	4	1/16

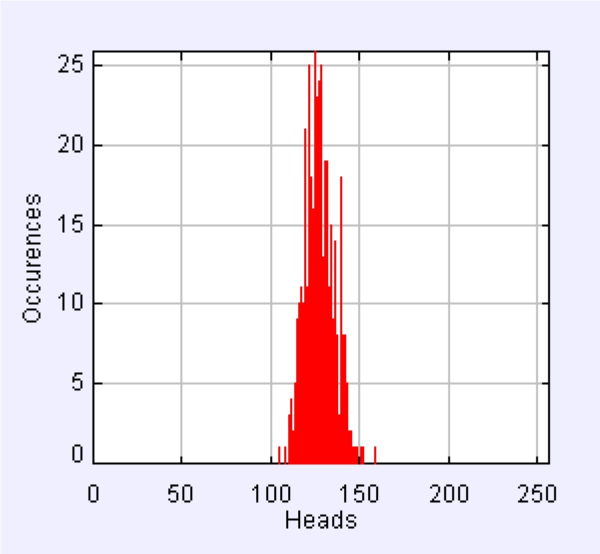
Simulation of coin tosses:
stp_CoinToss.jar

Distribution of outcomes for tossing N coins

N=4 -- 316 trials



N=256 -- 443 trials



Mathematics of probability theory

- For each event I, we assign a probability P(i) with the conditions: $P(i) \geq 0 \quad \text{and} \quad \sum_i P(i) = 1$
- For two independent events i and j, the probability of I or j is

$$P(i \text{ or } j) = P(i) + P(j)$$

- For two independent events I and j, the probability of I and j is

$$P(i \text{ and } j) = P(i)P(j)$$

Example (similar to 3.8 of your textbook)

Suppose that in a certain class of 100 students the grade distribution averaged over many years is given as shown in the second column. What is the probability of getting each grade assuming that the distribution remains the same:

Grade	100 student Distribution	Probabilty
A	16	0.16
B	32	0.32
C	48	0.48
D	4	0.04
F	0	0.00

Calculation of averages

$$\text{Define: } P(i) \equiv \text{probability of } x_i$$

$$\Rightarrow \text{Average} \left\langle x \right\rangle = \sum_{i=1}^{n} x_i P(i)$$

$$\Rightarrow \text{Moment} \left\langle x^m \right\rangle = \sum_{i=1}^{n} x_i^m P(i)$$

$$\Rightarrow \text{Standard deviation} \quad \sigma = \sqrt{\left\langle \Delta x^2 \right\rangle} \equiv \sqrt{\left\langle \left(x - \left\langle x \right\rangle\right)^2 \right\rangle}$$

$$\text{note that: } \quad \left\langle \Delta x^2 \right\rangle = \left\langle x^2 - \left\langle x \right\rangle^2 \right\rangle$$

Example:

Grade	100 student Distribution	Probabilty
A (4)	16	0.16
B (3)	32	0.32
C (2)	48	0.48
D (1)	4	0.04
F (0)	0	0.00

$$\Rightarrow \text{Average}\left\langle G \right\rangle = \sum_{i=1}^{n} G_i P(i) = 2.6$$

$$\Rightarrow \text{Standard deviation} \quad \sigma = \sqrt{\left\langle \Delta x^2 \right\rangle} \equiv \sqrt{\left\langle \left(x - \left\langle x \right\rangle\right)^2 \right\rangle} = 0.8$$

Example: Dice throws

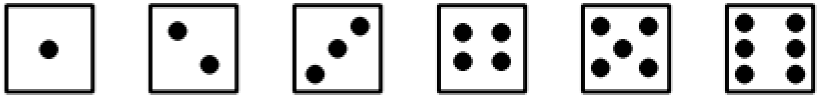


Figure 3.1: The six possible outcomes of the toss of a die.

Example 3.10. On average, how many times must a die be thrown until "4" appears?

Let $p =$ probability of getting 4 on one throw $(p = 1/6)$

Let $q =$ probability of not getting 4 on one throw

Probability of first getting 4 on nth throw: $P_n = pq^{n-1}$

$$\text{Mean \# of throws:} \quad m = \sum_{n=1}^{\infty} npq^{n-1} = p\frac{d}{dq}\sum_{n=0}^{\infty} q^n$$

$$= p\frac{d}{dq}\frac{1}{1\text{-}q} = p\frac{1}{(1-q)^2} = \frac{1}{p}$$