# PHY 712 Electrodynamics 11-11:50 AM MWF Olin 107

### Plan for Lecture 15:

Finish reading Chap. 5, start Chap. 6

- A. Magnetic susceptibility
- B. Magnetic boundary value problems
- C. Effects of time varying fields and sources

#### Course schedule for 2013

(Preliminary schedule -- subject to frequent adjustment.)

	Date	JDJ Reading	Topic	Assign.
1	01-16(Wed)	Chap. 1	Introduction, units and Poisson equation.	<u>#1</u>
2	01-18(Fri)	Chap. 1	Electronstatic energy calculations	<u>#2</u>
	01-21(Mon)	No class	MKL Holiday	
3	01-23(Wed)	Chap. 1	Poisson Equation and Green's Functions	<u>#3</u>
4	01-25(Fri)	Chap. 1 & 2	Green's Theorem and Functions	<u>#4</u>
5	01-28(Mon)	Chap. 1 & 2	Brief introduction to numerical methods	<u>#5</u>
6	01-30(Wed)	Chap. 2	Method of images	<u>#6</u>
7	02-01(Fri)	Chap. 3	Cylindrical and spherical geometries	<u>#7</u>
8	02-04(Mon)	Chap. 4	Multipole moments	<u>#8</u>
9	02-06(Wed)	Chap. 4	Dipoles and dielectrics	<u>#9</u>
10	02-08(Fri)	Chap. 4	Microscopic and macroscopic polarizability	
11	02-11(Mon)	Chap. 5	Magnetostatics	<u>#10</u>
12	02-13(Wed)	Chap. 5	Magnetostatic fields	
13	02-15(Fri)	Chap. 5	Magnetic dipole fields	Exam
14	02-18(Mon)	Chap. 5	Permeable media	Exam
15	02-20(Wed)	Chap. 5	Magnetic susceptibility and permeability	Exam
16	02-22(Fri)	Chap. 6	Maxwell's equations	Exam



#### WFU Physics Colloquium

TITLE: Needle in a Haystack: Sifting through Predicted Protein

Structures for Good Folds

**SPEAKER:** Professor Xiuping Tao,

Department of Chemistry/Physics, Winston-Salem State University, Winston-Salem, North Carolina

TIME: Wednesday February 20, 2013 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

#### **ABSTRACT**

Of the large number of sequence-solved proteins, only a small percentage are structure-solved. It makes protein structure prediction importance. Tremendous computational prediction efforts are applied to generate possible protein structures (decoys), often thousands or even millions of them, for a given sequence. Identifying good (near-native) structures among them can be tough. While the RMSD clustering method is often used to do the job, we propose a new method based on clustering amino-acid contacts in decoys. A comparison between our results and those in a CASP (Critical Assessment of protein Structure Prediction) experiment will be presented.

Summary of equations of magnetostatics:

$$\nabla \times \mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{J}_{total}(\mathbf{r})$$

$$\nabla \times \mathbf{H}(\mathbf{r}) = \mathbf{J}_{free}(\mathbf{r})$$

$$\mathbf{B}(\mathbf{r}) = \mu_0 (\mathbf{H}(\mathbf{r}) + \mathbf{M}(\mathbf{r}))$$

$$\nabla \cdot \mathbf{B}(\mathbf{r}) = 0$$

For the case that  $\mathbf{J}_{free}(\mathbf{r})$ :

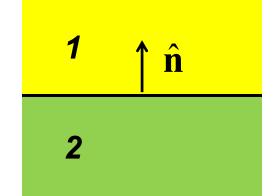
$$\nabla \times \mathbf{H}(\mathbf{r}) = 0$$

$$\nabla \cdot \mathbf{B}(\mathbf{r}) = 0$$

At boundary:

$$\mathbf{H}_1 \times \hat{\mathbf{n}} = \mathbf{H}_2 \times \hat{\mathbf{n}}$$

$$\mathbf{B}_1 \cdot \hat{\mathbf{n}} = \mathbf{B}_2 \cdot \hat{\mathbf{n}}$$



### Magnetism in materials

$$\mathbf{B}(\mathbf{r}) = \mu_0 (\mathbf{H}(\mathbf{r}) + \mathbf{M}(\mathbf{r}))$$

For materials with linear magnetism:

$$\mathbf{B} = \mu \mathbf{H}$$

 $\mu > \mu_0 \implies$  paramagnetic material

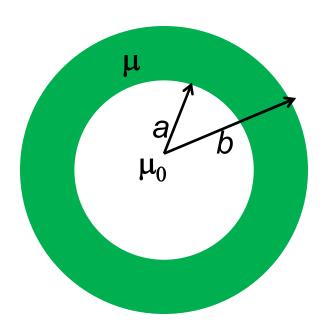
 $\mu < \mu_0 \implies$  diamagnetic material

For ferromagnetic, antiferromagnetic materials

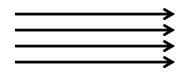
$$\mathbf{B} = f(\mathbf{H})$$
 (with hysteresis)

Example: permalloy, mumetal  $\mu/\mu_0 \sim 10^4$ 

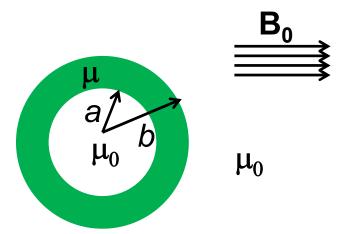
Spherical shell a < r < b:







 $\mu_0$ 



For this case:

$$\nabla \times \mathbf{H}(\mathbf{r}) = 0$$

$$\nabla \cdot \mathbf{B}(\mathbf{r}) = 0$$

$$B(r) = \mu H(r)$$

Continuity at boundaries:

$$\mathbf{H} \times \hat{\mathbf{n}} = \text{continuous}$$

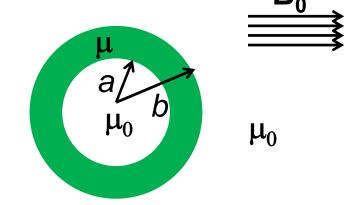
$$\mathbf{B} \cdot \hat{\mathbf{n}} = \text{continuous}$$

Let: 
$$\mathbf{H}(\mathbf{r}) = -\nabla \Phi_{H}(\mathbf{r})$$
  $\Rightarrow \nabla^{2} \Phi_{H}(\mathbf{r}) = 0$ 
For  $0 \le r \le a$   $\Phi_{H}(\mathbf{r}) = \sum_{l} \delta_{l} r^{l} P_{l}(\cos \theta)$ 
For  $r \ge b$   $\Phi_{H}(\mathbf{r}) = -\frac{B_{0}}{\mu_{0}} r \cos \theta + \sum_{l} \frac{\alpha_{l}}{r^{l+1}} P_{l}(\cos \theta)$ 

Applying boundary conditions

(only l = 1 terms contribute):

At 
$$r = a$$
  $\delta_1 = \frac{\mu}{\mu_0} \left( \beta_1 - 2 \frac{\gamma_1}{a^3} \right)$ 



$$a\delta_1 = a\beta_1 + \frac{\gamma_1}{a^2}$$

At 
$$r = b$$
  $\frac{\mu}{\mu_0} \left( \beta_1 - 2 \frac{\gamma_1}{b^3} \right) = -\frac{B_0}{\mu_0} - 2 \frac{\alpha_1}{b^3}$ 

$$b\beta_1 + \frac{\gamma_1}{b^2} = -b\frac{B_0}{\mu_0} + \frac{\alpha_1}{b^2}$$

 $\begin{array}{c} B_0 \\ \\ \\ \mu_0 \end{array}$   $\mu_0$ 

When the dust clears:

$$\delta_{1} = \left(\frac{-9\mu/\mu_{0}}{(2\mu/\mu_{0}+1)(\mu/\mu_{0}+2)-2(a/b)^{3}(\mu/\mu_{0}-1)^{2}}\right)\frac{B_{0}}{\mu_{0}}$$

$$\approx \frac{1}{\mu/\mu_{0}}\left(\frac{-9/2}{(1-(a/b)^{3})}\frac{B_{0}}{\mu_{0}}\right)$$

### Energy associated with magnetic fields

Note: We previously used without proof --

the force on a magnetic dipole m in an external B field is:

$$\mathbf{F} = \nabla (\mathbf{m} \cdot \mathbf{B})$$

This implies that energy associated with aligning a magnetic dipole **m** in an external **B** field is given by :

$$U = -\mathbf{m} \cdot \mathbf{B}$$

Macroscopic energies --

It can be shown that: 
$$W_B = \frac{1}{2} \int d^3 r \, \mathbf{B}(\mathbf{r}) \cdot \mathbf{H}(\mathbf{r})$$

In analogy to: 
$$W_E = \frac{1}{2} \int d^3 r \, \mathbf{E}(\mathbf{r}) \cdot \mathbf{D}(\mathbf{r})$$

### Full electrodynamics with time varying fields and sources

## Maxwell's equations

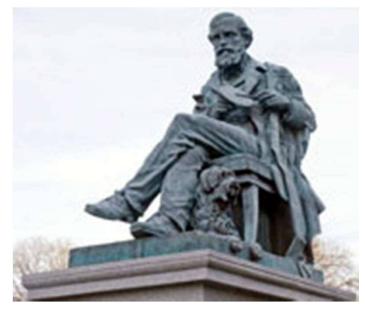


Image of statue in Edinburgh

JAMES Clerk-Maxwell was "the man who changed everything". He stands between Newton and Einstein in the triad of great scientists who shaped the modern world.

Duncan Macmillan -- 2008

http://www.clerkmaxwellfoundation.org/

## Maxwell's equations

Coulomb's law:

$$\nabla \cdot \mathbf{D} = \rho_{free}$$

Ampere-Maxwell's law: 
$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_{free}$$

Faraday's law:  $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$ 

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

No magnetic monopoles:

$$\nabla \cdot \mathbf{B} = 0$$

## Maxwell's equations

Microscopic or vacuum form  $(\mathbf{P} = 0; \mathbf{M} = 0)$ :

Coulomb's law:

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0$$

Ampere-Maxwell's law: 
$$\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$$

Faraday's law:

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

No magnetic monopoles:

$$\nabla \cdot \mathbf{B} = 0$$

$$\Rightarrow c^2 = \frac{1}{\mathcal{E}_0 \mu_0}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \Rightarrow \mathbf{B} = \nabla \times \mathbf{A}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \Rightarrow \nabla \times \left( \mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0$$

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla \Phi$$
or 
$$\mathbf{E} = -\nabla \Phi - \frac{\partial \mathbf{A}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 :$$

$$-\nabla^2 \Phi - \frac{\partial (\nabla \cdot \mathbf{A})}{\partial t} = \rho / \varepsilon_0$$

$$\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$$

$$\nabla \times (\nabla \times \mathbf{A}) + \frac{1}{c^2} \left( \frac{\partial (\nabla \Phi)}{\partial t} + \frac{\partial^2 \mathbf{A}}{\partial t^2} \right) = \mu_0 \mathbf{J}$$

Analysis of the scalar and vector potential equations:

$$-\nabla^2 \Phi - \frac{\partial (\nabla \cdot \mathbf{A})}{\partial t} = \rho / \varepsilon_0$$

$$\nabla \times (\nabla \times \mathbf{A}) + \frac{1}{c^2} \left( \frac{\partial (\nabla \Phi)}{\partial t} + \frac{\partial^2 \mathbf{A}}{\partial t^2} \right) = \mu_0 \mathbf{J}$$

Coulomb gauge form - - require  $\nabla \cdot \mathbf{A}_C = 0$ 

$$-\nabla^2\Phi_C = \rho/\varepsilon_0$$

$$-\nabla^2 \mathbf{A}_C + \frac{1}{c^2} \frac{\partial^2 \mathbf{A}_C}{\partial t^2} + \frac{1}{c^2} \frac{\partial (\nabla \Phi_C)}{\partial t} = \mu_0 \mathbf{J}$$

Note that  $\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_t$  with  $\nabla \times \mathbf{J}_1 = 0$  and  $\nabla \cdot \mathbf{J}_t = 0$ 

Coulomb gauge form - - require  $\nabla \cdot \mathbf{A}_C = 0$ 

$$-\nabla^2 \Phi_C = \rho / \varepsilon_0$$

$$-\nabla^2 \mathbf{A}_C + \frac{1}{c^2} \frac{\partial^2 \mathbf{A}_C}{\partial t^2} + \frac{1}{c^2} \frac{\partial (\nabla \Phi_C)}{\partial t} = \mu_0 \mathbf{J}$$

Note that  $\mathbf{J} = \mathbf{J}_l + \mathbf{J}_t$  with  $\nabla \times \mathbf{J}_l = 0$  and  $\nabla \cdot \mathbf{J}_t = 0$ 

Continuity equation for charge and current density:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J}_{l} = 0 \qquad \Rightarrow \frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{J}_{l} = -\varepsilon_{0} \nabla \cdot \frac{\partial (\nabla \Phi_{C})}{\partial t}$$
$$\Rightarrow \frac{1}{c^{2}} \frac{\partial (\nabla \Phi_{C})}{\partial t} = \varepsilon_{0} \mu_{0} \frac{\partial (\nabla \Phi_{C})}{\partial t} = \mu_{0} \mathbf{J}_{l}$$

$$-\nabla^{2}\mathbf{A}_{C} + \frac{1}{c^{2}} \frac{\partial^{2}\mathbf{A}_{C}}{\partial t^{2}} = \mu_{0}\mathbf{J}_{t}$$
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Analysis of the scalar and vector potential equations:

$$-\nabla^2 \Phi - \frac{\partial (\nabla \cdot \mathbf{A})}{\partial t} = \rho / \varepsilon_0$$

$$\nabla \times (\nabla \times \mathbf{A}) + \frac{1}{c^2} \left( \frac{\partial (\nabla \Phi)}{\partial t} + \frac{\partial^2 \mathbf{A}}{\partial t^2} \right) = \mu_0 \mathbf{J}$$

Lorentz gauge form - - require  $\nabla \cdot \mathbf{A}_L + \frac{1}{c^2} \frac{\partial \Phi_L}{\partial t} = 0$ 

$$-\nabla^2 \Phi_L + \frac{1}{c^2} \frac{\partial^2 \Phi_L}{\partial t^2} = \rho / \varepsilon_0$$

$$-\nabla^2 \mathbf{A}_L + \frac{1}{c^2} \frac{\partial^2 \mathbf{A}_L}{\partial t^2} = \mu_0 \mathbf{J}$$

Lorentz gauge form - - require 
$$\nabla \cdot \mathbf{A}_L + \frac{1}{c^2} \frac{\partial \Phi_L}{\partial t} = 0$$

$$-\nabla^2 \Phi_L + \frac{1}{c^2} \frac{\partial^2 \Phi_L}{\partial t^2} = \rho / \varepsilon_0$$

$$-\nabla^2 \mathbf{A}_L + \frac{1}{c^2} \frac{\partial^2 \mathbf{A}_L}{\partial t^2} = \mu_0 \mathbf{J}$$

Alternate potentials:  $\mathbf{A'}_L = \mathbf{A}_L + \nabla \Lambda$  and  $\Phi'_L = \Phi_L - \frac{\partial \Lambda}{\partial t}$ 

Yields same physics provided that:  $\nabla^2 \Lambda - \frac{1}{c^2} \frac{\partial^2 \Lambda}{\partial t^2} = 0$