PHY 712 Electrodynamics
11-11:50 AM MWF Olin 107

Plan for Lecture 19:

Continue reading Chapter 7

A. **Reflectance and transmittance of electromagnetic waves – extension to anisotropy and complexity**

B. **Frequency dependence of dielectric materials**

11	02-11(Mon)	Chap. 5	Magnetostatics	#10
12	02-13(Wed)	Chap. 5	Magnetostatic fields	
13	02-15(Fri)	Chap. 5	Magnetic dipole fields	Exam
14	02-18(Mon)	Chap. 5	Permeable media	Exam
15	02-20(Wed)	Chap. 5	Magnetic susceptibility and permeability	Exam
16	02-22(Fri)	Chap. 6	Maxwell's equations	Exam
17	02-25(Mon)	Chap. 6	Poynting Vector	#11
18	02-27(Wed)	Chap. 7	Reflectance and transmittance of electromagnetic plane waves	#12
19	02-28(Thur)	Chap. 7	Anisotropic media	#13
20	03-01(Fri)	Chap. 7	Dielectric models; Kramers-Kronig Relations	
21	03-04(Mon)			
22	03-06(Wed)			
23	03-07(Thur)			
24	03-08(Fri)			
	03-11(Mon)	*Spring Break*		
	03-13(Mon)	*Spring Break*		
	03-15(Fri)	*Spring Break*		
	03-18(Mon)	*APS Meeting*		Exam
	03-20(Wed)	*APS Meeting*		Exam
	03-22(Fri)	*APS Meeting*		Exam

Electromagnetic plane waves in isotropic medium with real permeability and permittivity: μ ε.

$$\mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}\cdot\mathbf{r}-ct\right)}\right)$$

$$\mathbf{B}(\mathbf{r},t) = \frac{n}{c}\hat{\mathbf{k}}\times\mathbf{E}(\mathbf{r},t) = \sqrt{\mu\varepsilon}\,\hat{\mathbf{k}}\times\mathbf{E}(\mathbf{r},t)$$

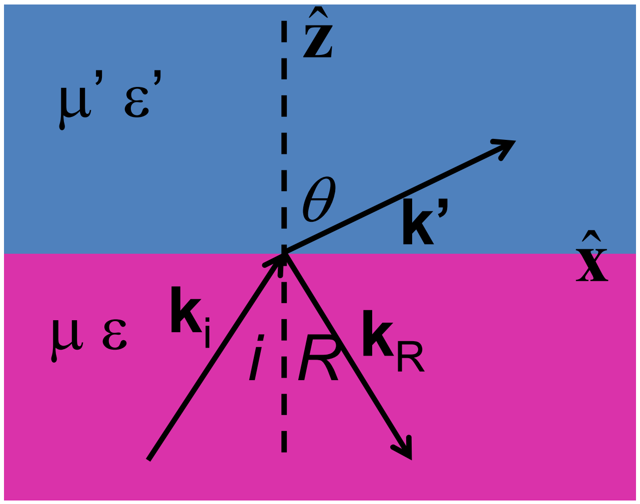
Poynting vector for plane electromagnetic waves:

$$\langle\mathbf{S}\rangle_{avg} = \frac{n|\mathbf{E}_0|^2}{2\mu c}\hat{\mathbf{k}} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|\mathbf{E}_0|^2\hat{\mathbf{k}}$$

Energy density for plane electromagnetic waves:

$$\langle u\rangle_{avg} = \frac{1}{2}\varepsilon|\mathbf{E}_0|^2$$

Reflection and refraction between two isotropic media



Reflectance, transmittance:

$$R = \frac{\mathbf{S}_R \cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}} = \left| \frac{E_{0R}}{E_{0i}} \right|^2 \qquad T = \frac{\mathbf{S}' \cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}} = \left| \frac{E'_0}{E_{0i}} \right|^2 \frac{n'}{n} \frac{\mu}{\mu'} \frac{\cos\theta}{\cos i}$$

Note that $\quad R + T = 1$

For s-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{n\cos i - \frac{\mu}{\mu'} n'\cos\theta}{n\cos i + \frac{\mu}{\mu'} n'\cos\theta} \qquad \frac{E'_0}{E_{0i}} = \frac{2n\cos i}{n\cos i + \frac{\mu}{\mu'} n'\cos\theta}$$

$$\text{Note that:}\quad n'\cos\theta = \sqrt{n'^2 - n^2\sin^2 i}$$

For p-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'} n'\cos i - n\cos\theta}{\frac{\mu}{\mu'} n'\cos i + n\cos\theta} \qquad \frac{E'_0}{E_{0i}} = \frac{2n\cos i}{\frac{\mu}{\mu'} n'\cos i + n\cos\theta}$$

$$\text{Note that:}\quad n'\cos\theta = \sqrt{n'^2 - n^2\sin^2 i}$$

Special case: normal incidence (i=0, θ=0)

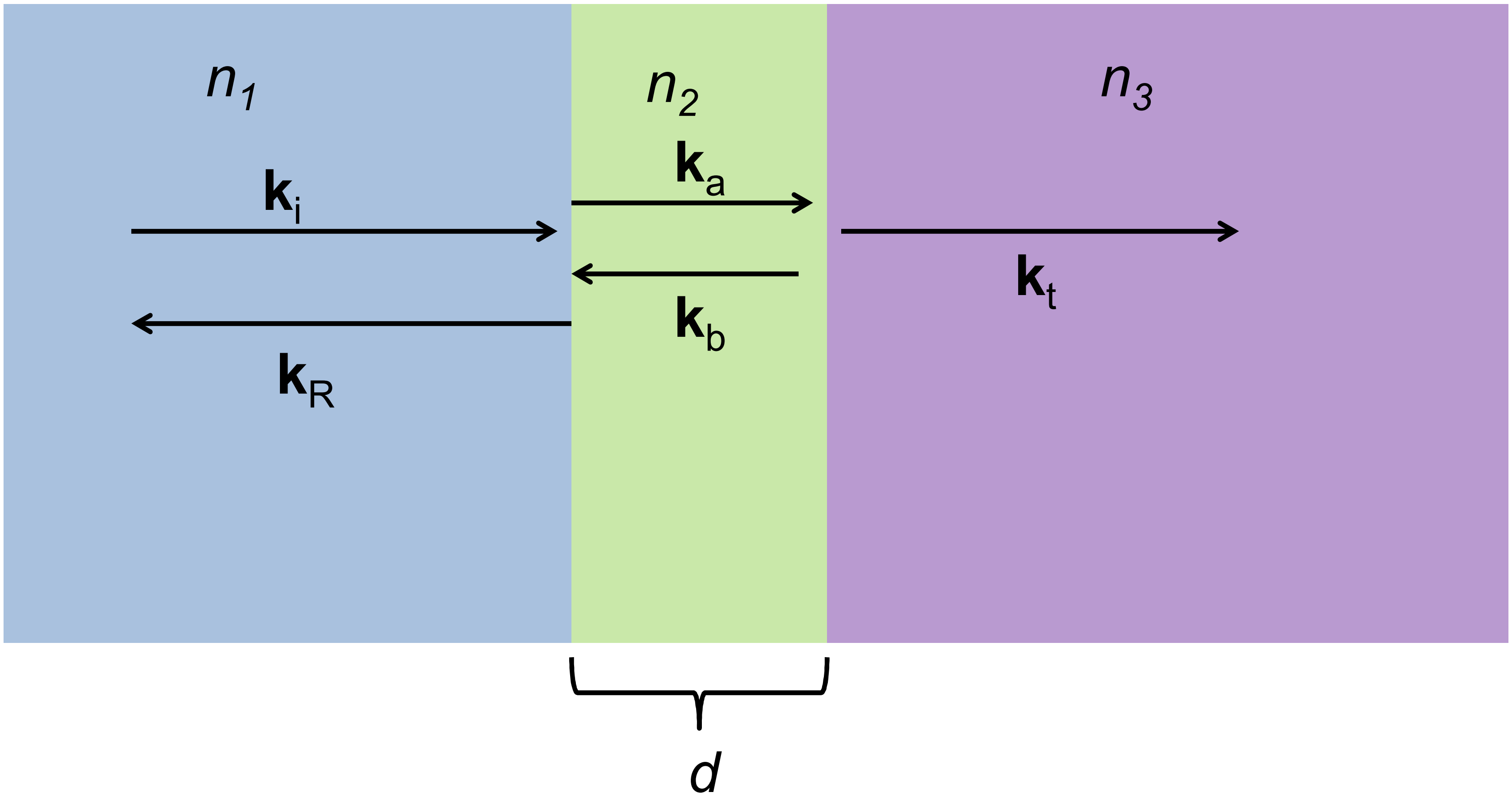
$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'}n' - n}{\frac{\mu}{\mu'}n' + n} \qquad \frac{E'_0}{E_{0i}} = \frac{2n}{\frac{\mu}{\mu'}n' + n}$$

Reflectance, transmittance:

$$\mathrm{R} = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n' - n}{\frac{\mu}{\mu'}n' + n}\right|^2$$

$$\mathrm{T} = \left|\frac{E'_0}{E_{0i}}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'} = \left|\frac{2n}{\frac{\mu}{\mu'}n' + n}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'}$$

Multilayer dielectrics   (Problem #7.2)



Extension of analysis to anisotropic media --

Extension of analysis to complex dielectric functions

For simplicity assume that $\mu = \mu_0$

Suppose the dielectric function is complex:

$$\varepsilon = \varepsilon_R + i\varepsilon_I \qquad \frac{\varepsilon}{\varepsilon_0} = \left(n_R + in_I\right)^2 \equiv \alpha + i\beta$$

$$n_R = \left(\frac{\sqrt{\alpha^2+\beta^2}+\alpha}{2}\right)^{1/2} \qquad n_I = \left(\frac{\sqrt{\alpha^2+\beta^2}-\alpha}{2}\right)^{1/2}$$

$$\mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}\cdot\mathbf{r}-ct\right)}\right) = \Re\left(\mathbf{E}_0 e^{i\frac{\omega}{c}\left(n_R\hat{\mathbf{k}}\cdot\mathbf{r}-ct\right)}\right) e^{-\frac{\omega}{c}n_I\hat{\mathbf{k}}\cdot\mathbf{r}}$$

Paul Karl Ludwig Drude 1863-1906

http://photos.aip.org/history/Thumbnails/drude_paul_a1.jpg

Drude model:

Vibrations of charged particles near equilibrium:

$$m\delta\ddot{\mathbf{r}} = q\mathbf{E}_0 e^{-i\omega t} - m\omega_0^2\delta\mathbf{r} - m\gamma\delta\dot{\mathbf{r}}$$

For $\quad \delta\mathbf{r} \equiv \delta\mathbf{r}_0 e^{-i\omega t}, \qquad \delta\mathbf{r}_0 = \frac{q\mathbf{E}_0}{m}\frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}$

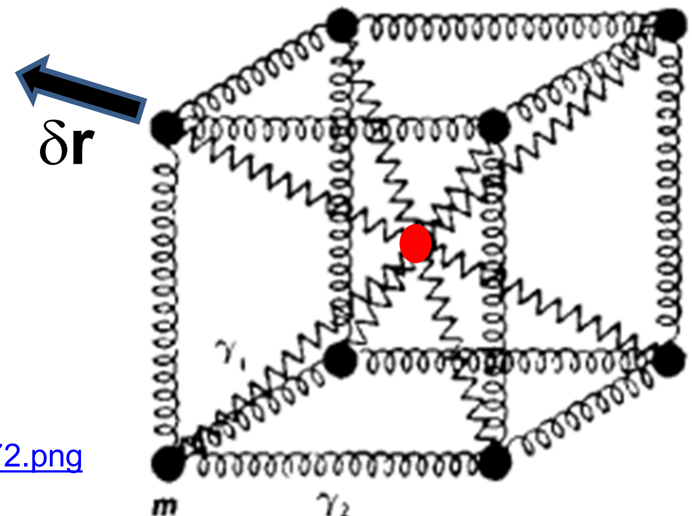
Induced dipole:

$$\mathbf{p} = q\,\delta\mathbf{r} = \frac{q^2\mathbf{E}_0}{m}\frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}e^{-i\omega t}$$

Displacement field:

$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\mathbf{E} + \mathbf{P}$$

$$\mathbf{P} = \sum_i \mathbf{p}_i \delta^3(\mathbf{r} - \mathbf{r}_i)$$



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