PHY 712 Electrodynamics
11-11:50 AM MWF Olin 107

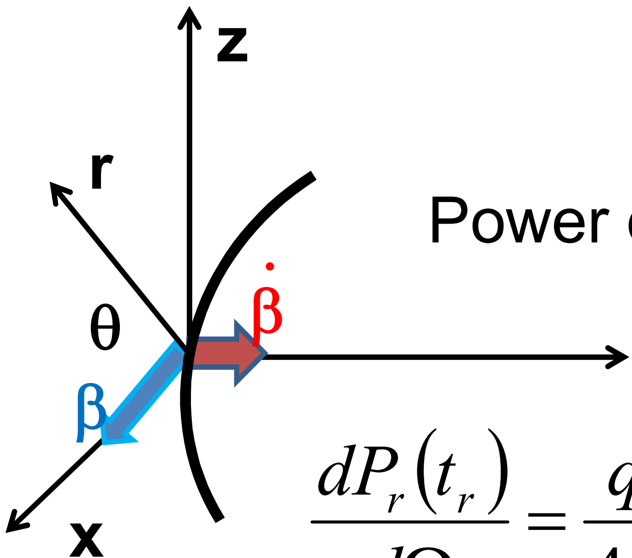
Plan for Lecture 30:

Continue reading Chap. 14 – Synchrotron radiation

1. **Radiation from electron synchrotron devices**
2. **Radiation from astronomical objects in circular orbits**

	03-22(Fri)	*APS Meeting*	(no class)	Exam
25	03-25(Mon)	Chap. 11	Lorentz transformations	#17
26	03-27(Wed)	Chap. 11	Transformations between electromagnetic fields	#18
27	03-28(Thur)	Chap. 11	Liénard-Wiechert potentials revisited	
	03-29(Fri)	*Good Friday*	(no class)	
28	04-01(Mon)	Chap. 14	Radiation by accelerated charges	#19
29	04-03(Wed)	Chap. 14	Radiation by accelerated charges	#20
30	04-05(Fri)	Chap. 14	Synchrotron radiation spectrum	#21
31	04-08(Mon)			
32	04-10(Wed)			
33	04-12(Fri)			
	04-15(Mon)		(no class -- presentation preparation)	
	04-17(Wed)		(no class -- presentation preparation)	
	04-19(Fri)		(no class -- presentation preparation)	
34	04-22(Mon)			
35	04-24(Wed)			
36	04-26(Fri)			
	04-29(Mon)		Student presentations I	
	05-01(Wed)		Student presentations II	
	05-02(Thurs)		Student presentations III	

Radiation from charged particle in circular path



Power distribution for circular acceleration

$$\frac{dP_r(t_r)}{d\Omega} = \frac{q^2}{4\pi c} \left. \frac{\left| \hat{\mathbf{R}} \times \left[\left(\hat{\mathbf{R}} - \boldsymbol{\beta} \right) \times \dot{\boldsymbol{\beta}} \right]^2 \right|}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}}\right)^5} \right|_{t_r = t - R/c}$$

$$= \frac{q^2}{4\pi c} \left. \frac{\left|\dot{\boldsymbol{\beta}}\right|^2 \left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}}\right)^2 - \left(\hat{\mathbf{R}} \cdot \dot{\boldsymbol{\beta}}\right)^2 \left(1 - \beta^2\right)}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}}\right)^5} \right|_{t_r = t - R/c}$$

$$P_r(t_r) = \int d\Omega \frac{dP_r(t_r)}{d\Omega} = \frac{2}{3} \frac{q^2}{c^3} \left|\dot{\mathbf{v}}\right|^2 \gamma^4$$

Spectral composition of electromagnetic radiation -- continued

When the dust clears, the spectral intensity depends on the following integral:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt_r \left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}(t_r)\right)\right] e^{i\omega\left(t_r - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)/c\right)}\right|^2$$

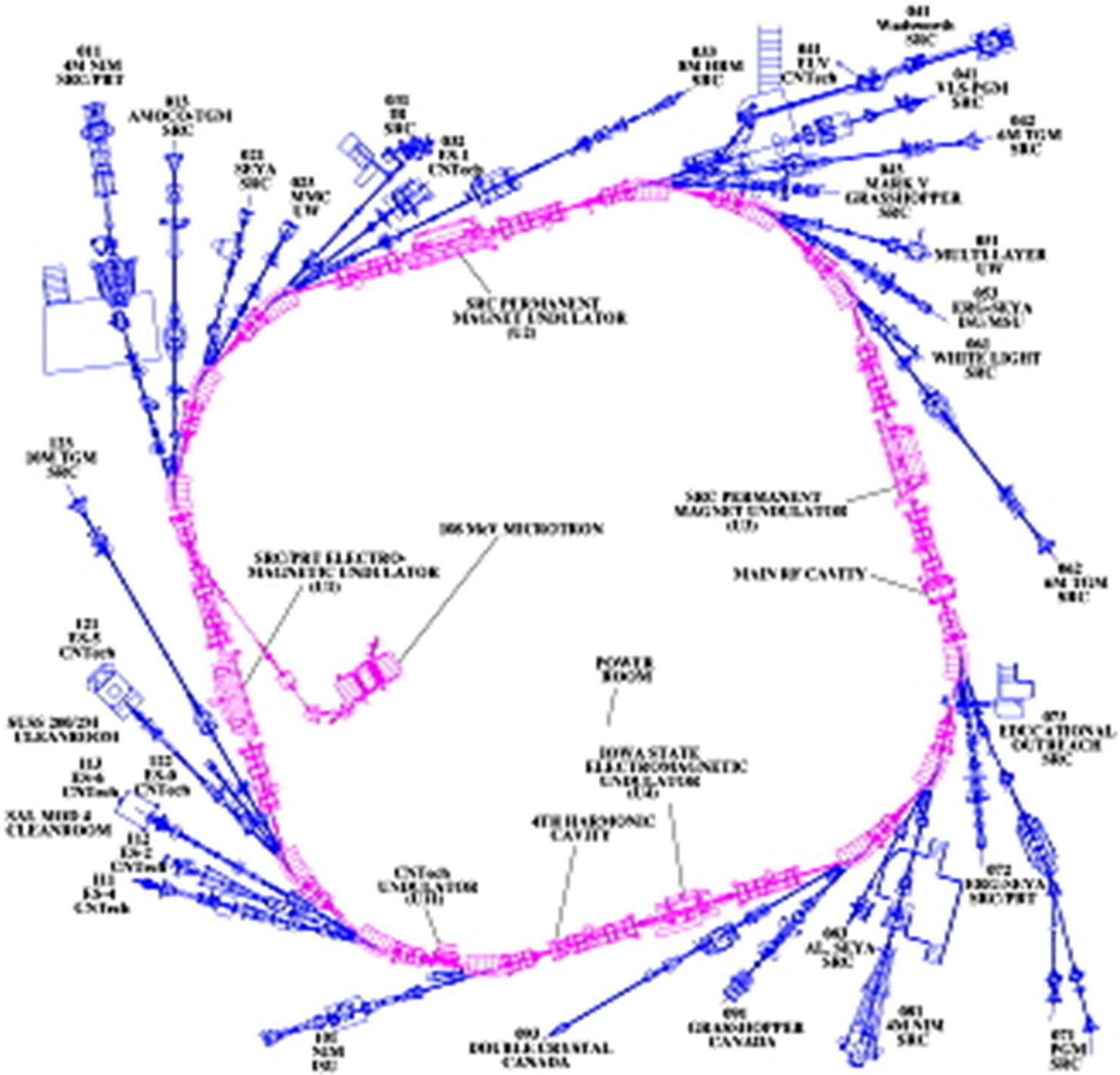
Synchrotron radiation light source installations

Synchrotron radiation center in Madison, Wisconsin

E_c=0.5 GeV and 1 GeV; λ_c = 20 Å and 10 Å

http://www.src.wisc.edu/

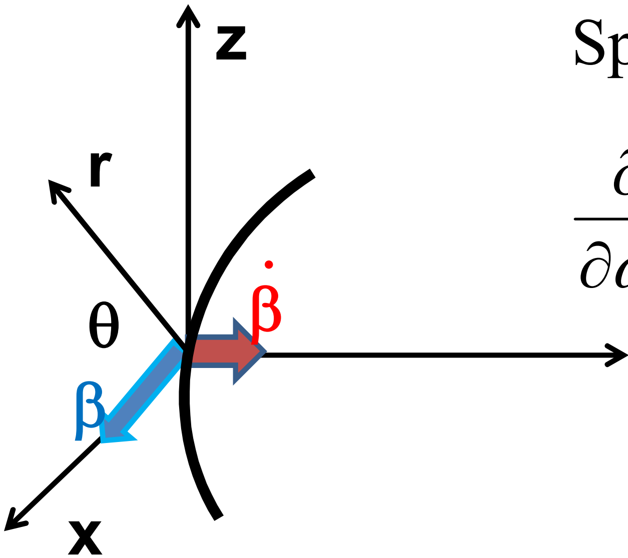
SRC –
"Aladdin" --
Madison
Wisconsin



Brookhaven National Laboratory – National Light Source

E_c=0.6 GeV; λ_c = 20 Å

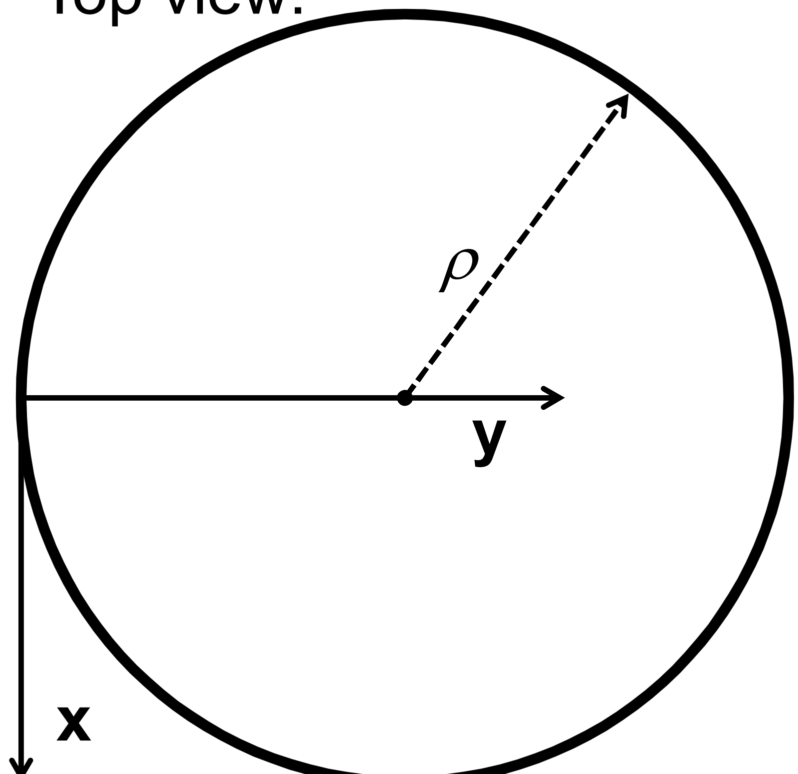
http://www.bnl.gov/ps/



Spectral intensity relationship:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt_r\left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}(t_r)\right)\right]e^{i\omega\left(t_r-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)/c\right)}\right|^2$$

Top view:

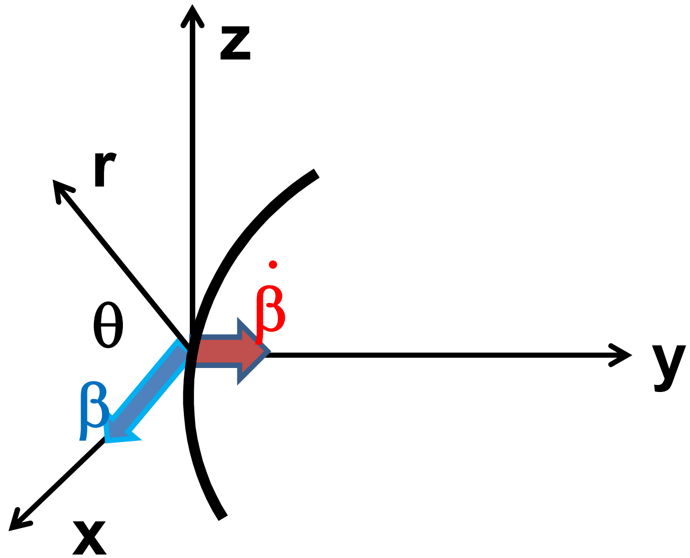


$$\mathbf{R}_q(t_r) = \rho\hat{\mathbf{x}}\sin(vt_r/\rho) + \rho\hat{\mathbf{y}}\left(1-\cos(vt_r/\rho)\right)$$

$$\boldsymbol{\beta}(t_r) = \beta\left(\hat{\mathbf{x}}\cos(vt_r/\rho) + \hat{\mathbf{y}}\sin(vt_r/\rho)\right)$$

For convenience, choose:

$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$



$$\mathbf{R}_q(t_r) = \rho\hat{\mathbf{x}}\sin(vt_r/\rho)$$
$$+ \; \rho\hat{\mathbf{y}}(1-\cos(vt_r/\rho))$$
$$\boldsymbol{\beta}(t_r) = \beta(\hat{\mathbf{x}}\cos(vt_r/\rho) + \hat{\mathbf{y}}\sin(vt_r/\rho))$$

For convenience, choose:

$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$

Note that we have previous shown that in the radiation zone, the Poynting vector is in the $\hat{\mathbf{r}}$ direction; we can then choose to analyze two orthogonal polarization directions:

$$\boldsymbol{\varepsilon}_{\parallel} = \hat{\mathbf{y}} \qquad\qquad \boldsymbol{\varepsilon}_{\perp} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$$

$$\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta}) = \beta(-\boldsymbol{\varepsilon}_{\parallel}\sin(vt_r/\rho) + \boldsymbol{\varepsilon}_{\perp}\sin\theta\cos(vt_r/\rho))$$

Modified Bessel functions

$$K_{1/3}(\xi)=\sqrt{3}\int_0^\infty dx\cos\left[\tfrac{3}{2}\xi\left(x+\tfrac{1}{3}x^3\right)\right] \qquad K_{2/3}(\xi)=\sqrt{3}\int_0^\infty dx\, x\sin\left[\tfrac{3}{2}\xi\left(x+\tfrac{1}{3}x^3\right)\right]$$

Exponential factor

$$\omega\left(t_r-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)\right)=\omega\left(t_r-\frac{\rho}{c}\cos\theta\sin\left(vt_r/\rho\right)\right)$$

$$\text{In the limit of}\quad t_r\approx 0,\quad \theta\approx 0,\ v\approx c\left(1-\frac{1}{2\gamma^2}\right)$$

$$\omega\left(t_r-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)\right)\approx\frac{\omega t_r}{2\gamma^2}\left(1+\gamma^2\theta^2\right)+\frac{\omega c^2 t_r^{\,3}}{6\rho^2}=\frac{3}{2}\xi\left(x+\frac{1}{3}x^3\right)$$

$$\text{where}\quad \xi=\frac{\omega\rho}{3c\gamma^3}\left(1+\gamma^2\theta^2\right)^{3/2}\ \text{and}\ x=\frac{c\gamma t_r}{\rho\left(1+\gamma^2\theta^2\right)^{1/2}}$$