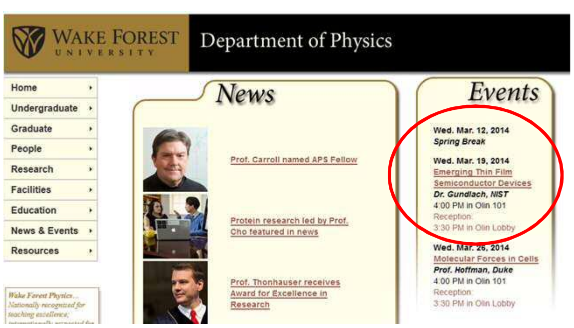
PHY 712 Electrodynamics
10-10:50 AM MWF Olin 107

Plan for Lecture 20:

Start reading Chap. 9

- **A. Electromagnetic waves due to specific sources**
- **B. Dipole radiation patterns**

7	Fri: 01/31/2014	Chap. 3	Cylindrical and spherical geometries	#7	02/05/2014
8	Mon: 02/03/2014	Chap. 4	Multipole analysis of charge distributions	#8	02/05/2014
9	Wed: 02/05/2014	Chap. 4	Dipoles and dielectrics	#9	02/07/2014
10	Fri: 02/07/2014	Chap. 4	Dipoles and dielectrics	#10	02/10/2014
11	Mon: 02/10/2014	Chap. 5	Magnetostatics	#11	02/12/2014
12	Wed: 02/12/2014	Chap. 5	Magnetostatics	#12	02/14/2014
	Fri: 02/14/2014		Class cancelled because of weather		
13	Mon: 02/17/2014	Chap. 5	Magnetostatics	#13	02/28/2014
14	Mon: 02/17/2014	Chap. 6	Maxwell's equations	#14	02/28/2014
15	Wed: 02/19/2014	Chap. 6	Electromagnetic energy and force	#15	02/28/2014
16	Fri: 02/21/2014	Chap. 7	Electromagnetic plane waves	#16	02/28/2014
17	Fri: 02/21/2014	Chap. 7	Dynamic dielectric media and their effects	#17	02/28/2014
	Mon: 02/24/2014		No class -- NAWH out of town		
	Wed: 02/26/2014		No class -- NAWH out of town		
18	Fri: 02/28/2014	Chap. 7	Complex dielectrics; TEM modes		
	Mon: 03/03/2014	*APS Meeting*	Take-home exam (no class meeting)		
	Wed: 03/05/2014	*APS Meeting*	Take-home exam (no class meeting)		
	Fri: 03/07/2014	*APS Meeting*	Take-home exam (no class meeting)		
	Mon: 03/10/2014	*Spring Break*			
	Wed: 03/12/2014	*Spring Break*			
	Fri: 03/14/2014	*Spring Break*			
19	Mon: 03/17/2014	Chap. 8	Wave guides; Take-home exam due	#18	3/21/2014
20	Wed: 03/19/2014	Chap. 9	Sources of Electromagnetic Waves	#19	3/21/2014
21	Fri: 03/21/2014	Chap. 9	Sources of Electromagnetic Waves	#20	3/28/2014



WFU Physics Colloquium

TITLE: Measurements to Advance Emerging Thin Film Semiconductor Devices

SPEAKER: Dr. David J. Gundlach,

National Institute of Standards and Technology, Gaithersburg, MD

TIME: Wednesday March 19, 2014 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

A key driver of pervasive and portable electronics is the continued advancement in function and performance of thin film electronics. One example is the active matrix flat panel display which utilizes thin film transistors to address individual pixels comprised of liquid crystal light valves or organic electroluminescent devices. Thin film electronics has matured to the point that continued economic growth of silicon (Si) metal-oxide-field effect transistor (MOSFET) technology is intimately tied to the success of the thin film electronic technology platform in which Si MOSFETS are heterogeneously integrated.

04/19/2014 PHY 712 Spring 2014 -- Lecture 24 4

Maxwell's equations

Microscopic or vacuum form $(\mathbf{P}=0;\ \mathbf{M}=0)$:

Coulomb's law: $\nabla\cdot\mathbf{E}=\rho/\varepsilon_0$

Ampere-Maxwell's law: $\nabla\times\mathbf{B}-\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}=\mu_0\mathbf{J}$

Faraday's law: $\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$

No magnetic monopoles: $\nabla\cdot\mathbf{B}=0$

$$\Rightarrow c^2=\frac{1}{\varepsilon_0\mu_0}$$

04/19/2014 PHY 712 Spring 2014 -- Lecture 24 5

Formulation of Maxwell's equations in terms of vector and scalar potentials

$$\nabla\cdot\mathbf{B}=0 \qquad \Rightarrow \mathbf{B}=\nabla\times\mathbf{A}$$

$$\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0 \qquad \Rightarrow \nabla\times\left(\mathbf{E}+\frac{\partial\mathbf{A}}{\partial t}\right)=0$$

$$\mathbf{E}+\frac{\partial\mathbf{A}}{\partial t}=-\nabla\Phi$$

$$\text{or}\quad \mathbf{E}=-\nabla\Phi-\frac{\partial\mathbf{A}}{\partial t}$$

04/19/2014 PHY 712 Spring 2014 -- Lecture 24 6

Formulation of Maxwell's equations in terms of vector and scalar potentials -- continued

Lorentz gauge form - - require : $\nabla \cdot \mathbf{A}_L + \frac{1}{c^2}\frac{\partial \Phi_L}{\partial t} = 0$

$$-\nabla^2 \Phi_L + \frac{1}{c^2}\frac{\partial^2 \Phi_L}{\partial t^2} = \rho / \varepsilon_0$$

$$-\nabla^2 \mathbf{A}_L + \frac{1}{c^2}\frac{\partial^2 \mathbf{A}_L}{\partial t^2} = \mu_0 \mathbf{J}$$

General equation form :

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi = -4\pi f$$

04/19/2014 PHY 712 Spring 2014 -- Lecture 24 7

Solution of Maxwell's equations in the Lorentz gauge -- continued

$$G(\mathbf{r}, t; \mathbf{r}', t') = \frac{1}{|\mathbf{r} - \mathbf{r}'|}\delta\left(t' - \left(t - |\mathbf{r} - \mathbf{r}'|/c\right)\right)$$

Solution for field $\Psi(\mathbf{r}, t)$:

$$\Psi(\mathbf{r}, t) = \Psi_{f=0}(\mathbf{r}, t) + \int d^3r' \int dt' \frac{1}{|\mathbf{r} - \mathbf{r}'|}\delta\left(t' - \left(t - \frac{1}{c}|\mathbf{r} - \mathbf{r}'|\right)\right) f(\mathbf{r}', t')$$

04/19/2014 PHY 712 Spring 2014 -- Lecture 24 8

Electromagnetic waves from time harmonic sources

Charge density : $\rho(\mathbf{r}, t) = \Re\left(\tilde{\rho}(\mathbf{r}, \omega) e^{-i\omega t}\right)$

Current density : $\mathbf{J}(\mathbf{r}, t) = \Re\left(\tilde{\mathbf{J}}(\mathbf{r}, \omega) e^{-i\omega t}\right)$

Note that the continuity condition :

$$\frac{\partial \rho(\mathbf{r}, t)}{\partial t} + \nabla \cdot \mathbf{J}(\mathbf{r}, t) = 0 \quad \Rightarrow -i\omega \tilde{\rho}(\mathbf{r}, \omega) + \nabla \cdot \tilde{\mathbf{J}}(\mathbf{r}, \omega) = 0$$

General source : $f(\mathbf{r}, t) = \Re\left(\tilde{f}(\mathbf{r}, \omega) e^{-i\omega t}\right)$

For $\tilde{f}(\mathbf{r}, \omega) = \frac{1}{4\pi\varepsilon_0}\tilde{\rho}(\mathbf{r}, \omega)$

or $\tilde{f}(\mathbf{r}, \omega) = \frac{\mu_0}{4\pi}\tilde{J}_i(\mathbf{r}, \omega)$

04/19/2014 PHY 712 Spring 2014 -- Lecture 24 9

Electromagnetic waves from time harmonic sources – continued:

$$\Psi(\mathbf{r},t) = \Psi_{f=0}(\mathbf{r},t) + \int d^3r' \int dt' \frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\left(t' - \left(t - \frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right) f(\mathbf{r}',t')$$

$$\tilde{\Psi}(\mathbf{r},\omega)e^{-i\omega t} = \tilde{\Psi}_{f=0}(\mathbf{r},\omega)e^{-i\omega t} + \int d^3r' \int dt' \frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\left(t' - \left(t - \frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right)\tilde{f}(\mathbf{r}',\omega)e^{-i\omega t'}$$

$$= \tilde{\Psi}_{f=0}(\mathbf{r},\omega)e^{-i\omega t} + \int d^3r' \frac{e^{i\frac{\omega}{c}|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{f}(\mathbf{r}',\omega)e^{-i\omega t}$$

Electromagnetic waves from time harmonic sources – continued:

For scalar potential (Lorentz gauge, $k \equiv \frac{\omega}{c}$)

$$\tilde{\Phi}(\mathbf{r},\omega) = \tilde{\Phi}_0(\mathbf{r},\omega) + \frac{1}{4\pi\varepsilon_0}\int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\rho}(\mathbf{r}',\omega)$$

For vector potential (Lorentz gauge, $k \equiv \frac{\omega}{c}$)

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) = \tilde{\mathbf{A}}_0(\mathbf{r},\omega) + \frac{\mu_0}{4\pi}\int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

Electromagnetic waves from time harmonic sources – continued:

Useful expansion :

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} = ik\sum_{lm} j_l(kr_<)h_l(kr_>)Y_{lm}(\hat{\mathbf{r}})Y^*_{lm}(\hat{\mathbf{r}}')$$

Spherical Bessel function : $j_l(kr)$

Spherical Hankel function : $h_l(kr) = j_l(kr) + in_l(kr)$

$$\tilde{\Phi}(\mathbf{r},\omega) = \tilde{\Phi}_0(\mathbf{r},\omega) + \sum_{lm}\tilde{\phi}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

$$\tilde{\phi}_{lm}(r,\omega) = \frac{ik}{\varepsilon_0}\int d^3r' \tilde{\rho}(\mathbf{r}',\omega) j_l(kr_<)h_l(kr_>)Y^*_{lm}(\hat{\mathbf{r}}')$$

Electromagnetic waves from time harmonic sources – continued:

Useful expansion :

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} = ik\sum_{lm} j_l(kr_<)h_l(kr_>)Y_{lm}(\hat{\mathbf{r}})Y^*_{lm}(\hat{\mathbf{r}}')$$

Spherical Bessel function : $j_l(kr)$

Spherical Hankel function : $h_l(kr) = j_l(kr) + in_l(kr)$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) = \tilde{\mathbf{A}}_0(\mathbf{r},\omega) + \sum_{lm} \tilde{\mathbf{a}}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

$$\tilde{\mathbf{a}}_{lm}(r,\omega) = ik\mu_0\int d^3r'\,\tilde{\mathbf{J}}(\mathbf{r}',\omega)j_l(kr_<)h_l(kr_>)Y^*_{lm}(\hat{\mathbf{r}}')$$

04/19/2014 PHY 712 Spring 2014 -- Lecture 24 13

Electromagnetic waves from time harmonic sources – continued:

$$\tilde{\Phi}(\mathbf{r},\omega) = \tilde{\Phi}_0(\mathbf{r},\omega) + \sum_{lm} \tilde{\phi}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

$$\tilde{\phi}_{lm}(r,\omega) = \frac{ik}{\varepsilon_0}\int d^3r'\,\tilde{\rho}(\mathbf{r}',\omega)j_l(kr_<)h_l(kr_>)Y^*_{lm}(\hat{\mathbf{r}}')$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) = \tilde{\mathbf{A}}_0(\mathbf{r},\omega) + \sum_{lm} \tilde{\mathbf{a}}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

$$\tilde{\mathbf{a}}_{lm}(r,\omega) = ik\mu_0\int d^3r'\,\tilde{\mathbf{J}}(\mathbf{r}',\omega)j_l(kr_<)h_l(kr_>)Y^*_{lm}(\hat{\mathbf{r}}')$$

For r >> (extent of source)

$$\tilde{\phi}_{lm}(r,\omega) \approx \frac{ik}{\varepsilon_0}h_l(kr)\int d^3r'\,\tilde{\rho}(\mathbf{r}',\omega)j_l(kr')Y^*_{lm}(\hat{\mathbf{r}}')$$

$$\tilde{\mathbf{a}}_{lm}(r,\omega) \approx ik\mu_0 h_l(kr)\int d^3r'\,\tilde{\mathbf{J}}(\mathbf{r}',\omega)j_l(kr')Y^*_{lm}(\hat{\mathbf{r}}')$$

04/19/2014 PHY 712 Spring 2014 -- Lecture 24 14

Electromagnetic waves from time harmonic sources – continued:

For r >> (extent of source)

$$\tilde{\phi}_{lm}(r,\omega) \approx \frac{ik}{\varepsilon_0}h_l(kr)\int d^3r'\,\tilde{\rho}(\mathbf{r}',\omega)j_l(kr')Y^*_{lm}(\hat{\mathbf{r}}')$$

$$\tilde{\mathbf{a}}_{lm}(r,\omega) \approx ik\mu_0 h_l(kr)\int d^3r'\,\tilde{\mathbf{J}}(\mathbf{r}',\omega)j_l(kr')Y^*_{lm}(\hat{\mathbf{r}}')$$

Note that $\tilde{\rho}(\mathbf{r}',\omega)$ and $\tilde{\mathbf{J}}(\mathbf{r}',\omega)$ are connected via the continuity condition : $-i\omega\,\tilde{\rho}(\mathbf{r},\omega) + \nabla\cdot\tilde{\mathbf{J}}(\mathbf{r},\omega) = 0$

$$\tilde{\phi}_{lm}(r,\omega) \approx \frac{ik}{\varepsilon_0}h_l(kr)\int d^3r'\,\tilde{\rho}(\mathbf{r}',\omega)j_l(kr')Y^*_{lm}(\hat{\mathbf{r}}')$$

$$= -\frac{k}{\omega\varepsilon_0}h_l(kr)\int d^3r'\,\tilde{\mathbf{J}}(\mathbf{r}',\omega)\cdot\nabla'\left(j_l(kr')Y^*_{lm}(\hat{\mathbf{r}}')\right)$$

04/19/2014 PHY 712 Spring 2014 -- Lecture 24 15

Electromagnetic waves from time harmonic sources – continued:

Various approximations :

$$kr >> 1 \quad \Rightarrow h_l(kr) \approx (-i)^{l+1} \frac{e^{ikr}}{kr}$$

$$kr' << 1 \quad \Rightarrow j_l(kr') \approx \frac{(kr')^l}{(2l+1)!!}$$

Lowest (non - trivial) contributions in l expansions :

$$\tilde{\phi}_{1m}(r,\omega) \approx -\frac{ik}{\varepsilon_0}\frac{e^{ikr}}{kr}\int d^3r'\, \tilde{\rho}(\mathbf{r}',\omega)\frac{kr'}{3} Y^*_{1m}(\hat{\mathbf{r}}')$$

$$\tilde{\mathbf{a}}_{00}(r,\omega) \approx ik\mu_0\left((-i)\frac{e^{ikr}}{kr}\right)\int d^3r'\, \tilde{\mathbf{J}}(\mathbf{r}',\omega) Y^*_{00}(\hat{\mathbf{r}}')$$

Electromagnetic waves from time harmonic sources – continued:

Lowest order contribution; dipole radiation :

Define dipole moment at frequency ω :

$$\mathbf{p}(\omega) \equiv \int d^3r\, \mathbf{r}\tilde{\rho}(\mathbf{r},\omega) = -\frac{1}{i\omega}\int d^3r\, \tilde{\mathbf{J}}(\mathbf{r},\omega)$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) = -\frac{i\mu_0\omega}{4\pi}\mathbf{p}(\omega)\frac{e^{ikr}}{r}$$

$$\tilde{\Phi}(\mathbf{r},\omega) = -\frac{i}{4\pi\omega\varepsilon_0}\mathbf{p}(\omega)\cdot\hat{\mathbf{r}}\left(1+\frac{i}{kr}\right)\frac{e^{ikr}}{r}$$

Note: in this case we have only assumed the throughout the extent of the source *kr'<<1*.

Electromagnetic waves from time harmonic sources – continued:

$$\tilde{\mathbf{E}}(\mathbf{r},\omega) = -\nabla\tilde{\Phi}(\mathbf{r},\omega) + i\omega\tilde{\mathbf{A}}(\mathbf{r},\omega)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{e^{ikr}}{r}\left(k^2((\hat{\mathbf{r}}\times\mathbf{p}(\omega))\times\hat{\mathbf{r}}) + \left(\frac{3\hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\mathbf{p}(\omega)) - \mathbf{p}(\omega)}{r^2}\right)(1-ikr)\right)$$

$$\tilde{\mathbf{B}}(\mathbf{r},\omega) = \nabla\times\tilde{\mathbf{A}}(\mathbf{r},\omega)$$

$$= \frac{1}{4\pi\varepsilon_0 c^2}\frac{e^{ikr}}{r}k^2(\hat{\mathbf{r}}\times\mathbf{p}(\omega))\left(1-\frac{1}{ikr}\right)$$

Power radiated for $kr >> 1$:

$$\frac{dP}{d\Omega} = r^2\hat{\mathbf{r}}\cdot\langle\mathbf{S}\rangle_{avg} = \frac{r^2\hat{\mathbf{r}}}{2\mu_0}\hat{\mathbf{r}}\cdot\Re\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)\times\tilde{\mathbf{B}}^*(\mathbf{r},\omega)\right)$$

$$= \frac{c^2k^4}{32\pi^2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left|(\hat{\mathbf{r}}\times\mathbf{p}(\omega))\times\hat{\mathbf{r}}\right|^2$$