PHY 712 Electrodynamics
10-10:50 AM MWF Olin 107

Plan for Lecture 23:

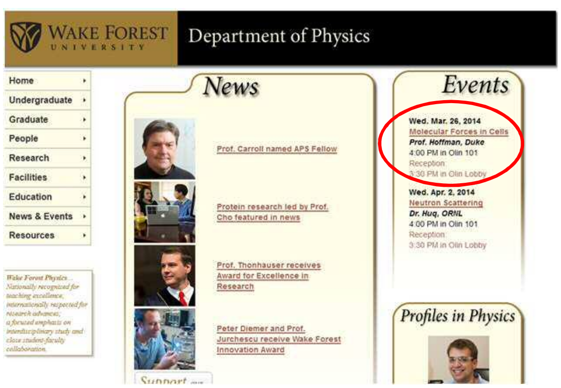
Continue reading Chap. 11 –

Theory of Special Relativity

A. Lorentz transformation relations

B. Energy and momentum

C. Electromagnetic field transformations



WFU Physics Colloquium

TITLE: Measuring Molecular Forces Across Specific Proteins in Living Cells

SPEAKER: Professor Brent D. Hoffman,

Department of Biomedical Engineering, Duke University

TIME: Wednesday March 26, 2014 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

In vivo, cells adhere to the deformable extracellular matrix (ECM) that is both a source of applied forces and a means of mechanical support. Cells detect and interpret mechanical signals, such as force and rigidity, from the ECM through mechanotransduction. While the connections between cells and the ECM, mediated by structures called focal adhesions (FAs), are primary determinants of mechanotransduction, the molecular mechanisms mediating this process are largely unknown. Progress has been limited by an inability to measure dynamic forces across proteins in living cells. Therefore we developed an experimentally calibrated Forster resonance energy transfer (FRET)-based biosensor that measures forces across specific proteins with pico-Newton sensitivity. The sensor has been applied to vinculin, a critical linker protein in the connections between the integrins and

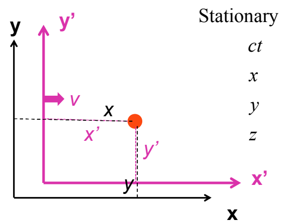
10	Fri: 02/07/2014	Chap. 4	Dipoles and dielectrics	#10	02/10/2014
11	Mon: 02/10/2014	Chap. 5	Magnetostatics	#11	02/12/2014
12	Wed: 02/12/2014	Chap. 5	Magnetostatics	#12	02/14/2014
	Fri: 02/14/2014		Class cancelled because of weather		
13	Mon: 02/17/2014	Chap. 5	Magnetostatics	#13	02/28/2014
14	Mon: 02/17/2014	Chap. 6	Maxwell's equations	#14	02/28/2014
15	Wed: 02/19/2014	Chap. 6	Electromagnetic energy and force	#15	02/28/2014
16	Fri: 02/21/2014	Chap. 7	Electromagnetic plane waves	#16	02/28/2014
17	Fri: 02/21/2014	Chap. 7	Dynamic dielectric media and their effects	#17	02/28/2014
	Mon: 02/24/2014		No class -- NAWH out of town		
	Wed: 02/26/2014		No class -- NAWH out of town		
18	Fri: 02/28/2014	Chap. 7	Complex dielectrics; TEM modes		
	Mon: 03/03/2014	*APS Meeting*	Take-home exam (no class meeting)		
	Wed: 03/05/2014	*APS Meeting*	Take-home exam (no class meeting)		
	Fri: 03/07/2014	*APS Meeting*	Take-home exam (no class meeting)		
	Mon: 03/10/2014	*Spring Break*			
	Wed: 03/12/2014	*Spring Break*			
	Fri: 03/14/2014	*Spring Break*			
19	Mon: 03/17/2014	Chap. 8	Wave guides;Take-home exam due	#18	3/21/2014
20	Wed: 03/19/2014	Chap. 9	Sources of Electromagnetic Waves	#19	3/21/2014
21	Fri: 03/21/2014	Chap. 9	Sources of Electromagnetic Waves	#20	3/28/2014
22	Mon: 03/24/2014	Chap. 11	Special Theory of Relativity	#21	3/28/2014
23	Wed: 03/26/2014	Chap. 11	Special Theory of Relativity	#22	3/28/2014
24	Fri: 03/28/2014	Chap. 11	Special Theory of Relativity	#23	4/04/2014

Lorentz transformations

Convenient notation :

$$\beta_v \equiv \frac{v}{c}$$

$$\gamma_v \equiv \frac{1}{\sqrt{1-\beta_v^2}}$$



Stationary frame		Moving frame
ct	$=$	$\gamma(ct'+\beta x')$
x	$=$	$\gamma(x'+\beta ct')$
y	$=$	y'
z	$=$	z'

Lorentz transformations -- continued

For the moving frame with $\mathbf{v} = v\hat{\mathbf{x}}$:

$$\mathcal{L}_v = \begin{pmatrix} \gamma_v & \gamma_v\beta_v & 0 & 0 \\ \gamma_v\beta_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathcal{L}_v^{-1} = \begin{pmatrix} \gamma_v & -\gamma_v\beta_v & 0 & 0 \\ -\gamma_v\beta_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \mathcal{L}_v \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} \qquad \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \mathcal{L}_v^{-1} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

Notice :

$$c^2t^2 - x^2 - y^2 - z^2 = c^2t'^2 - x'^2 - y'^2 - z'^2$$

Lorentz transformation of the velocity

Define: $u_x \equiv \frac{dx}{dt} \quad u_y \equiv \frac{dy}{dt} \quad u_z \equiv \frac{dz}{dt}$

$u'_x \equiv \frac{dx'}{dt'} \quad u'_y \equiv \frac{dy'}{dt'} \quad u'_z \equiv \frac{dz'}{dt'}$

$$u_x = \frac{u'_x + v}{1 + vu'_x/c^2}$$

$$u_y = \frac{u'_y}{\gamma_v\left(1 + vu'_x/c^2\right)} \qquad u_z = \frac{u'_z}{\gamma_v\left(1 + vu'_x/c^2\right)}$$

Where: $\gamma_v \equiv \frac{1}{\sqrt{1-(v/c)^2}} \equiv \frac{1}{\sqrt{1-\beta_v^2}}$

Note that the velocity components themselves do not obviously transform according to a Lorentz transformation.

03/26/2014 PHY 712 Spring 2014 -- Lecture 26 7

Velocity transformations continued:

Consider: $u_x = \frac{u'_x + v}{1 + vu'_x/c^2} \quad u_y = \frac{u'_y}{\gamma_v\left(1 + vu'_x/c^2\right)} \quad u_z = \frac{u'_z}{\gamma_v\left(1 + vu'_x/c^2\right)}.$

Note that $\gamma_u \equiv \frac{1}{\sqrt{1-(u/c)^2}} = \frac{1 + vu'_x/c^2}{\sqrt{1-(u'/c)^2}\sqrt{1-(v/c)^2}}$

$$\Rightarrow \gamma_u c = \gamma_v\left(\gamma_{u'} c + \beta_v \gamma_{u'} u'_x\right)$$

$$\Rightarrow \gamma_u u_x = \gamma_v\left(\gamma_{u'} u'_x + \gamma_{u'} v\right) = \gamma_v\left(\gamma_{u'} u'_x + \beta_v \gamma_{u'} c\right)$$

$$\Rightarrow \gamma_u u_y = \gamma_{u'} u'_y \qquad \gamma_u u_z = \gamma_{u'} u'_z$$

Velocity 4-vector: $\begin{pmatrix}\gamma_u c\\ \gamma_u u_x\\ \gamma_u u_y\\ \gamma_u u_z\end{pmatrix} = \mathcal{L}_v \begin{pmatrix}\gamma_{u'} c\\ \gamma_{u'} u'_x\\ \gamma_{u'} u'_y\\ \gamma_{u'} u'_z\end{pmatrix}$

03/26/2014 PHY 712 Spring 2014 -- Lecture 26 8

Significance of 4-velocity vector: $\begin{pmatrix}\gamma_u c\\ \gamma_u u_x\\ \gamma_u u_y\\ \gamma_u u_z\end{pmatrix}$

Introduce the “rest” mass *m* of particle characterized by velocity **u**:

$$mc\begin{pmatrix}\gamma_u c\\ \gamma_u u_x\\ \gamma_u u_y\\ \gamma_u u_z\end{pmatrix} = \begin{pmatrix}\gamma_u mc^2\\ \gamma_u mu_x c\\ \gamma_u mu_y c\\ \gamma_u mu_z c\end{pmatrix} = \begin{pmatrix}E\\ p_x c\\ p_y c\\ p_z c\end{pmatrix}$$

Properties of energy-moment 4-vector:

$$\begin{pmatrix}E\\ p_x c\\ p_y c\\ p_z c\end{pmatrix} = \mathcal{L}\begin{pmatrix}E'\\ p'_x c\\ p'_y c\\ p'_z c\end{pmatrix} \qquad \begin{pmatrix}E'\\ p'_x c\\ p'_y c\\ p'_z c\end{pmatrix} = \mathcal{L}^{-1}\begin{pmatrix}E\\ p_x c\\ p_y c\\ p_z c\end{pmatrix}$$

Note: $E^2 - p^2c^2 = E'^2 - p'^2c^2$

03/26/2014 PHY 712 Spring 2014 -- Lecture 26 9

Properties of Energy-momentum 4-vector -- continued

$$\begin{pmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{pmatrix} = \begin{pmatrix} \gamma_u mc^2 \\ \gamma_u m u_x c \\ \gamma_u m u_y c \\ \gamma_u m u_z c \end{pmatrix}$$

Note: $E^2 - p^2c^2 = \frac{(mc^2)^2}{1-\beta_u^2}\left(1 - \left(\frac{u_x}{c}\right)^2 - \left(\frac{u_y}{c}\right)^2 - \left(\frac{u_z}{c}\right)^2\right) = (mc^2)^2 = E'^2 - p'^2 c^2$

Notion of "rest energy": For $\mathbf{p} \equiv 0$, $E = mc^2$

Define kinetic energy: $E_K \equiv E - mc^2 = \sqrt{p^2c^2 + m^2c^4} - mc^2$

Non-relativistic limit: If $\frac{p}{mc} << 1$, $E_K = mc^2\left(\sqrt{1+\left(\frac{p}{mc}\right)^2} - 1\right)$

$$\approx \frac{p^2}{2m}$$

Summary of relativistic energy relationships

$$\begin{pmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{pmatrix} = \begin{pmatrix} \gamma_u mc^2 \\ \gamma_u m u_x c \\ \gamma_u m u_y c \\ \gamma_u m u_z c \end{pmatrix}$$

$E = \sqrt{p^2c^2 + m^2c^4} = \gamma_u mc^2$

Check: $\sqrt{p^2c^2 + m^2c^4} = mc^2\sqrt{\gamma_u^2\beta_u^2 + 1} = \gamma_u mc^2$

Example: for an electron $mc^2 = 0.5\,\text{MeV}$

for $E = 200\,\text{GeV}$

$$\gamma_u = \frac{E}{mc^2} = 4\times 10^5$$

$$\beta_u = \sqrt{1 - \frac{1}{\gamma_u^2}} \approx 1 - \frac{1}{2\gamma_u^2} \approx 1 - 3\times 10^{-12}$$

Special theory of relativity and Maxwell's equations

Continuity equation: $\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$

Lorentz gauge condition: $\frac{1}{c}\frac{\partial \Phi}{\partial t} + \nabla \cdot \mathbf{A} = 0$

Potential equations: $\frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} + \nabla^2 \Phi = 4\pi\rho$

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} + \nabla^2 \mathbf{A} = \frac{4\pi}{c}\mathbf{J}$$

Field relations: $\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

More 4-vectors:

Time and position : $\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \Rightarrow x^\alpha$

Charge and current : $\begin{pmatrix} c\rho \\ J_x \\ J_y \\ J_z \end{pmatrix} \Rightarrow J^\alpha$

Vector and scalar potentials : $\begin{pmatrix} \Phi \\ A_x \\ A_y \\ A_z \end{pmatrix} \Rightarrow A^\alpha$

Lorentz transformations

$$\mathcal{L}_v = \begin{pmatrix} \gamma_v & \gamma_v\beta_v & 0 & 0 \\ \gamma_v\beta_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Time and space : $x^\alpha = \mathcal{L}_v x'^\alpha \equiv \mathcal{L}_v^{\alpha\beta} x'^\beta$

Charge and current : $J^\alpha = \mathcal{L}_v J'^\alpha \equiv \mathcal{L}_v^{\alpha\beta} J'^\beta$

Vector and scalar potential : $A^\alpha = \mathcal{L}_v A'^\alpha \equiv \mathcal{L}_v^{\alpha\beta} A'^\beta$

4-vector relationships

$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \Leftrightarrow \begin{pmatrix} A^0 \\ A^1 \\ A^2 \\ A^3 \end{pmatrix} \Leftrightarrow (A^0, \mathbf{A})$: upper index 4 - vector A^α for $(\alpha = 0,1,2,3)$

Keeping track of signs - - lower index 4 - vector $A_\alpha = (A^0, -\mathbf{A})$

Derivative operators :

$$\partial^\alpha = \left(\frac{\partial}{c\partial t}, -\nabla\right) \qquad \partial_\alpha = \left(\frac{\partial}{c\partial t}, \nabla\right)$$

Special theory of relativity and Maxwell's equations

Continuity equation: $\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$ $\qquad \partial_\alpha J^\alpha = 0$

Lorentz gauge condition: $\frac{1}{c}\frac{\partial \Phi}{\partial t} + \nabla \cdot \mathbf{A} = 0$ $\qquad \partial_\alpha A^\alpha = 0$

Potential equations:
$$\frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} - \nabla^2 \Phi = 4\pi\rho$$
$$\frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2 \mathbf{A} = \frac{4\pi}{c}\mathbf{J} \qquad \partial_\alpha \partial^\alpha A^\alpha = \frac{4\pi}{c} J^\alpha$$

Field relations:
$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$
$$\mathbf{B} = \nabla \times \mathbf{A}$$

Electric and Magnetic field relationships

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$
$$E_x = -\frac{\partial \Phi}{\partial x} - \frac{\partial A_x}{c\partial t} = -\left(\partial^0 A^1 - \partial^1 A^0\right)$$
$$E_y = -\frac{\partial \Phi}{\partial y} - \frac{\partial A_y}{c\partial t} = -\left(\partial^0 A^2 - \partial^2 A^0\right)$$
$$E_z = -\frac{\partial \Phi}{\partial z} - \frac{\partial A_z}{c\partial t} = -\left(\partial^0 A^3 - \partial^3 A^0\right)$$
$$\mathbf{B} = \nabla \times \mathbf{A}$$
$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = -\left(\partial^2 A^3 - \partial^3 A^2\right)$$
$$B_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = -\left(\partial^3 A^1 - \partial^1 A^3\right)$$
$$B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = -\left(\partial^1 A^2 - \partial^2 A^1\right)$$

Field strength tensor $F^{\alpha\beta} \equiv \left(\partial^\alpha A^\beta - \partial^\beta A^\alpha\right)$

$$F^{\alpha\beta} \equiv \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

Transformation of field strength tensor

$$F^{\alpha\beta} = \boldsymbol{\mathcal{L}}_v^{\ \alpha\gamma} F'^{\gamma\delta} \boldsymbol{\mathcal{L}}_v^{\ \delta\beta} \qquad \boldsymbol{\mathcal{L}}_v = \begin{pmatrix} \gamma_v & \gamma_v\beta_v & 0 & 0 \\ \gamma_v\beta_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E'_x & -\gamma_v\left(E'_y + \beta_v B'_z\right) & -\gamma_v\left(E'_z - \beta_v B'_y\right) \\ E'_x & 0 & -\gamma_v\left(B'_z + \beta_v E'_y\right) & \gamma_v\left(B'_y - \beta_v E'_z\right) \\ \gamma_v\left(E'_y + \beta_v B'_z\right) & \gamma_v\left(B'_z + \beta_v E'_y\right) & 0 & -B'_x \\ \gamma_v\left(E'_z - \beta_v B'_y\right) & -\gamma_v\left(B'_y - \beta_v E'_z\right) & B'_x & 0 \end{pmatrix}$$

Lorentz transformation of electromagnetic fields:

$$E_x = E'_x$$
$$E_y = \gamma_v \left(E'_y + \beta_v B'_z \right)$$
$$E_z = \gamma_v \left(E'_z - \beta_v B'_y \right)$$

$$B_x = B'_x$$
$$B_y = \gamma_v \left(B'_y - \beta_v E'_z \right)$$
$$B_z = \gamma_v \left(B'_z + \beta_v E'_y \right)$$