PHY 712 Electrodynamics
10-10:50 AM MWF Olin 107

Plan for Lecture 25:
Start reading Chap. 14 –
Radiation by moving charges
1. Motion in a line
2. Motion in a circle

WFU Joint Physics and Chemistry Colloquium

TITLE: Neutron Scattering Tools for Materials Research

SPEAKER: Dr. Ashifa Huq,
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Spallation Neutron Source, Oak Ridge National Laboratory,
Oak Ridge, TN

TIME: Wednesday April 2, 2014 at 4:00 PM
PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT
Historically, Oak Ridge National Laboratory has played a very important role in developing neutron scattering techniques for crystallography. Clifford Shull, a winner of the 1996 Nobel Prize in Physics started his pioneering work in neutron diffraction in 1946 at Oak Ridge. In keeping with this strong tradition, the lab currently hosts two facilities, High Flux Isotope Reactor and the Spallation Neutron Source, which supports scattering studies in physical, chemical and biological sciences. In this talk, I will present an overview of the neutron scattering technique and its applications in materials science. The talk will also cover a description of neutron powder diffraction with some examples of how this technique has been applied to solve structural questions of some energy related materials.
Radiation from a moving charged particle

Variables (notation):
\[ R(t) = \frac{dR(t)}{dt} = v \]
\[ R(t) = r - R(t) = R \]

Liénard-Wiechert fields (cgs Gaussian units):
\[ E(r,t) = \frac{q}{(r-R)^2} \left[ \left( R - \frac{vR}{c} \right) \left( 1 - \frac{v^2}{c^2} \right) + \left( R \times \left( R - \frac{vR}{c} \right) \times \frac{\hat{\mathbf{v}}}{c^2} \right) \right] \]
\[ B(r,t) = \frac{q}{c} \left[ \frac{-R \times \mathbf{v}}{(r-R)^2} \left( 1 - \frac{v^2}{c^2} \right) - \frac{\mathbf{R} \times \mathbf{v}/c}{(r-R)^2} \right] \]

In this case, the electric and magnetic fields are related according to
\[ B(r,t) = \frac{R \times E(r,t)}{R} \]

Electric field far from source:
\[ E(r,t) = \frac{q}{cR(1-\beta \cdot \mathbf{R})} \left[ \mathbf{R} \times \left( \mathbf{R} - \frac{vR}{c} \right) \times \frac{\mathbf{v}}{c^2} \right] \]
\[ B(r,t) = \frac{R \times E(r,t)}{R} \]

Let \[ \hat{\mathbf{R}} = \frac{R}{R} \quad \beta = \frac{v}{c} \quad \hat{\beta} = \frac{\mathbf{v}}{c} \]
\[ E(r,t) = \frac{q}{cR(1-\beta \cdot \mathbf{R})} \left[ \mathbf{R} \times [\hat{\mathbf{R}} - \beta \cdot \mathbf{R}] \times \hat{\beta} \right] \]
\[ B(r,t) = \hat{\mathbf{R}} \times E(r,t) \]
Poynting vector:

\[
S(r,t) = \frac{c}{4\pi} \left( \mathbf{E} \times \mathbf{B} \right)
\]

\[
\mathbf{E}(r,t) = \frac{q}{\epsilon_0 r} \left[ \mathbf{R} \times \left[ (\mathbf{\dot{R}} - \mathbf{\beta}) \times \mathbf{\beta} \right] \right]
\]

\[
\mathbf{B}(r,t) = \mathbf{R} \times \mathbf{E}(r,t)
\]

\[
S(r,t) = \frac{c}{4\pi} \mathbf{R} \left[ \mathbf{E} \times \mathbf{B} \right] = \frac{q^2}{4\pi R^2} \left[ \mathbf{R} \times \left[ (\mathbf{\dot{R}} - \mathbf{\beta}) \times \mathbf{\beta} \right] \right]
\]

Note: We have assumed that \( \mathbf{R} \cdot \mathbf{E}(r,t) = 0 \)

Power radiated

\[
S(r,t) = \frac{c}{4\pi} \mathbf{\dot{R}} \left[ \mathbf{E}(r,t) \right]^2 = \frac{q^2}{4\pi R^2} \left[ \mathbf{R} \times \left[ (\mathbf{\dot{R}} - \mathbf{\beta}) \times \mathbf{\beta} \right] \right]
\]

\[
\frac{dP}{d\Omega} = S \cdot R^2 = \frac{q^2}{4\pi} \left[ \mathbf{R} \times \left[ (\mathbf{\dot{R}} - \mathbf{\beta}) \times \mathbf{\beta} \right] \right]
\]

In the non-relativistic limit: \( \beta << 1 \)

\[
\frac{dP}{d\Omega} = \frac{q^2}{4\pi} \left| \mathbf{\dot{R}} \times \mathbf{\beta} \right|^2 = \frac{q^2}{4\pi} |\mathbf{v}|^2 \sin^2 \Theta
\]

Radiation from a moving charged particle

Variables (notation):

\[
\mathbf{R}_i(t) = \frac{d\mathbf{R}_i(t)}{dt}, \quad \mathbf{v}
\]

\[
\mathbf{R}(t) = \mathbf{r} - \mathbf{R}_i(t) = \mathbf{R}
\]

\[
\frac{dP}{d\Omega} = \frac{q^2}{4\pi} |\mathbf{v}|^2 \sin^2 \Theta
\]
Radiation power in non-relativistic case -- continued

\[ \frac{dP}{d\Omega} = -\frac{q^2}{4\pi} |v| \sin^2 \Theta \]

\[ P = \int d\Omega \frac{dP}{d\Omega} = \frac{2q^2}{3c^2} |v|^2 \]

Radiation distribution in the relativistic case

\[ \frac{dP}{d\Omega} = S \cdot \hat{R} \hat{R}^2 = \frac{q^2}{4\pi} \left[ \hat{R} \times \left( \hat{R} - \hat{\beta} \right) \right] \left( 1 - \hat{\beta} \cdot \hat{R} \right) \]

This expression gives us the energy per unit field time \( t \). We are often interested in the power per unit retarded time \( t = t - R/c \):

\[ \frac{dP}{d\Omega} = \frac{dP}{d\Omega} \left( \frac{dt}{dt} \right) \frac{dt}{dr} = 1 - \hat{\beta} \cdot \hat{R} \]

\[ dP(r) = q^2 \left[ \hat{R} \times \left( \hat{R} - \hat{\beta} \right) \right] \left( 1 - \hat{\beta} \cdot \hat{R} \right) \]

Radiation distribution in the relativistic case -- continued

\[ \frac{dP}{d\Omega} = \frac{q^2}{4\pi} \left[ \hat{R} \times \left( \hat{R} - \hat{\beta} \right) \right] \left( 1 - \hat{\beta} \cdot \hat{R} \right) \]

For linear acceleration: \( \hat{\beta} \times \hat{\beta} = 0 \)

\[ \frac{dP}{d\Omega} = \frac{q^2}{4\pi} \left[ \hat{R} \times \left( \hat{R} - \hat{\beta} \right) \right] \left( 1 - \hat{\beta} \cdot \hat{R} \right) = \frac{q^2}{4\pi}|v|^2 \sin^2 \Theta \left( 1 - \hat{\beta} \cdot \hat{R} \right) \]
Power from linearly accelerating particle

\[
\frac{dP(t)}{d\Omega} = \frac{q^2}{4\pi c^3} \left| \frac{\mathbf{\dot{R}} \times (\mathbf{\dot{R}} \times \mathbf{\beta})}{(1 - \mathbf{\beta} \cdot \mathbf{R})} \right| = \frac{q^2}{4\pi c^3} |\mathbf{v}|^2 \sin^2 \theta \left(1 - \beta \cos \theta\right)^{-1}
\]

\[P(\gamma) = \int \frac{dP(t)}{d\Omega} d\Omega = \frac{2}{3} \frac{q^2}{c} |\mathbf{v}|^3 \gamma^6 \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}
\]

Power distribution for linear acceleration -- continued
Power distribution for circular acceleration

\[
\frac{dP(t)}{d\Omega} = \frac{q^2}{4\pi c} \left[ |\vec{R} \times ( \vec{R} - \vec{\beta} ) \times \vec{x} | \right] \left[ (1 - \beta \cdot \vec{R}) \right]
\]

\[
= \frac{q^2}{4\pi c} \left[ |\vec{R} |^2 \left( 1 - \beta \cdot \vec{R} \right) \left( 1 - \beta \cdot \vec{\beta} \right) \right]
\]

\[
P(t) = \int d\Omega \frac{dP(t)}{d\Omega} = \frac{2q^2}{3c^2} |\vec{v}| \gamma
\]

Power distribution for circular acceleration

\[
\frac{dP(t)}{d\Omega} = \frac{q^2}{4\pi c} \left| \vec{R} \times ( \vec{R} - \vec{\beta} ) \times \vec{x} \right|
\]

\[
= \frac{q^2}{4\pi c} \left| \vec{v} \right| \gamma \left( 1 - \beta \cdot \vec{R} \right) \left( 1 - \beta \cdot \vec{\beta} \right)
\]

\[
P(t) = \int d\Omega \frac{dP(t)}{d\Omega} = \frac{2q^2}{3c^2} \left| \vec{v} \right| \gamma
\]