PHY 712 Electrodynamics
10-10:50 AM MWF Olin 107

Plan for Lecture 26:

Start reading Chap. 14 –

Radiation by moving charges

-- spectral analysis

	Wed: 03/05/2014	*APS Meeting*	Take-home exam (no class meeting)		
	Fri: 03/07/2014	*APS Meeting*	Take-home exam (no class meeting)		
	Mon: 03/10/2014	*Spring Break*			
	Wed: 03/12/2014	*Spring Break*			
	Fri: 03/14/2014	*Spring Break*			
19	Mon: 03/17/2014	Chap. 8	Wave guides;Take-home exam due	#18	3/21/2014
20	Wed: 03/19/2014	Chap. 9	Sources of Electromagnetic Waves	#19	3/21/2014
21	Fri: 03/21/2014	Chap. 9	Sources of Electromagnetic Waves	#20	3/28/2014
22	Mon: 03/24/2014	Chap. 11	Special Theory of Relativity	#21	3/28/2014
23	Wed: 03/26/2014	Chap. 11	Special Theory of Relativity	#22	3/28/2014
24	Fri: 03/28/2014	Chap. 11	Special Theory of Relativity	#23	4/04/2014
25	Mon: 03/31/2014	Chap. 14	Radiation from moving charges	#24	4/04/2014
26	Wed: 04/02/2014	Chap. 14	Radiation from moving charges	#25	4/04/2014
27	Fri: 04/04/2014	Chap. 14	Radiation from moving charges	#26	4/11/2014
28	Mon: 04/07/2014				
29	Wed: 04/09/2014				
30	Fri: 04/11/2014				
31	Mon: 04/14/2014				
32	Wed: 04/16/2014				
	Fri: 04/18/2014	Good Friday			
33	Mon: 04/21/2014				
34	Wed: 04/23/2014				
35	Fri: 04/23/2014				
	Mon: 04/28/2014		Presentations Part I		
	Wed: 04/30/2014		Presentations Part II		

WFU Joint Physics and Chemistry Colloquium

TITLE: Neutron Scattering Tools for Materials Research

SPEAKER: Dr. Ashfia Huq,

Chemical and Engineering Materials Division,
Spallation Neutron Source, Oak Ridge National Laboratory,
Oak Ridge, TN

TIME: Wednesday April 2, 2014 at 4:00 PM

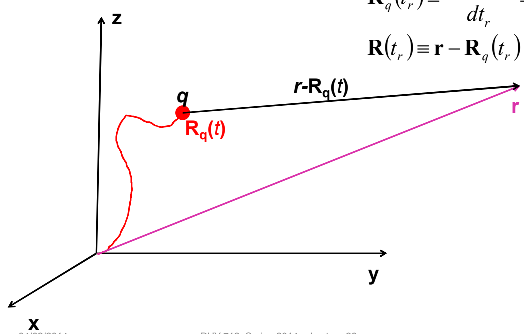
PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

Historically Oak Ridge National Laboratory has played a very important role in developing neutron scattering techniques for crystallography. Clifford Shull, the winner of the 1994 Noble Prize in Physics started his pioneering work in neutron diffraction in 1946 at Oak Ridge. In keeping with this strong tradition, the lab currently hosts two facilities; High Flux Isotope Reactor and the Spallation Neutron Source, which supports scattering studies in physical, chemical and biological sciences. In this talk I will present an overview of the various different neutron scattering techniques for materials research. A more focused description of neutron powder diffraction will follow with some examples of how this technique has been applied to solve structural questions of some energy related materials.

Radiation from a moving charged particle



Variables (notation):

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R}$$

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Spectral composition of electromagnetic radiation

Previously we determined the power distribution from a charged particle:

$$\frac{dP(t)}{d\Omega} = \mathbf{S}\cdot\hat{\mathbf{R}}R^2 = \frac{q^2}{4\pi c}\frac{\left|\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|^2}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^6}\Bigg|_{t_r=t-R/c}$$

$$\equiv |\boldsymbol{a}(t)|^2$$

where $$\boldsymbol{a}(t) \equiv \sqrt{\frac{q^2}{4\pi c}}\frac{\left|\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^3}\Bigg|_{t_r=t-R/c}$$

Time integrated power per solid angle:

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} dt\,\frac{dP(t)}{d\Omega} = \int_{-\infty}^{\infty} dt\,|\boldsymbol{a}(t)|^2 = \int_{-\infty}^{\infty} d\omega\,|\tilde{\boldsymbol{a}}(\omega)|^2$$

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Spectral composition of electromagnetic radiation -- continued

Time integrated power per solid angle:

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} dt\,\frac{dP(t)}{d\Omega} = \int_{-\infty}^{\infty} dt\,|\boldsymbol{a}(t)|^2 = \int_{-\infty}^{\infty} d\omega\,|\tilde{\boldsymbol{a}}(\omega)|^2$$

Fourier amplitude:

$$\tilde{\boldsymbol{a}}(\omega) \equiv \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dt\,\boldsymbol{a}(t)e^{i\omega t} \qquad \boldsymbol{a}(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} d\omega\,\tilde{\boldsymbol{a}}(\omega)e^{-i\omega t}$$

Parseval's theorem

Marc-Antoine Parseval des Chênes 1755-1836

http://www-history.mcs.st-andrews.ac.uk/Biographies/Parseval.html

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Spectral composition of electromagnetic radiation -- continued

Consequences of Parseval's analysis:

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} dt \frac{dP(t)}{d\Omega} = \int_{-\infty}^{\infty} dt |\boldsymbol{a}(t)|^2 = \int_{-\infty}^{\infty} d\omega |\tilde{\boldsymbol{a}}(\omega)|^2$$

Note that: $\tilde{\boldsymbol{a}}(\omega) = \tilde{\boldsymbol{a}}^*(-\omega)$

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} d\omega |\tilde{\boldsymbol{a}}(\omega)|^2 = \int_{0}^{\infty} d\omega \left(|\tilde{\boldsymbol{a}}(\omega)|^2 + |\tilde{\boldsymbol{a}}(-\omega)|^2 \right) \equiv \int_{0}^{\infty} d\omega \frac{\partial^2 I}{\partial\Omega\partial\omega}$$

$$\frac{\partial^2 I}{\partial\Omega\partial\omega} \equiv 2|\tilde{\boldsymbol{a}}(\omega)|^2$$

Spectral composition of electromagnetic radiation -- continued

For our case: $\boldsymbol{a}(t) \equiv \sqrt{\frac{q^2}{4\pi c}} \left. \frac{|\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]|}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^3} \right|_{t_r = t - R/c}$

Fourier amplitude:

$$\tilde{\boldsymbol{a}}(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\, \boldsymbol{a}(t) e^{i\omega t}$$

$$= \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt \left. \frac{|\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]|}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^3} \right|_{t_r = t - R/c} e^{i\omega t}$$

Spectral composition of electromagnetic radiation -- continued

Fourier amplitude:

$$\tilde{\boldsymbol{a}}(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\, \boldsymbol{a}(t) e^{i\omega t}$$

$$= \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt \left. \frac{|\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]|}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^3} \right|_{t_r = t - R/c} e^{i\omega t}$$

$$= \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt_r \frac{dt}{dt_r} \left. \frac{|\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]|}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^3} \right|_{t_r = t - R/c} e^{i\omega(t_r + R(t_r)/c)}$$

$$= \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt_r \left. \frac{|\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]|}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^2} \right|_{t_r = t - R/c} e^{i\omega(t_r + R(t_r)/c)}$$

Spectral composition of electromagnetic radiation -- continued

Exact expression :

$$\tilde{a}(\omega)=\sqrt{\frac{q^2}{8\pi^2 c}}\int_{-\infty}^{\infty} dt_r \left.\frac{\left|\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^2}\right|_{t_r=t-R/c} e^{i\omega\left(t_r+R(t_r)/c\right)}$$

Recall : $\dot{\mathbf{R}}_q(t_r)\equiv\frac{d\mathbf{R}_q(t_r)}{dt_r}\equiv\mathbf{v}$ $\quad\mathbf{R}(t_r)\equiv\mathbf{r}-\mathbf{R}_q(t_r)\equiv\mathbf{R}$

For $r>>R_q(t_r)$ $\quad R(t_r)\approx r-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)$ where $\hat{\mathbf{r}}\equiv\frac{\mathbf{r}}{r}$

At the same level of approximation : $\hat{\mathbf{R}}\approx\hat{\mathbf{r}}$

Spectral composition of electromagnetic radiation -- continued

Exact expression:

$$\tilde{a}(\omega)=\sqrt{\frac{q^2}{8\pi^2 c}}\int_{-\infty}^{\infty} dt_r \left.\frac{\left|\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^2}\right|_{t_r=t-R/c} e^{i\omega\left(t_r+R(t_r)/c\right)}$$

Approximate expression:

$$\tilde{a}(\omega)=\sqrt{\frac{q^2}{8\pi^2 c}}e^{i\omega(r/c)}\int_{-\infty}^{\infty} dt_r \left.\frac{\left|\hat{\mathbf{r}}\times\left[\left(\hat{\mathbf{r}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{r}}\right)^2}\right|_{t_r=t-R/c} e^{i\omega\left(t_r-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)/c\right)}$$

Resulting spectral intensity expression:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega}=\frac{q^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt_r \left.\frac{\left|\hat{\mathbf{r}}\times\left[\left(\hat{\mathbf{r}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{r}}\right)^2}\right|_{t_r=t-R/c} e^{i\omega\left(t_r-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)/c\right)}\right|^2$$

Example – radiation from a collinear acceleration burst

$$\frac{\partial^2 I}{\partial\omega\partial\Omega}=\frac{q^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt_r \left.\frac{\left|\hat{\mathbf{r}}\times\left[\left(\hat{\mathbf{r}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{r}}\right)^2}\right|_{t_r=t-R/c} e^{i\omega\left(t_r-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)/c\right)}\right|^2$$

Suppose that $\dot{\boldsymbol{\beta}}=\begin{cases}\frac{\hat{\boldsymbol{\beta}}\Delta v}{c\tau} & 0<t_r<\tau\\ 0 & \text{otherwise}\end{cases}$

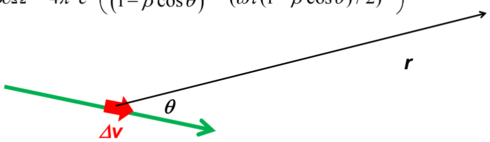
$$\frac{\partial^2 I}{\partial\omega\partial\Omega}=\frac{q^2}{4\pi^2 c^3}\left|\frac{\left|\hat{\mathbf{r}}\times\left[\hat{\mathbf{r}}\times\hat{\boldsymbol{\beta}}\right]\right|\Delta v}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{r}}\right)^2\tau}\right|^2\left|\int_0^{\tau} dt_r e^{i\omega\left(t_r-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}t_r\right)}\right|^2 \quad \text{Let } \boldsymbol{\beta}\cdot\hat{\mathbf{r}}=\beta\cos\theta$$

$$\frac{\partial^2 I}{\partial\omega\partial\Omega}=\frac{q^2}{4\pi^2 c^3}\left(\frac{\Delta v\sin\theta}{\left(1-\beta\cos\theta\right)^2}\frac{\sin(\omega\tau(1-\beta\cos\theta)/2)}{(\omega\tau(1-\beta\cos\theta)/2)}\right)^2$$

Example:

Suppose that $\dot{\boldsymbol{\beta}} = \begin{cases} \dfrac{\hat{\boldsymbol{\beta}}\Delta v}{c\tau} & 0 < t_r < \tau \\ 0 & \text{otherwise} \end{cases}$

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2}{4\pi^2 c^3}\left(\frac{\Delta v \sin\theta}{(1-\beta\cos\theta)^2}\frac{\sin(\omega\tau(1-\beta\cos\theta)/2)}{(\omega\tau(1-\beta\cos\theta)/2)}\right)^2$$



Spectral composition of electromagnetic radiation -- continued

Alternative expression --
It can be shown that:

$$\frac{\hat{\mathbf{r}}\times\left[(\hat{\mathbf{r}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]}{(1-\boldsymbol{\beta}\cdot\hat{\mathbf{r}})^2} = \frac{d}{dt_r}\left(\frac{\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta})}{(1-\boldsymbol{\beta}\cdot\hat{\mathbf{r}})}\right)$$

Integration by parts and assumptions about the integration limit behavior shows that the spectral intensity depends on the following integral:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt_r \left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}(t_r)\right)\right]e^{i\omega\left(t_r-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)/c\right)}\right|^2$$