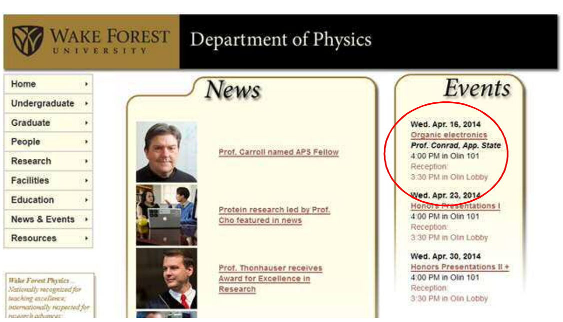
PHY 712 Electrodynamics
10-10:50 AM MWF Olin 107

Plan for Lecture 32:

Special Topics in Electrodynamics:

Electromagnetic aspects of superconductivity -- continued

19	Mon: 03/17/2014	Chap. 8	Wave guides;Take-home exam due	#18	3/21/2014
20	Wed: 03/19/2014	Chap. 9	Sources of Electromagnetic Waves	#19	3/21/2014
21	Fri: 03/21/2014	Chap. 9	Sources of Electromagnetic Waves	#20	3/28/2014
22	Mon: 03/24/2014	Chap. 11	Special Theory of Relativity	#21	3/28/2014
23	Wed: 03/26/2014	Chap. 11	Special Theory of Relativity	#22	3/28/2014
24	Fri: 03/28/2014	Chap. 11	Special Theory of Relativity	#23	4/04/2014
25	Mon: 03/31/2014	Chap. 14	Radiation from moving charges	#24	4/04/2014
26	Wed: 04/02/2014	Chap. 14	Radiation from moving charges	#25	4/04/2014
27	Fri: 04/04/2014	Chap. 14	Radiation from moving charges	#26	4/11/2014
28	Mon: 04/07/2014	Chap. 14	Radiation from moving charges	#27	4/11/2014
29	Wed: 04/09/2014	Chap. 15	Radiation due to collision processes	#28	4/16/2014
30	Fri: 04/11/2014	Chap. 13	Cherenkov radiation	#29	4/16/2014
31	Mon: 04/14/2014		Special topic -- E&M of superconductivity		
32	Wed: 04/16/2014		Special topic -- E&M of superconductivity		
	Fri: 04/18/2014		Good Friday Holiday -- no class		
33	Mon: 04/21/2014				
34	Wed: 04/23/2014				
35	Fri: 04/25/2014				
	Mon: 04/28/2014		Presentations Part I		
	Wed: 04/30/2014		Presentations Part II		



WFU Physics Colloquium

TITLE: Advances in Organic Electronics

SPEAKER: Professor Brad Conrad,

Department of Physics and Astronomy, Appalachian State University

TIME: Wednesday April 16, 2014 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

Organic semiconductors offer the promise of low-cost devices on arbitrary surfaces, transparent displays, efficient OLEDs, and even biodegradable photovoltaics. Through chemical engineering, the physical and electronic properties of these new materials can be tailored to meet specific requirements and investigate interesting condensed matter physics. In this colloquium I will have a lively introduction to organic semiconductors, explore some recent advances, and briefly discuss our recent work on small organic molecule solar cells.

Behavior of superconducting material – exclusion of magnetic field according to the London model

Penetration length for superconductor: $\lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$

$B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

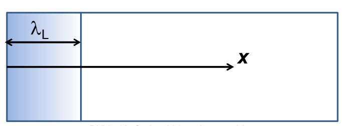
Vector potential for $\nabla \cdot \mathbf{A} = 0$:

$\mathbf{A} = \hat{\mathbf{y}} A_y(x) \qquad A_y(x) = -\lambda_L B_z(0) e^{-x/\lambda_L}$

Current density: $J_y(x) = \lambda_L \frac{ne^2}{mc} \mathrm{B}_z(0)\mathrm{e}^{-x/\lambda_L}$

$\Rightarrow \mathbf{J} + \frac{ne^2}{mc}\mathbf{A} = 0 \quad \text{or} \quad \frac{ne}{m}\left(m\mathbf{v} + \frac{e}{c}\mathbf{A}\right) = 0$

Typically, $\lambda_L \approx 10^{-7} m$



Magnetization field

Treating London current in terms of corresponding magnetization field **M**:

$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$

$\Rightarrow$ For $x >> \lambda_L$, $\quad \mathbf{H} = -4\pi\mathbf{M}$

Gibbs free energy associated with magnetization for superconductor:

$G_S(H_a) = G_S(H=0) - \int_0^{H_a} dH M(H) = G_S(0) + \frac{1}{8\pi} H_a^2$

Gibbs free energy associated with magnetization for normal conductor:

$G_N(H_a) \approx G_N(H=0)$

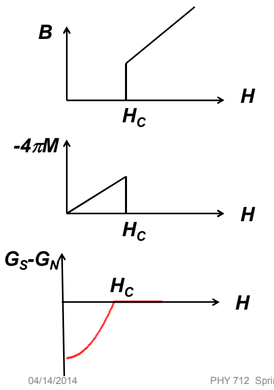
Condition at phase boundary between normal and superconducting states:

$G_N(H_C) \approx G_N(0) = G_S(H_C) = G_S(0) + \frac{1}{8\pi} H_C^2$

$\Rightarrow G_S(0) - G_N(0) = -\frac{1}{8\pi} H_C^2$

$$G_S(H_a) - G_N(H_a) = \begin{cases} -\frac{1}{8\pi}\left(H_C^2 - H_a^2\right) & \text{for } H_a < H_C \\ 0 & \text{for } H_a > H_C \end{cases}$$

Magnetization field (for "type I" superconductor)



Examples of type I superconductors

http://hyperphysics.phy-astr.gsu.edu/hbase/tables/supcon.html#c1

Superconductivity Transition Temperatures and Critical Fields

Li	Be 0.026 ...	Superconductivity parameters for elements Transition temperature in Kelvin Critical magnetic field in gauss (10^{-4} tesla)										B	C	N	O	F	Ne ... ...
Na	Mg	...										Al 1.140 105	Si* 7 ...	P* 5 ...	S*	Cl	Ar ... ...
K	Ca	Sc	Ti 0.39 100	V 5.38 1420	Cr*	Mn	Fe	Co	Ni	Cu	Zn 0.875 53	Ga 1.091 51	Ge* 5 ...	As* 0.5 ...	Se* 7 ...	Br	Kr ... ...
Rb	Sr	Y*	Zr 0.546 47	Nb 9.50 1980	Mo 0.90 95	Tc 7.77 1410	Ru 0.51 70	Rh 0.0003 0.049	Pd	Ag	Cd 0.56 30	In 3.4035 293	Sn(w) 3.722 309	Sb* 3.5 ...	Te* 4 ...	I	Xe ... ...
Cs* 1.5 ...	Ba* 5 ...	La(fcc) 6.00 1100	Hf 0.12 ...	Ta 4.483 830	W 0.012 1.07	Re 1.4 198	Os 0.655 65	Ir 0.14 19	Pt	Au	Hg 4.153 412	Tl 2.39 171	Pb 7.193 803	Bi* 8 ...	Po	At	Rn ... ...

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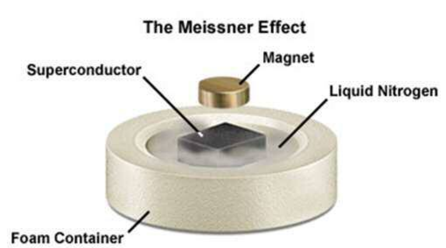
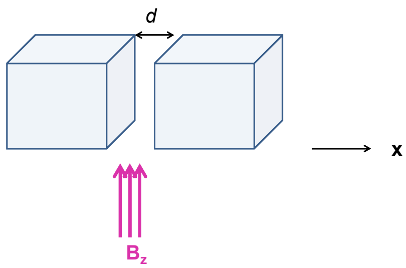
Data from Kittel, Introduction to Solid State Physics, 7th Ed., Ch 12
*Superconducting only in thin films or under high pressure in a crystal modification not normally stable. Critical temperatures for those elements from Myers, Ch 13.

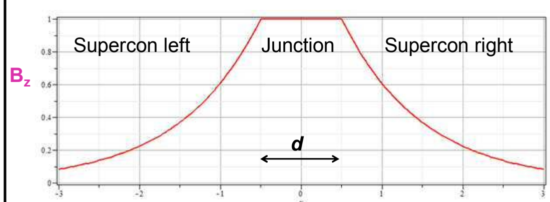
It is notable that in the range of data covered by this table, the best conductors like Cu do not become superconducting at all. Neither the noble metals or the magnetic materials become superconducting. That is not to be taken as a statement that they cannot be made superconducting, it is just that the transitions to superconductivity must be at such low temperatures and require such great purity of material that they have not been demonstrated conclusively.



Josephson junction -- tunneling current between two superconductors

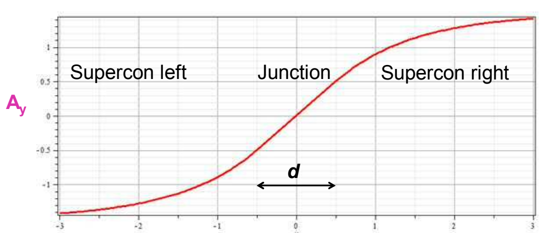


Josephson junction -- continued



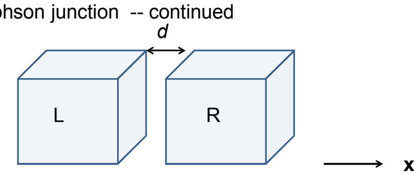
$$B_z(x) = \begin{cases} B_0 e^{(x+d/2)/\lambda_L} & x < -d/2 \\ B_0 & -d/2 < x < d/2 \\ B_0 e^{(-x+d/2)/\lambda_L} & x > d/2 \end{cases}$$

Josephson junction -- continued



$$A_y(x) = \begin{cases} B_0\left(\lambda_L e^{(x+d/2)/\lambda_L} - \left(\lambda_L + d/2\right)\right) & x < -d/2 \\ B_0 x & -d/2 < x < d/2 \\ B_0\left(-\lambda_L e^{(-x+d/2)/\lambda_L} + \left(\lambda_L + d/2\right)\right) & x > d/2 \end{cases}$$

Josephson junction -- continued



Quantum mechanical model of tunnelling current

Let $\Psi_L = \Psi_L{}^0 e^{i\phi_L}$ denote a wavefunction for a Cooper pair on left

Let $\Psi_R = \Psi_R{}^0 e^{i\phi_R}$ denote a wavefunction for a Cooper pair on right

$$-i\hbar \frac{\partial \Psi_L}{\partial t} = E_L \Psi_L + \varepsilon \Psi_R$$

$$-i\hbar \frac{\partial \Psi_R}{\partial t} = E_R \Psi_R + \varepsilon \Psi_L$$

Josephson junction -- continued

Solving for wavefunctions

$$\frac{1}{2}\frac{\partial \left|\Psi_L{}^0\right|^2}{\partial t} + i\left|\Psi_L{}^0\right|^2 \frac{\partial \phi_L}{\partial t} = -\frac{i}{\hbar}\left(E_L \left|\Psi_L{}^0\right|^2 + \varepsilon \Psi_L{}^0 \Psi_R{}^0 e^{i(\phi_R - \phi_L)}\right)$$

$$\frac{1}{2}\frac{\partial \left|\Psi_R{}^0\right|^2}{\partial t} + i\left|\Psi_R{}^0\right|^2 \frac{\partial \phi_R}{\partial t} = -\frac{i}{\hbar}\left(E_R \left|\Psi_R{}^0\right|^2 + \varepsilon \Psi_L{}^0 \Psi_R{}^0 e^{-i(\phi_R - \phi_L)}\right)$$

$$\left|\Psi_L{}^0\right|^2 \equiv n_L \qquad \left|\Psi_R{}^0\right|^2 \equiv n_R \qquad \phi_{LR} \equiv \phi_L - \phi_R$$

$$\frac{\partial n_L}{\partial t} = -\frac{\partial n_R}{\partial t} = -\frac{2\varepsilon}{\hbar}\sqrt{(n_L n_R)}\sin\phi_{LR}$$

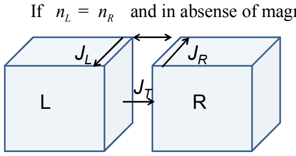
$$\frac{\partial \phi_L}{\partial t} = -\frac{E_L}{\hbar} - \varepsilon\sqrt{\frac{n_R}{n_L}}\cos\phi_{LR}$$

$$\frac{\partial \phi_R}{\partial t} = -\frac{E_R}{\hbar} - \varepsilon\sqrt{\frac{n_L}{n_R}}\cos\phi_{LR}$$

Josephson junction -- continued

Tunneling current: $J_T = 2e\frac{\partial n_L}{\partial t} = -\frac{4e\varepsilon}{\hbar}\sqrt{(n_L n_R)}\sin\phi_{LR}$

If $n_L = n_R$ and in absense of magnetic field, $\phi_{LR}(t) = \phi_{LR}(0) + \frac{E_R - E_L}{\hbar}t$



$$\Rightarrow J_L = \frac{2e}{2m}\left|\Psi_L{}^0\right|^2\left(\hbar\nabla\phi_L - \frac{2e}{c}\mathbf{A}\right)$$

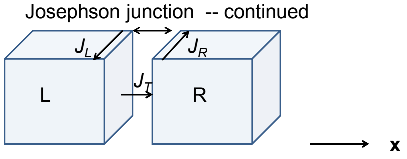
$$\Rightarrow J_R = \frac{2e}{2m}\left|\Psi_R{}^0\right|^2\left(\hbar\nabla\phi_R - \frac{2e}{c}\mathbf{A}\right)$$

x

Relationship between superconductor currents J_L and J_R and tunneling current. Within the superconductor, denote the generalize current operator acting on pair wavefunction $\Psi = \Psi^0 e^{i\phi}$

$$\hat{\mathbf{v}} \equiv \frac{1}{2m}\left(-i\hbar\nabla - \frac{2e}{c}\mathbf{A}\right) \quad \text{with current } J = \frac{2e}{2}\left(\Psi^*\left(\hat{\mathbf{v}}\Psi\right) + \Psi\left(\hat{\mathbf{v}}\Psi\right)^*\right)$$

Josephson junction -- continued



$$\Rightarrow J_L = \frac{2e}{2m}\left|\Psi_L{}^0\right|^2\left(\hbar\nabla\phi_L - \frac{2e}{c}\mathbf{A}\right) \equiv 2en_L\mathbf{v}_L$$

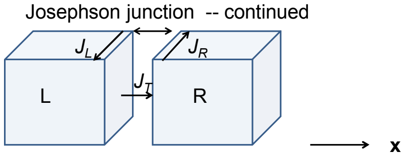
$$\Rightarrow J_R = \frac{2e}{2m}\left|\Psi_R{}^0\right|^2\left(\hbar\nabla\phi_R - \frac{2e}{c}\mathbf{A}\right) \equiv 2en_R\mathbf{v}_R$$

$$\nabla\phi_L = \frac{2m\mathbf{v}_L}{\hbar} + \frac{2e}{\hbar c}\mathbf{A} \qquad \nabla\phi_R = \frac{2m\mathbf{v}_R}{\hbar} + \frac{2e}{\hbar c}\mathbf{A}$$

Tunneling current: $J_T = 2e\frac{\partial n_L}{\partial t} = -\frac{4e\varepsilon}{\hbar}\sqrt{(n_L n_R)}\sin\phi_{LR}$

Need to evaluate ϕ_{LR} in presence of magnetic field

Josephson junction -- continued



$$\nabla\phi_L = \frac{2m\mathbf{v}_L}{\hbar} + \frac{2e}{\hbar c}\mathbf{A} \qquad \nabla\phi_R = \frac{2m\mathbf{v}_R}{\hbar} + \frac{2e}{\hbar c}\mathbf{A}$$

Recall that for $x \to -\infty$ $\mathbf{v}_L \to 0$ and $\mathbf{A} \to -\left(\lambda_L + d/2\right)B_0\hat{\mathbf{y}}$

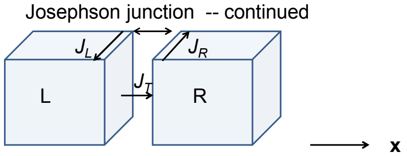
for $x \to \infty$ $\mathbf{v}_R \to 0$ and $\mathbf{A} \to \left(\lambda_L + d/2\right)B_0\hat{\mathbf{y}}$

Integrating the difference of the phase angles along y:

$\phi_{LR} = \phi^0_{LR} + B_0(2\lambda_L + d)y$

Tunneling current: $J_T = -\frac{4e\varepsilon}{\hbar}\sqrt{(n_L n_R)}\sin\phi_{LR}$

Josephson junction -- continued



Integrating the difference of the phase angles along y:

$\phi_{LR} = \phi^0_{LR} + \frac{2e}{\hbar c}B_0(2\lambda_L + d)y$

Tunneling current density: $J_T = \frac{4e\varepsilon}{\hbar}n_L\sin\phi_{LR}$

Integrating current density throughout width w of superconductors

$$I_T = h\int_{-w/2}^{w/2} J_T\,dy = hwJ_{T0}\sin(\phi^0_{LR})\frac{\sin(\pi\Phi/\Phi^0)}{\pi\Phi/\Phi^0}$$

where $\Phi = B_0 w(2\lambda_L + d)$ and $\Phi^0 = \frac{2\pi\hbar c}{2e}$