PHY 712 Electrodynamics
11-11:50 AM MWF Olin 107

Plan for Lecture 9:

Continue reading Chapter 4

Dipolar fields and dielectrics

A. Electric field due to a dipole

B. Electric polarization **P**

C. Electric displacement **D** and dielectric functions

PHY 712 Electrodynamics

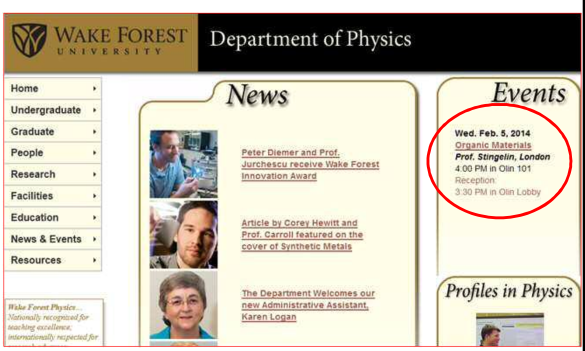
MWF 10-10:50 AM | OPL 107 | http://www.wfu.edu/~natalie/s14phy712/

Instructor: Natalie Holzwarth **Phone:** 758-5510 **Office:** 300 OPL **e-mail:** natalie@wfu.edu

Course schedule for Spring 2014

(Preliminary schedule -- subject to frequent adjustment.) Please note that makeup lectures (indicated in red) are scheduled for Tuesdays or Thursdays at 11 AM - 12:15 PM in Olin 107.

	Lecture date	JDJ Reading	Topic	Assign.	Due date
1	Wed: 01/15/2014	Chap. 1	Introduction, units and Poisson equation	#1	01/31/2014
2	Thu: 01/16/2014	Chap. 1	Electrostatic energy calculations	#2	01/31/2014
3	Fri: 01/17/2014	Chap. 1	Poisson equation and Green's theorm	#3	01/31/2014
	Mon: 01/20/2014		MLK Holiday - no class		
4	Wed: 01/22/2014	Chap. 1	Green's functions for cartesian coordinates	#4	01/31/2014
5	Thu: 01/23/2014	Chap. 1	Brief introduction to numerical methods	#5	01/31/2014
6	Fri: 01/24/2014	Chap. 2	Method of images	#6	01/31/2014
	Mon: 01/27/2014		NAWH out of town - no class		
	Wed: 01/29/2014		NAWH out of town - no class		
7	Fri: 01/31/2014	Chap. 3	Cylindrical and spherical geometries	#7	02/05/2014
8	Mon: 02/03/2014	Chap. 4	Multipole analysis of charge distributions	#8	02/05/2014
9	Wed: 02/05/2014	Chap. 4	Dipoles and dielectrics	#9	02/07/2014



WFU Physics Colloquium

TITLE: Unravelling electronic processes and phenomena in organic materials through polymer scientists' tools

SPEAKER: Dr. Natalie Stingelin,

Department of Materials and Centre for Plastic Electronics, Imperial College London, UK

TIME: Wednesday February 5, 2014 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

In the past decade, significant progress has been made in the fabrication of organic semiconductor thin-film devices predominantly due to important improvements of existing materials and the creation of a wealth of novel compounds. Many challenges, however, still exist. Key to commercial success is to make it technological practice to exploit the touted potential for low-cost manufacturing of these functional materials. This requires intimate knowledge of relevant structure/processing/performance interrelations. Here, examples are

General results for a multipole analysis of the electrostatic potential due to an isolated charge distribution:

General form of electrostatic potential with boundary value $r \rightarrow \infty$, for isolated charge density $\rho(\mathbf{r})$:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

$$= \frac{1}{4\pi\varepsilon_0}\int d^3r' \rho(\mathbf{r}')\left(\sum_{lm}\frac{4\pi}{2l+1}\frac{r_<^l}{r_>^{l+1}} Y_{lm}(\theta,\varphi) Y_{lm}^*(\theta',\varphi')\right)$$

Suppose that $\rho(\mathbf{r}) = \sum_{lm}\rho_{lm}(r) Y_{lm}(\theta,\varphi)$

$$\Rightarrow \Phi(\mathbf{r}) = \frac{1}{\varepsilon_0}\sum_{lm}\frac{1}{2l+1} Y_{lm}(\theta,\varphi)\left(\frac{1}{r^{l+1}}\int_0^r r'^{2+l}\, dr' \rho_{lm}(r') + r^l \int_r^\infty r'^{1-l}\, dr' \rho_{lm}(r')\right)$$

Notion of multipole moment:

In the spherical harmonic representation --

define the moment q_{lm} of the (confined) charge distribution $\rho(\mathbf{r})$:

$$q_{lm} \equiv \int d^3r'\, r'^l\, Y_{lm}^*(\theta',\phi')\rho(\mathbf{r}')$$

In the Cartesian representation --

define the monopole moment q:

$$q \equiv \int d^3r'\, \rho(\mathbf{r}')$$

define the dipole moment $\mathbf{p}$:

$$\mathbf{p} \equiv \int d^3r'\, \mathbf{r}'\, \rho(\mathbf{r}')$$

define the quadrupole moment components Q_{ij} $(i,j \rightarrow x,y,z)$:

$$Q_{ij} \equiv \int d^3r' \left(3r'_i r'_j - r'^2 \delta_{ij}\right)\rho(\mathbf{r}')$$

General form of electrostatic potential in terms of multipole moments:

For r outside the extent of $\rho(\mathbf{r})$:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{lm}\frac{4\pi}{2l+1}\frac{Y_{lm}(\theta,\varphi)}{r^{l+1}}\left(\int d^3r'\, r'^l\, Y_{lm}^*(\theta',\varphi')\rho(\mathbf{r}')\right)$$
$$= \frac{1}{4\pi\varepsilon_0}\sum_{lm}\frac{4\pi\, q_{lm}}{2l+1}\frac{Y_{lm}(\theta,\varphi)}{r^{l+1}}$$

In terms of Cartesian expansion:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}+\frac{\mathbf{p}\cdot\mathbf{r}}{r^3}+\frac{1}{2}\sum_{i,j}Q_{ij}\frac{r_i r_j}{r^5}\cdots\right)$$

Focus on dipolar contributions:

For r outside the extent of $\rho(\mathbf{r})$:

Electrostatic potential:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{\mathbf{p}\cdot\mathbf{r}}{r^3}\right)$$

Electrostatic field:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{3\mathbf{r}(\mathbf{p}\cdot\mathbf{r})-r^2\mathbf{p}}{r^5}-\frac{4\pi}{3}\mathbf{p}\delta^3(\mathbf{r})\right)$$

Coarse grain representation of macroscopic distribution of dipoles:

Electric polarization $\mathbf{P}(\mathbf{r})$:

$$\mathbf{P}(\mathbf{r}) \equiv \sum_i \mathbf{p}_i\delta^3(\mathbf{r}-\mathbf{r}_i)$$

Mono electric charge density $\rho_{\text{mono}}(\mathbf{r})$:

$$\rho_{\text{mono}}(\mathbf{r}) \equiv \sum_i q_i\delta^3(\mathbf{r}-\mathbf{r}_i)$$

Electrostatic potential for a single monopole charge q and a single dipole $\mathbf{p}$:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}+\frac{\mathbf{p}\cdot\mathbf{r}}{r^3}\right)$$

Coarse grain representation of macroscopic distribution of dipoles -- continued:

Electrostatic potential for a single monopole charge q and a single dipole $\mathbf{p}$:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{\mathbf{p}\cdot\mathbf{r}}{r^3}\right)$$

Electrostatic potential for collections of monopole charges q_i and dipoles $\mathbf{p}_i$:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\int d^3r' \frac{\rho_{mono}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} + \int d^3r' \frac{\mathbf{P}(\mathbf{r}')\cdot(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\right)$$

Note: $\int d^3r' \frac{\mathbf{P}(\mathbf{r}')\cdot(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} = \int d^3r' \, \mathbf{P}(\mathbf{r}')\cdot\nabla' \frac{1}{|\mathbf{r}-\mathbf{r}'|} = -\int d^3r' \frac{\nabla'\cdot\mathbf{P}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$

Coarse grain representation of macroscopic distribution of dipoles -- continued:

Electrostatic potential for collections of monopole charges q_i and dipoles $\mathbf{p}_i$:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\int d^3r' \frac{\rho_{mono}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} - \int d^3r' \frac{\nabla'\cdot\mathbf{P}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\right)$$

$$-\nabla^2\Phi(\mathbf{r}) = \nabla\cdot\mathbf{E}(\mathbf{r}) = \frac{1}{\varepsilon_0}\left(\rho_{mono}(\mathbf{r}) - \nabla\cdot\mathbf{P}(\mathbf{r})\right)$$

$$\Rightarrow \nabla\cdot\left(\varepsilon_0\mathbf{E}(\mathbf{r}) + \mathbf{P}(\mathbf{r})\right) = \rho_{mono}(\mathbf{r})$$

Define Displacement field: $\mathbf{D}(\mathbf{r}) \equiv \varepsilon_0\mathbf{E}(\mathbf{r}) + \mathbf{P}(\mathbf{r})$

Macroscopic form of Gauss's law: $\nabla\cdot\mathbf{D}(\mathbf{r}) = \rho_{mono}(\mathbf{r})$

Coarse grain representation of macroscopic distribution of dipoles -- continued:

Many materials are polarizable and produce a polarization field in the presence of an electric field with a proportionality constant χ_e :

$$\mathbf{P}(\mathbf{r}) = \varepsilon_0\chi_e\mathbf{E}(\mathbf{r})$$

$$\mathbf{D}(\mathbf{r}) \equiv \varepsilon_0\mathbf{E}(\mathbf{r}) + \mathbf{P}(\mathbf{r}) = \varepsilon_0(1+\chi_e)\mathbf{E}(\mathbf{r}) \equiv \varepsilon\mathbf{E}(\mathbf{r})$$

ε represents the dielectric function of the material

Boundary value problems in the presence of dielectrics

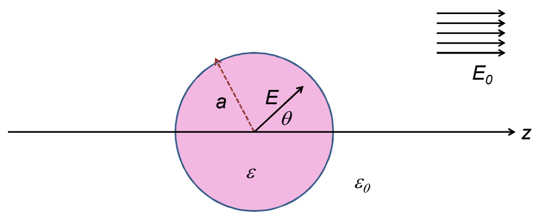
For $\rho_{mono}(\mathbf{r}) = 0$

$\nabla\cdot\mathbf{D}(\mathbf{r}) = 0$ and $\nabla\times\mathbf{E}(\mathbf{r}) = 0$

$\Rightarrow$ At a surface between two dielectrics, in terms of surface normal $\hat{\mathbf{r}}$:

$\hat{\mathbf{r}}\cdot\mathbf{D}(\mathbf{r}) = \text{continuous} = \hat{\mathbf{r}}\times\mathbf{E}(\mathbf{r})$

Boundary value problems in the presence of dielectrics – example:



$$\nabla\cdot\mathbf{D}(\mathbf{r})=0 \quad \text{and} \quad \nabla\times\mathbf{E}(\mathbf{r})=0 \quad \text{At } r=a: \quad \varepsilon\frac{\partial\Phi_<(\mathbf{r})}{\partial r}=\varepsilon_0\frac{\partial\Phi_>(\mathbf{r})}{\partial r}$$

$$\text{For } r\le a \quad \mathbf{D}(\mathbf{r})=-\varepsilon\nabla\Phi(\mathbf{r}) \qquad \frac{\partial\Phi_<(\mathbf{r})}{\partial\theta}=\frac{\partial\Phi_>(\mathbf{r})}{\partial\theta}$$

$$\text{For } r> a \quad \mathbf{D}(\mathbf{r})=-\varepsilon_0\nabla\Phi(\mathbf{r})$$

Boundary value problems in the presence of dielectrics – example -- continued:

$$\Phi_<(\mathbf{r})=\sum_{l=0}^{\infty}A_l r^l P_l(\cos\theta)$$

$$\Phi_>(\mathbf{r})=\sum_{l=0}^{\infty}\left(B_l r^l+\frac{C_l}{r^{l+1}}\right)P_l(\cos\theta)$$

$$\text{At } r=a: \quad \varepsilon\frac{\partial\Phi_<(\mathbf{r})}{\partial r}=\varepsilon_0\frac{\partial\Phi_>(\mathbf{r})}{\partial r}$$

$$\frac{\partial\Phi_<(\mathbf{r})}{\partial\theta}=\frac{\partial\Phi_>(\mathbf{r})}{\partial\theta}$$

$$\text{For } r\to\infty \quad \Phi_>(\mathbf{r})=-E_0 r\cos\theta$$

Solution -- only $l=1$ contributes

$$B_1=-E_0$$

$$A_1=-\left(\frac{3}{2+\varepsilon/\varepsilon_0}\right)E_0 \qquad C_1=\left(\frac{\varepsilon/\varepsilon_0-1}{2+\varepsilon/\varepsilon_0}\right)a^3E_0$$

Boundary value problems in the presence of dielectrics – example -- continued:

$$\Phi_<(\mathbf{r})=-\left(\frac{3}{2+\varepsilon/\varepsilon_0}\right)E_0\, r\cos\theta$$

$$\Phi_>(\mathbf{r})=-\left(r-\left(\frac{\varepsilon/\varepsilon_0-1}{2+\varepsilon/\varepsilon_0}\right)\frac{a^3}{r^2}\right)E_0\cos\theta$$