PHY 770 -- Statistical Mechanics
12:00* - 1:45 PM TR Olin 107

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Course Webpage: http://www.wfu.edu/~natalie/s14phy770

Lecture 21

Chap. 9 – Transport coefficients

- ❑ **Conserved quantities associated with the Boltzmann equation**
- ❑ **Zero order solutions**
- ❑ **Linearized Bolzmann equation**

***Partial make-up lecture -- early start time**

11	Tue: 02/18/2014	Chap. 5	Equilibrium Statistical Mechanics	#11	02/27/2014
12	Thur: 02/20/2014	Chap. 5	The Ising model (class 12-1:45 PM)	#12	02/27/2014
	Tue: 02/25/2014		NAWH out of town -- no class		
13	Thur: 02/27/2014	Chap. 6	Grand partition function (class 12-1:45 PM)		
	Tue: 03/04/2014	*APS Meeting*	Take-home exam (no class meeting)		
	Thur: 03/06/2014	*APS Meeting*	Take-home exam (no class meeting)		
	Tue: 03/11/2014		Spring break (no class meeting)		
	Thur: 03/13/2014		Spring break (no class meeting)		
14	Tue: 03/18/2014	Chap. 6	Fermi and Bose particles (class 12-1:45 PM; Exam due)	#13	03/25/2014
15	Thur: 03/20/2014	Chap. 6	Interacting particles (class 12-1:45 PM)	#14	03/25/2014
16	Tue: 03/25/2014	Chap. 7	Langevin equation (class 12-1:45 PM)	#15	04/01/2014
17	Thur: 03/27/2014	Chap. 7	Fokker-Planck equation (class 12-1:45 PM)	#16	04/03/2014
18	Tue: 04/01/2014	Chap. 7	Linear Response (class 12-1:45 PM)	#17	04/10/2014
19	Thur: 04/03/2014	Chap. 9	Transport theory (class 12-1:45 PM)	#18	4/10/2014
20	Tue: 04/08/2014	Chap. 9	The Boltzmann Equation (class 12-1:45 PM)	#19	4/10/2014
21	Thur: 04/10/2014	Chap. 9	The Boltzmann Equation (class 12-1:45 PM)	#20	4/17/2014
22	Tue: 04/15/2014		(class 12-1:45 PM)		
23	Thur: 04/17/2014		(class 12-1:45 PM)		
24	Tue: 04/22/2014		(class 12-1:45 PM)		
	Thur: 04/24/2014		Presentations Part I (class 11-1:45 PM)		
	Tue: 04/29/2014		Presentations Part II (class 11-1:45 PM)		

The Boltzmann equation:

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla_r+\frac{\mathbf{F}}{m}\cdot\nabla_v\right)f(\mathbf{r},\mathbf{v},t)=\left(\frac{\partial f}{\partial t}\right)_{coll}$$

With:

$$\left(\frac{\partial f}{\partial t}\right)_{coll}=$$

$$\int d\Omega_{CM}\int d^3v_2\left|\mathbf{v}_2-\mathbf{v}\right|\frac{d\sigma_{CM}}{d\Omega_{CM}}\left(f\left(\mathbf{r},\mathbf{v}',t\right)f\left(\mathbf{r},\mathbf{v}_2',t\right)-f\left(\mathbf{r},\mathbf{v},t\right)f\left(\mathbf{r},\mathbf{v}_2,t\right)\right)$$

In the notation of your textbook:

$$\left(\frac{\partial f}{\partial t}\right)_{coll}=\int d\Omega_{CM}\int d^3v_2 g\sigma_{cm}(b,g)\left(f\left(\mathbf{r},\mathbf{v}',t\right)f\left(\mathbf{r},\mathbf{v}_2',t\right)-f\left(\mathbf{r},\mathbf{v},t\right)f\left(\mathbf{r},\mathbf{v}_2,t\right)\right)$$

Conservation laws implied by Boltzmann equation

Consider some quantity (mass, momentum, etc) associated with a particle of velocity $\mathbf{v}$ and position $\mathbf{r}$: $\chi(\mathbf{r},\mathbf{v})$

If $\chi(\mathbf{r},\mathbf{v})$ is conserved during a collision $\left(\mathbf{v}_1,\mathbf{v}_2 \rightarrow \mathbf{v}_1',\mathbf{v}_2'\right)$,

so that $\chi(\mathbf{r},\mathbf{v}_1)+\chi(\mathbf{r},\mathbf{v}_2)=\chi(\mathbf{r},\mathbf{v}_1')+\chi(\mathbf{r},\mathbf{v}_2')$, it is possible to show that:

$$\int d^3v\chi(\mathbf{r},\mathbf{v})\left(\frac{\partial f}{\partial t}\right)_{coll}=0$$

These results lead to identities involving the distribution function $f(\mathbf{r},\mathbf{v},t)$

Ref: K. Huang, *Statistical Mechanics*

Evaluation of collision integral $I \equiv \int d^3v\chi(\mathbf{r},\mathbf{v})\left(\frac{\partial f}{\partial t}\right)_{coll}$

Where $\left(\frac{\partial f}{\partial t}\right)_{coll}=$

$$\int d\Omega_{CM}\int d^3v_2\left|\mathbf{v}_2-\mathbf{v}\right|\frac{d\sigma_{CM}}{d\Omega_{CM}}\left(f\left(\mathbf{r},\mathbf{v}',t\right)f\left(\mathbf{r},\mathbf{v}_2',t\right)-f\left(\mathbf{r},\mathbf{v},t\right)f\left(\mathbf{r},\mathbf{v}_2,t\right)\right)$$

Note that because of the symmetry of the processes associated with the integrand we can change the variables of the integrand:

$\mathbf{v}\leftrightarrow\mathbf{v}_2$ or $\mathbf{v},\mathbf{v}_2\leftrightarrow\mathbf{v}',\mathbf{v}_2'$ or $\mathbf{v},\mathbf{v}_2\leftrightarrow\mathbf{v}_2',\mathbf{v}'$

Evaluating all 4 expressions for I and adding them, we find:

$$4I=\int d^3v\int d\Omega_{CM}\int d^3v_2\left|\mathbf{v}_2-\mathbf{v}\right|\frac{d\sigma_{CM}}{d\Omega_{CM}}\times$$
$$\left(f\left(\mathbf{r},\mathbf{v}',t\right)f\left(\mathbf{r},\mathbf{v}_2',t\right)-f\left(\mathbf{r},\mathbf{v},t\right)f\left(\mathbf{r},\mathbf{v}_2,t\right)\right)\times$$
$$\left(\chi(\mathbf{r},\mathbf{v},t)+\chi(\mathbf{r},\mathbf{v}_2,t)-\chi(\mathbf{r},\mathbf{v}',t)-\chi(\mathbf{r},\mathbf{v}_2',t)\right)$$

$$\Rightarrow 4I=0$$

Conservation laws implied by Boltzmann equation -- continued

$$\int d^3v\chi(\mathbf{r},\mathbf{v})\left(\frac{\partial f}{\partial t}\right)_{coll}=0$$

Since $\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla_r+\frac{\mathbf{F}}{m}\cdot\nabla_v\right)f(\mathbf{r},\mathbf{v},t)=\left(\frac{\partial f}{\partial t}\right)_{coll}$

$$\int d^3v\chi(\mathbf{r},\mathbf{v})\left(\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla_r+\frac{\mathbf{F}}{m}\cdot\nabla_v\right)f(\mathbf{r},\mathbf{v},t)\right)=0$$

Define: $n(\mathbf{r},t)\equiv\int d^3v f(\mathbf{r},\mathbf{v},t)$

$$\langle A\rangle\equiv\frac{1}{n(\mathbf{r},t)}\int d^3v A(\mathbf{r},\mathbf{v},t)f(\mathbf{r},\mathbf{v},t)$$

General identity:

$$\frac{\partial\left(n\langle\chi\rangle\right)}{\partial t}+\nabla_{\mathbf{r}}\cdot\left(n\langle\mathbf{v}\chi\rangle\right)-n\langle\mathbf{v}\cdot\nabla_{\mathbf{r}}\chi\rangle-\frac{n}{m}\langle\mathbf{F}\cdot\nabla_{\mathbf{v}}\chi\rangle=0$$

Conservation laws implied by Boltzmann equation -- continued

General identity:

$$\frac{\partial\left(n\langle\chi\rangle\right)}{\partial t}+\nabla_{\mathbf{r}}\cdot\left(n\langle\mathbf{v}\chi\rangle\right)-n\langle\mathbf{v}\cdot\nabla_{\mathbf{r}}\chi\rangle-\frac{n}{m}\langle\mathbf{F}\cdot\nabla_{\mathbf{v}}\chi\rangle=0$$

Example: $\chi=m$

$$\frac{\partial(nm)}{\partial t}+\nabla_{\mathbf{r}}\cdot\left(nm\langle\mathbf{v}\rangle\right)=0$$

Defining mass density: $\rho(\mathbf{r},t)\equiv mn(\mathbf{r},t)$

$$\frac{\partial\rho}{\partial t}+\nabla_{\mathbf{r}}\cdot\left(\rho\langle\mathbf{v}\rangle\right)=0 \qquad \text{continuity equation}$$

Conservation laws implied by Boltzmann equation -- continued

Example: $\chi=m\mathbf{v}$ (conservation of momentum)

$$\rho\left(\frac{\partial}{\partial t}+\langle\mathbf{v}\rangle\cdot\nabla\right)\langle\mathbf{v}\rangle=\frac{\rho}{m}\mathbf{F}-\nabla\cdot\mathbf{P}$$

where $\mathbf{P}=\rho\left\langle\left(\mathbf{v}-\langle\mathbf{v}\rangle\right)\left(\mathbf{v}-\langle\mathbf{v}\rangle\right)\right\rangle$ (dyad notation)

Example: $\chi=\frac{1}{2}m\left|\mathbf{v}-\langle\mathbf{v}\rangle\right|^2$ (conservation of energy)

$$\rho\left(\frac{\partial}{\partial t}+\langle\mathbf{v}\rangle\cdot\nabla\right)\theta=-\frac{2}{3}\nabla\cdot\mathbf{q}-\frac{2}{3}\mathbf{P}\cdot\mathbf{\Lambda}$$

where $\theta\equiv kT=\frac{1}{3}m\left\langle\left|\mathbf{v}-\langle\mathbf{v}\rangle\right|^2\right\rangle$ (temperature)

$\mathbf{q}=\frac{1}{2}m\left\langle\left(\mathbf{v}-\langle\mathbf{v}\rangle\right)\left|\mathbf{v}-\langle\mathbf{v}\rangle\right|^2\right\rangle$ (heat flux)

$$\Lambda_{ij}=\frac{1}{2}m\left(\frac{\partial\langle v_i\rangle}{\partial x_j}+\frac{\partial\langle v_j\rangle}{\partial x_i}\right)$$

Summary of conservation results

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\left(\rho\langle\mathbf{v}\rangle\right)=0$$

$$\rho\left(\frac{\partial}{\partial t}+\langle\mathbf{v}\rangle\cdot\nabla\right)\langle\mathbf{v}\rangle=\frac{\rho}{m}\mathbf{F}-\nabla\cdot\mathbf{P}$$

$$\rho\left(\frac{\partial}{\partial t}+\langle\mathbf{v}\rangle\cdot\nabla\right)\theta=-\frac{2}{3}\nabla\cdot\mathbf{q}-\frac{2}{3}\mathbf{P}\cdot\mathbf{\Lambda}$$

Note that we have integrated out the velocity dependence, but all variables are position and time dependent.

Approximate treatment valid for functions which vary slowly over length scales large compared to mean free path and times compared to collision time

$$f\left(\mathbf{r},\mathbf{v},t\right)\approx f^0\left(\mathbf{r},\mathbf{v},t\right)=n\left(\frac{m}{2\pi\theta}\right)^{3/2}e^{-\frac{m\left|\mathbf{v}-\langle\mathbf{v}\rangle\right|^2}{2\theta}}$$

Approximate distribution function -- continued

$$f(\mathbf{r},\mathbf{v},t) \approx f^0(\mathbf{r},\mathbf{v},t) = n\left(\frac{m}{2\pi\theta}\right)^{3/2} e^{-\frac{m|\mathbf{v}-\langle\mathbf{v}\rangle|^2}{2\theta}}$$

Zero order functions:

$$n^0(\mathbf{r},t) \equiv \int d^3v f^0(\mathbf{r},\mathbf{v},t) = n(\mathbf{r},t)$$

$$\theta^0(\mathbf{r},t) = \frac{1}{3}m\int d^3v f^0(\mathbf{r},\mathbf{v},t)\left|\mathbf{v}-\langle\mathbf{v}\rangle\right|^2 = \theta(\mathbf{r},t)$$

$$\mathbf{q}^0(\mathbf{r},t) = \frac{1}{2}m^2 n(\mathbf{r},t)\int d^3v f^0(\mathbf{r},\mathbf{v},t)\left(\mathbf{v}-\langle\mathbf{v}\rangle\right)\left|\mathbf{v}-\langle\mathbf{v}\rangle\right|^2 = 0$$

$$P_{ij}^0(\mathbf{r},t) = mn(\mathbf{r},t)\int d^3v f^0(\mathbf{r},\mathbf{v},t)\left(v_i-\langle v_i\rangle\right)\left(v_j-\langle v_j\rangle\right) = \delta_{ij}P(\mathbf{r},t)$$
$$= \delta_{ij}n(\mathbf{r},t)\theta(\mathbf{r},t)$$

Approximate distribution function -- continued

Exact relationships:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\left(\rho\langle\mathbf{v}\rangle\right) = 0$$

$$\rho\left(\frac{\partial}{\partial t} + \langle\mathbf{v}\rangle\cdot\nabla\right)\langle\mathbf{v}\rangle = \frac{\rho}{m}\mathbf{F} - \nabla\cdot\mathbf{P}$$

$$\rho\left(\frac{\partial}{\partial t} + \langle\mathbf{v}\rangle\cdot\nabla\right)\theta = -\frac{2}{3}\nabla\cdot\mathbf{q} - \frac{2}{3}\mathbf{P}\cdot\mathbf{\Lambda}$$

Approximate relationships:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\left(\rho\langle\mathbf{v}\rangle\right) = 0$$

$$\rho\left(\frac{\partial}{\partial t} + \langle\mathbf{v}\rangle\cdot\nabla\right)\langle\mathbf{v}\rangle = \frac{\rho}{m}\mathbf{F} - \nabla P$$

$$\left(\frac{\partial}{\partial t} + \langle\mathbf{v}\rangle\cdot\nabla\right)\theta = -\frac{2}{3}\theta\left(\nabla\cdot\langle\mathbf{v}\rangle\right)$$

Approximate distribution function -- continued

Approximate relationships:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\left(\rho\langle\mathbf{v}\rangle\right) = 0$$

$$\rho\left(\frac{\partial}{\partial t} + \langle\mathbf{v}\rangle\cdot\nabla\right)\langle\mathbf{v}\rangle = \frac{\rho}{m}\mathbf{F} - \nabla P$$

$$\left(\frac{\partial}{\partial t} + \langle\mathbf{v}\rangle\cdot\nabla\right)\theta = -\frac{2}{3}\theta\left(\nabla\cdot\langle\mathbf{v}\rangle\right)$$

Some consequences:

$$\left(\frac{\partial}{\partial t} + \langle\mathbf{v}\rangle\cdot\nabla\right)\rho\theta^{-3/2} = 0$$

$$\Rightarrow \rho(\mathbf{r},t)\left(\theta(\mathbf{r},t)\right)^{-3/2} = K \quad \text{(constant)}$$

Note that: $P = n\theta$

$$P\rho^{-5/3} = K'$$

Approximate distribution function -- continued

Approximate relationships:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \langle \mathbf{v} \rangle \right) = 0$$

$$\rho \left(\frac{\partial}{\partial t} + \langle \mathbf{v} \rangle \cdot \nabla \right) \langle \mathbf{v} \rangle = \frac{\rho}{m} \mathbf{F} - \nabla P$$

$$\left(\frac{\partial}{\partial t} + \langle \mathbf{v} \rangle \cdot \nabla \right) \theta = -\frac{2}{3} \theta \left(\nabla \cdot \langle \mathbf{v} \rangle \right)$$

Assuming that $\langle \mathbf{v} \rangle$ and time and space derivatives of $\langle \mathbf{v} \rangle, \rho, \theta,$ and P are small; in absence of force $\mathbf{F}$, lowest order terms of the equations are :

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \langle \mathbf{v} \rangle = 0$$

$$\rho \frac{\partial \langle \mathbf{v} \rangle}{\partial t} = -\nabla P$$

$$\frac{\partial \theta}{\partial t} = -\frac{2}{3} \theta \left(\nabla \cdot \langle \mathbf{v} \rangle \right)$$

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Approximate distribution function -- continued

First order equations:

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \langle \mathbf{v} \rangle = 0$$

$$\rho \frac{\partial \langle \mathbf{v} \rangle}{\partial t} = -\nabla P$$

$$\frac{\partial \theta}{\partial t} = -\frac{2}{3} \theta \left(\nabla \cdot \langle \mathbf{v} \rangle \right)$$

Further linearized analysis:

$$\nabla^2 P - \frac{\partial^2 \rho}{\partial t^2} = 0$$

Also assuming $P = P(\rho(\mathbf{r}, t))$:

$$\nabla^2 P \approx \nabla \cdot \left(\left(\frac{\partial P}{\partial \rho} \right)_S \nabla \rho \right) \approx \left(\frac{\partial P}{\partial \rho} \right)_S \nabla^2 \rho$$

Using relation for adiabatic compressibility:

$$\kappa_S = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial P} \right)_S = \frac{3}{5} \frac{m}{\rho \theta}$$

$$\left(\frac{\partial P}{\partial \rho} \right)_S = \frac{1}{\rho \kappa_S} \equiv c^2$$

$$\Rightarrow \nabla^2 \rho - \frac{1}{c^2} \frac{\partial^2 \rho}{\partial t^2} = 0$$

Wave equation for sound

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Approximate distribution function -- continued

Euler equation for fluids:

$$\rho \left(\frac{\partial}{\partial t} + \langle \mathbf{v} \rangle \cdot \nabla \right) \langle \mathbf{v} \rangle = \frac{\rho}{m} \mathbf{F} - \nabla P$$

$$\left(\frac{\partial}{\partial t} + \langle \mathbf{v} \rangle \cdot \nabla \right) \langle \mathbf{v} \rangle = \frac{1}{m} \mathbf{F} - \frac{\nabla P}{\rho}$$

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First order correction to zero-order results

$$f^0(\mathbf{r},\mathbf{v},t) = n\left(\frac{m}{2\pi\theta}\right)^{3/2} e^{-\frac{m|\mathbf{v}-\langle\mathbf{v}\rangle|^2}{2\theta}}$$

Define: $f(\mathbf{r},\mathbf{v},t) = f^0(\mathbf{r},\mathbf{v},t) + w(\mathbf{r},\mathbf{v},t)$

$$\left(\frac{\partial f}{\partial t}\right)_{coll} =$$

$$\int d\Omega_{CM} \int d^3v_2 |\mathbf{v}_2 - \mathbf{v}| \frac{d\sigma_{CM}}{d\Omega_{CM}} \left(f(\mathbf{r},\mathbf{v}',t) f(\mathbf{r},\mathbf{v}_2',t) - f(\mathbf{r},\mathbf{v},t) f(\mathbf{r},\mathbf{v}_2,t) \right)$$

$$\approx \int d\Omega_{CM} \int d^3v_2 |\mathbf{v}_2 - \mathbf{v}| \frac{d\sigma_{CM}}{d\Omega_{CM}} \left(f^0{}' w_2' + f^{02}{}' w' - f^0 w_2 - f^{02} w \right)$$

where $f^0{}' \equiv f^0(\mathbf{r},\mathbf{v}',t)$, $f^{02}{}' \equiv f^0(\mathbf{r},\mathbf{v}_2',t)$ etc..

$w' \equiv w(\mathbf{r},\mathbf{v}',t)$, $w^2{}' \equiv w(\mathbf{r},\mathbf{v}_2',t)$ etc..

First order correction to zero-order results -- continued

$$\left(\frac{\partial f}{\partial t}\right)_{coll} \approx \int d\Omega_{CM} \int d^3v_2 |\mathbf{v}_2 - \mathbf{v}| \frac{d\sigma_{CM}}{d\Omega_{CM}} \left(f^0{}' w_2' + f^{02}{}' w' - f^0 w_2 - f^{02} w \right)$$

$$\approx -\frac{f(\mathbf{r},\mathbf{v},t) - f^0(\mathbf{r},\mathbf{v},t)}{\tau} = -\frac{w(\mathbf{r},\mathbf{v},t)}{\tau}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_r + \frac{\mathbf{F}}{m}\cdot\nabla_v\right) f(\mathbf{r},\mathbf{v},t) = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

$$\Rightarrow \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_r + \frac{\mathbf{F}}{m}\cdot\nabla_v\right) f^0(\mathbf{r},\mathbf{v},t) \approx -\frac{w(\mathbf{r},\mathbf{v},t)}{\tau}$$

We can use this expression to evaluate *w(**r**,**v**,t)*

First order correction to zero-order results -- continued

$$w(\mathbf{r},\mathbf{v},t) = -\tau\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_r + \frac{\mathbf{F}}{m}\cdot\nabla_v\right) f^0(\mathbf{r},\mathbf{v},t)$$

$$f^0(\mathbf{r},\mathbf{v},t) = n\left(\frac{m}{2\pi\theta}\right)^{3/2} e^{-\frac{m|\mathbf{v}-\langle\mathbf{v}\rangle|^2}{2\theta}}$$

Defining: $\mathbf{U} \equiv \mathbf{v} - \langle\mathbf{v}\rangle$

$$\frac{\partial f^0}{\partial \rho} = \frac{f^0}{\rho}$$

$$\frac{\partial f^0}{\partial \theta} = \frac{f^0}{\theta}\left(\frac{mU^2}{2\theta} - \frac{3}{2}\right)$$

$$\frac{\partial f^0}{\partial v_i} = -\frac{f^0}{\theta} m U_i$$

First order correction to zero-order results -- continued

Estimation of effects of first order correction:

$$\mathbf{q} = \frac{m^2}{2}\int d^3v\left(\mathbf{v}-\langle\mathbf{v}\rangle\right)\left|\mathbf{v}-\langle\mathbf{v}\rangle\right|^2 w(\mathbf{r},\mathbf{v},t)$$
$$= -K\nabla\theta$$

where approximate coefficient of thermal conductivity is given by

$$K = \frac{5}{2}\tau\theta n$$

This result (obtained after MANY steps) is consistent with the approximation that q is small if mean free path is small compared to sample size

Reichl's treatment of linearized Boltzmann equation

Assume $\langle\mathbf{v}\rangle = 0$ and $\mathbf{F}=0$

$$f(\mathbf{r},\mathbf{v},t) \approx f^0(\mathbf{r},\mathbf{v},t)\left(1+h(\mathbf{r},\mathbf{v},t)\right)$$

For free particle moving in 3 dimensions in thermal equilibrium:

$$f^0(\mathbf{r},\mathbf{v},t) = f^0(\mathbf{v}) = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/(2kT)}$$

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla_r\right)f(\mathbf{r},\mathbf{v},t) \approx f^0(\mathbf{v})\left(\frac{\partial h}{\partial t}+\mathbf{v}\cdot\nabla_r h\right)$$

$$\left(\frac{\partial f}{\partial t}\right)_{coll} = \int d\Omega_{CM}\int d^3v_2 g\sigma_{cm}(b,g)\ \left(f(\mathbf{r},\mathbf{v}',t)f(\mathbf{r},\mathbf{v}_2',t)-f(\mathbf{r},\mathbf{v},t)f(\mathbf{r},\mathbf{v}_2,t)\right)$$
$$\approx f^0(\mathbf{v})\int d\Omega_{CM}\int d^3v_2 g\sigma_{cm}(b,g)f^0(\mathbf{v}_2)\left(h(\mathbf{r},\mathbf{v}',t)+h(\mathbf{r},\mathbf{v}_2',t)-h(\mathbf{r},\mathbf{v},t)-h(\mathbf{r},\mathbf{v}_2,t)\right)$$

$$\left(\frac{\partial h}{\partial t}+\mathbf{v}\cdot\nabla_r h\right) = \hat{\mathbf{C}}h$$

Reichl's treatment of linearized Boltzmann equation

$$\left(\frac{\partial h}{\partial t}+\mathbf{v}\cdot\nabla_r h\right) = \hat{\mathbf{C}}h$$

$$\hat{\mathbf{C}}h = \int d\Omega_{CM}\int d^3v_2 g\sigma_{cm}(b,g)f^0(\mathbf{v}_2)\left(h(\mathbf{r},\mathbf{v}',t)+h(\mathbf{r},\mathbf{v}_2',t)-h(\mathbf{r},\mathbf{v},t)-h(\mathbf{r},\mathbf{v}_2,t)\right)$$

For spatially isotropic system $h(\mathbf{r},\mathbf{v},t) = h(\mathbf{v},t)$

$\hat{\mathbf{C}}$ is a linear scalar operator which is isotropic in $\mathbf{v}$

$$\hat{\mathbf{C}}\Psi_{\nu lm}(\mathbf{v}) = \lambda_{\nu l}\Psi_{\nu lm}(\mathbf{v}) \qquad \Psi_{\nu lm}(\mathbf{v}) = R_{\nu l}(v)Y_{lm}(\hat{\mathbf{v}})$$

Can show that $\lambda_{\nu l} \leq 0$

$$\Rightarrow h(\mathbf{v},t) = \sum_{\nu lm} A_{\nu lm}e^{\lambda_{\nu l}t}\ \Psi_{\nu lm}(\mathbf{v})$$

Reichl's treatment of linearized Boltzmann equation -- continued

$$\left(\frac{\partial h}{\partial t}+\mathbf{v}\cdot\nabla_r h\right)=\hat{\mathbf{C}}h$$

More general solution:

$$h(\mathbf{r},\mathbf{v},t)=\Psi_\nu(\mathbf{v},\mathbf{k})\,\mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}\,e^{-i\omega_\nu t}$$

$$\left(\hat{\mathbf{C}}-i\mathbf{k}\cdot\mathbf{v}\right)\Psi_\nu(\mathbf{v},\mathbf{k})=-i\omega_\nu\Psi_\nu(\mathbf{v},\mathbf{k})$$