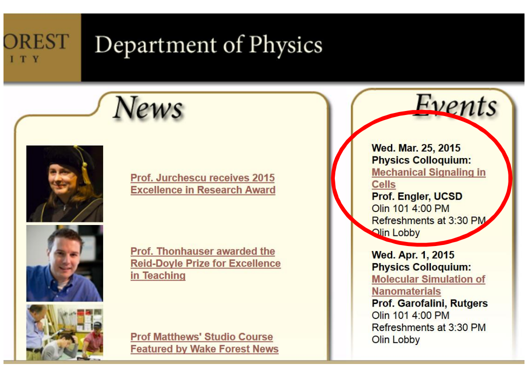
PHY 752 Solid State Physics
11-11:50 AM MWF Olin 107

Plan for Lecture 23:

- Transport phenomena – Chap. 17 & 18 in Marder
  - Hall effect
  - Magnetoresistance
  - Microscopic picture

Contains materials from Marder's lecture notes

22	Fri: 03/20/2015	Chap. 16	Electron Transport	#21	03/23/2015
23	Mon: 03/23/2015	Chap. 17	Electron Transport	#22	03/25/2015
24	Wed: 03/25/2015	Chap. 17 & 18	Electron Transport		
25	Fri: 03/27/2015	Chap. 18	Microscopic picture of transport		03/30/2015
26	Mon: 03/30/2015				04/01/2015
27	Wed: 04/01/2015				04/06/2015
	Fri: 04/03/2015	Good Friday	No class		
28	Mon: 04/06/2015				04/08/2015
29	Wed: 04/08/2015				04/10/2015
30	Fri: 04/10/2015				04/13/2015
31	Mon: 04/13/2015				04/15/2015
32	Wed: 04/15/2015				04/17/2015
33	Fri: 04/17/2015				04/20/2015
34	Mon: 04/20/2015				
35	Wed: 04/22/2015				
36	Fri: 04/24/2015				
	Mon: 04/27/2015		Presentations I		
	Wed: 04/29/2015		Presentations II		
	Fri: 05/01/2015		Presentations III & Take home exam		



WFU Physics Colloquium

TITLE: Mechanical Signaling and its Role in Differentiation and Aging

SPEAKER: Dr. Adam Engler,

Department of Bioengineering
University of California, San Diego

TIME: Wednesday March 25, 2015 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

Cells respond to the passive and active mechanics of their surrounding niche from the onset of fertilization through senescence. Their response is regulated by cell contractility but ultimately interpreted by a variety of nuclear- and adhesion-based mechanisms that convert biophysical information, e.g. niche stiffness, to biochemical cues. Here I will describe how one adhesion protein--vinculin--acts as a "molecular strain gauge," i.e. it opens and closes under force. I will highlight vinculin's role in regulating cell responses

Transport theory in the relaxation time approximation

Boltzmann's equation:

$$\frac{\partial g}{\partial t} = -\dot{\vec{r}} \cdot \frac{\partial}{\partial \vec{r}} g - \dot{\vec{k}} \frac{\partial}{\partial \vec{k}} g + \left.\frac{dg}{dt}\right|_{\text{coll.}}$$

Relaxation time approximation to collision term

$$\left.\frac{dg}{dt}\right|_{\text{coll.}} = -\frac{1}{\tau}\left[g_{\vec{r}\vec{k}} - f_{\vec{r}\vec{k}}\right]$$

Fermi-Dirac distribution

$$f_{\vec{r}\vec{k}} = \frac{1}{e^{\beta_{\vec{r}}(\mathcal{E}_{\vec{k}} - \mu_{\vec{r}})} + 1}$$

Transport theory in the relaxation time approximation

General solution to linear order:

$$g = f - \int_{-\infty}^{t} dt' e^{-(t-t')/\tau_{\mathcal{E}}} \vec{v}_{\vec{k}} \cdot \left\{ e\vec{E} + \vec{\nabla}\mu + \frac{\mathcal{E}_{\vec{k}} - \mu}{T} \vec{\nabla} T \right\} \frac{\partial f(t')}{\partial \mu}$$

Estimation of resulting current:

$$\vec{j} = \frac{\vec{J}}{\mathcal{V}} = -e \int [d\vec{k}]\, \vec{v}_{\vec{k}}\, g_{\vec{r}\vec{k}}.$$

Calculation of response coefficients

$$X_\alpha = \sum_\beta L_{\alpha\beta} x_\beta$$

General form of response coeffients:

$$L_{\alpha\beta} = \int [d\vec{k}_t]\, d\vec{r}_t \int_{-\infty}^{t} dt' \frac{d\dot{Q}(t)}{dx_\alpha} e^{-(t-t')/\tau_{\mathcal{E}}} \left[\frac{\partial}{\partial\mu} \ln f(t')\right] \frac{d\dot{Q}(t')}{dx_\beta}$$

where:

$$\dot{Q}_{\vec{r}\vec{k}} = \left[-e\vec{E} - \vec{\nabla}\mu - \frac{\vec{\nabla}T}{T}\left(\mathcal{E}_{\vec{k}} - \mu\right)\right] \cdot \vec{v}_{\vec{k}} f_{\vec{k}}$$

3/25/2015 PHY 752 Spring 2015 -- Lecture 24 7

Responses involving thermal and electrical gradients

Electrochemical force: $\vec{G} = \vec{E} + \frac{\vec{\nabla}\mu}{e}$

Electrochemical flux: $\vec{j} = -e\vec{J}_N/\mathcal{V} \qquad = -e\int \frac{d\vec{r}}{\mathcal{V}} \int [d\vec{k}]\, \vec{v}_{\vec{r}\vec{k}}\, g_{\vec{r}\vec{k}}$

Thermal force: $\frac{-\vec{\nabla}T}{T}$

Thermal flux: $\vec{j}_Q = (\vec{J}_{\mathcal{E}} - \mu\vec{J}_N)/\mathcal{V} \quad = \int \frac{d\vec{r}}{\mathcal{V}} \int [d\vec{k}]\, (\mathcal{E}_{\vec{k}} - \mu)\vec{v}_{\vec{r}\vec{k}} g_{\vec{r}\vec{k}}$

3/25/2015 PHY 752 Spring 2015 -- Lecture 24 8

Responses involving thermal and electrical gradients – continued

Linear coefficients:

$$\vec{j} = \mathbf{L}^{11}\vec{G} + \mathbf{L}^{12}\left(\frac{-\vec{\nabla}T}{T}\right)$$

$$\vec{j}_Q = \mathbf{L}^{21}\vec{G} + \mathbf{L}^{22}\left(\frac{-\vec{\nabla}T}{T}\right)$$

Note that these can be calculated from:

$$\mathbf{L}^{11} = \mathcal{L}^{(0)},\ \mathbf{L}^{12} = \mathbf{L}^{21} = -\frac{1}{e}\mathcal{L}^{(1)},\ \mathbf{L}^{22} = \frac{1}{e^2}\mathcal{L}^{(2)}.$$

where $\mathcal{L}^{(\nu)}_{\alpha\beta} = e^2 \int [d\vec{k}]\, \tau_{\mathcal{E}} \frac{\partial f}{\partial\mu} v_\alpha v_\beta \left(\mathcal{E}_{\vec{k}} - \mu\right)^\nu.$

3/25/2015 PHY 752 Spring 2015 -- Lecture 24 9

Responses involving thermal and electrical gradients – continued

Define: $$\sigma_{\alpha\beta}(\mathcal{E}) = \tau_{\mathcal{E}} e^2 \int [d\vec{k}]\, v_\alpha v_\beta\, \delta(\mathcal{E} - \mathcal{E}_{\vec{k}})$$

Note that: $$\mathcal{L}^{(\nu)}_{\alpha\beta} = \int d\mathcal{E} \frac{\partial f}{\partial \mu} (\mathcal{E} - \mu)^\nu \sigma_{\alpha\beta}(\mathcal{E}).$$

$$\frac{\partial f}{\partial \mu} \approx \delta(\mathcal{E} - \mathcal{E}_F)$$

$$\begin{aligned} \mathcal{L}^{(0)}_{\alpha\beta} &= \sigma_{\alpha\beta}(\mathcal{E}_F) \\ \mathcal{L}^{(1)}_{\alpha\beta} &= \frac{\pi^2}{3}(k_B T)^2 \sigma'_{\alpha\beta}(\mathcal{E}_F) \\ \mathcal{L}^{(2)}_{\alpha\beta} &= \frac{\pi^2}{3}(k_B T)^2 \sigma_{\alpha\beta}(\mathcal{E}_F). \end{aligned}$$

$$\sigma'_{\alpha\beta}(\mathcal{E}_F) \equiv \left.\frac{d\sigma(\mathcal{E})}{d\mathcal{E}}\right|_{\mathcal{E}=\mathcal{E}_F}$$

(See Chap. 6)

3/25/2015 PHY 752 Spring 2015 -- Lecture 24 10

Example: Thermal conductivity

$$\begin{aligned} \vec{j} &= \mathbf{L}^{11}\vec{G} + \mathbf{L}^{12}\left(\frac{-\vec{\nabla} T}{T}\right) \\ \vec{j}_Q &= \mathbf{L}^{21}\vec{G} + \mathbf{L}^{22}\left(\frac{-\vec{\nabla} T}{T}\right) \end{aligned}$$

Consider the case where there is heat flow but no current:

$$\vec{j} = 0 \qquad \vec{j}_Q = \kappa\left(-\vec{\nabla} T\right)$$

$$\begin{aligned} 0 &= \mathbf{L}^{11}\vec{G} + \mathbf{L}^{12}\left(\frac{-\vec{\nabla} T}{T}\right) \\ \vec{G} &= (\mathbf{L}^{11})^{-1}\mathbf{L}^{12}\frac{\vec{\nabla} T}{T}, \end{aligned}$$

3/25/2015 PHY 752 Spring 2015 -- Lecture 24 11

Example: Thermal conductivity -- continued

$$\begin{aligned} \vec{j}_Q &= \left[\mathbf{L}^{21}(\mathbf{L}^{11})^{-1}\mathbf{L}^{12} - \mathbf{L}^{22}\right]\left(\frac{\vec{\nabla} T}{T}\right) \\ \Rightarrow \kappa &= \frac{\mathbf{L}^{22}}{T} + \mathcal{O}\left(\frac{k_B T}{\mathcal{E}_F}\right)^2 \\ \Rightarrow \kappa_{\alpha\beta} &= \frac{\pi^2}{3}\frac{k_B^2 T}{e^2}\sigma_{\alpha\beta}. \end{aligned}$$

3/25/2015 PHY 752 Spring 2015 -- Lecture 24 12

Example: Hall effect

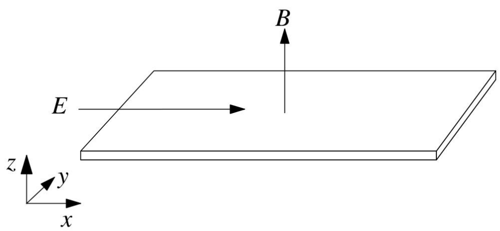


Figure 17.3: Geometry of the Hall effect.

Example: Hall effect -- continued

$$\hbar\dot{\vec{k}} = -e\frac{\vec{v}}{c}\times\vec{B} - e\vec{E}$$

$$\Rightarrow \vec{B}\times\hbar\dot{\vec{k}} + e\vec{B}\times\vec{E} = -e\vec{B}\times(\frac{\vec{v}}{c}\times\vec{B}) = -\frac{e}{c}\vec{v}_\perp B^2$$

$$\Rightarrow \vec{v}_\perp = -\frac{\hbar c}{e}\frac{\vec{B}\times\dot{\vec{k}}}{B^2} - c\frac{\vec{B}\times\vec{E}}{B^2}.$$

Example: Hall effect -- continued

$$g - f = \int_{-\infty}^{t} dt'\, e^{-(t-t')/\tau_\varepsilon}\left[\frac{c\hbar}{e}\frac{\vec{B}\times\dot{\vec{k}}}{B^2}\right]\cdot e\vec{E}\frac{\partial f}{\partial\mu}$$

$$= \int_{-\infty}^{t} dt'\, e^{-(t-t')/\tau_\varepsilon}\frac{c\hbar}{B^2}\dot{\vec{k}}\cdot\left[\vec{E}\times\vec{B}\right]\frac{\partial f}{\partial\mu}$$

$$= \frac{c\hbar}{B^2}\left(\vec{k} - \langle\vec{k}\rangle\right)\cdot\left[\vec{E}\times\vec{B}\right]\frac{\partial f}{\partial\mu}$$

$$\langle\vec{k}\rangle = \frac{1}{\tau_\varepsilon}\int_{-\infty}^{t} dt'\, e^{-(t-t')/\tau_\varepsilon}\vec{k}_{(t')}.$$

Example: Hall effect -- continued

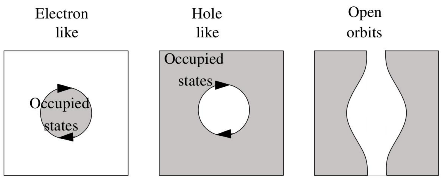
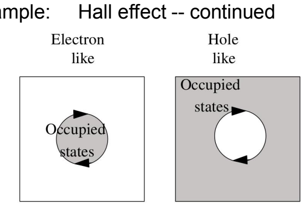


Figure 17.4: Electron-like, hole-like, and open orbits for the Hall effect.

$$
\begin{aligned}
\vec{j} &= -e\int[d\vec{k}]\,\vec{v}_{\vec{k}}\frac{\partial f}{\partial \mu}\frac{\hbar c}{B^2}\vec{k}\cdot(\vec{E}\times\vec{B}) \\
&= e\int[d\vec{k}]\,\frac{\partial f}{\partial \hbar\vec{k}}\frac{\hbar c}{B^2}\vec{k}\cdot(\vec{E}\times\vec{B}) \\
&= \left\{\frac{ec}{B^2}\int[d\vec{k}]\,\frac{\partial}{\partial\vec{k}}\left(f\vec{k}\cdot(\vec{E}\times\vec{B})\right)\right\} - \frac{nec}{B^2}(\vec{E}\times\vec{B})
\end{aligned}
$$

Example: Hall effect -- continued



$$\vec{j} = -\frac{nec}{B^2}(\vec{E}\times\vec{B}). \quad \vec{j} = \frac{pec}{B^2}(\vec{E}\times\vec{B}),$$

where: $$p = \int[d\vec{k}]\,(1-f_{\vec{k}})$$

Example: Hall effect -- continued

Hall coefficient:

$$R_H = -\frac{E_x}{Bj_y}.$$

$$R_H = \begin{cases} -\dfrac{1}{nec} & \text{for electron carriers} \\ \dfrac{1}{pec} & \text{for hole carriers} \end{cases}$$

Microscopic models of conduction (Chap. 18 of Marder)

Probability of electron scattering event $k \rightarrow k'$

$$\mathcal{P}(\vec{k} \to \vec{k}', t) = g_{\vec{k}}\left[1 - g_{\vec{k}'}\right]\delta_{\sigma\sigma'} W_{\vec{k}\vec{k}'}$$

distribution functions

transition probability

$$\left.\frac{dg}{dt}\right|_{\text{coll.}} = \frac{\mathcal{V}}{2}\int [d\vec{k}']\, g_{\vec{k}'}[1 - g_{\vec{k}}] W_{\vec{k}'\vec{k}} - g_{\vec{k}}[1 - g_{\vec{k}'}] W_{\vec{k}\vec{k}'}$$

Microscopic models of conduction -- continued

Estimate transition rate using Fermi Golden rule:

$$\begin{aligned} W_{\vec{k}\vec{k}'} &= \sum_{\text{final states}} \frac{2\pi}{\hbar}\delta(\mathcal{E}^{\text{f}} - \mathcal{E}^{\text{i}})|\langle \Psi^{\text{f}}|\hat{U}_{\text{tot}}|\Psi^{\text{i}}\rangle|^2 \\ &= 2\pi S^{\text{i}}(\vec{q}, \omega_e)|U(\vec{q})|^2 \frac{N_s}{\mathcal{V}^2}. \end{aligned}$$

Electron scattering potential

Number of scatters

Assume:

$$g_{\vec{k}} = f_{\vec{k}} + \vec{\mathcal{C}} \cdot \vec{k}.$$

Microscopic models of conduction -- continued

Approximate evaluation of integral:

$$\begin{aligned} \left.\frac{dg}{dt}\right|_{\text{coll.}} &= -\vec{\mathcal{C}} \cdot \frac{1}{2}\mathcal{V}\int [d\vec{k}']\,(\vec{k} - \vec{k}') W_{\vec{k}\vec{k}'}, \\ \left.\frac{dg}{dt}\right|_{\text{coll.}} &= -\frac{g - f}{\tau_{\mathcal{E}}}, \\ \frac{1}{\tau_{\mathcal{E}}} &= \frac{1}{2}\mathcal{V}\int [d\vec{k}']\, W_{\vec{k}\vec{k}'}(1 - \hat{k} \cdot \hat{k}'). \end{aligned}$$

$$(1 - \hat{k} \cdot \hat{k}') = 2\left(\frac{q}{2k_F}\right)^2,$$

$$\frac{1}{\tau_{\mathcal{E}}} = \frac{\pi}{2}\frac{N_s}{\mathcal{V}}\int [d\vec{q}]\, S(\vec{q}, \omega_e)|U(\vec{q})|^2 \frac{q^2}{k_F^2}$$