PHY 752 Solid State Physics
11-11:50 AM MWF Olin 107

Plan for Lecture 26:

- Chap. 19 in Marder
  - Properties of semiconductors
  - Diodes and other electronic devices

Some of the lecture materials are from slides prepared by Marder

21	Wed: 03/18/2015	Chap. 16	Electron Transport	#20	03/20/2015
22	Fri: 03/20/2015	Chap. 16	Electron Transport	#21	03/23/2015
23	Mon: 03/23/2015	Chap. 17	Electron Transport	#22	03/25/2015
24	Wed: 03/25/2015	Chap. 17 & 18	Electron Transport		
25	Fri: 03/27/2015	Chap. 18	Microscopic picture of transport	#23	03/30/2015
26	Mon: 03/30/2015	Chap. 19	Semiconductor devices	#24	04/01/2015
27	Wed: 04/01/2015				04/06/2015
	Fri: 04/03/2015	Good Friday	No class		
28	Mon: 04/06/2015				04/08/2015
29	Wed: 04/08/2015				04/10/2015
30	Fri: 04/10/2015				04/13/2015
31	Mon: 04/13/2015				04/15/2015
32	Wed: 04/15/2015				04/17/2015
33	Fri: 04/17/2015				04/20/2015
34	Mon: 04/20/2015				
35	Wed: 04/22/2015				
36	Fri: 04/24/2015				
	Mon: 04/27/2015		Presentations I		
	Wed: 04/29/2015		Presentations II		
	Fri: 05/01/2015		Presentations III & Take home exam		

Charge and voltage profile near a semiconductor p-n junction

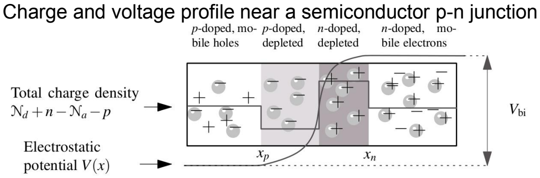


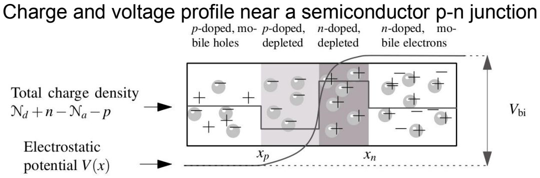
Figure 19.15: Illustration of the redistribution of mobile charges near a p–n junction. The mobile carriers abandon the region between x_n and x_p, leaving nonzero ionic charge density behind.

Equilibrium distributions

$$\begin{aligned} n(x) &= n_i e^{\beta(\mu + eV(x) - \mu_i)} \\ p(x) &= n_i e^{\beta(\mu_i - eV(x) - \mu)}. \end{aligned}$$

$$\begin{aligned} n(\infty)p(-\infty) &= \mathcal{N}_d\mathcal{N}_a = n_i^2 e^{\beta(eV(\infty) - eV(-\infty))} \\ \Rightarrow eV_{bi} &\equiv e[V(\infty) - V(-\infty)] \\ &= k_B T \ln \frac{\mathcal{N}_d\mathcal{N}_a}{n_i^2} = \mathcal{E}_g + k_B T \ln[\frac{\mathcal{N}_d\mathcal{N}_a}{\mathcal{N}_c\mathcal{N}_v}], \end{aligned}$$

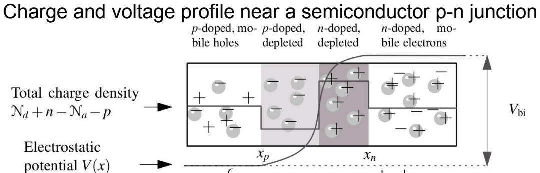
Charge and voltage profile near a semiconductor p-n junction



Internal voltage:

$$\frac{\partial^2 V}{\partial x^2} = -4\pi e[\mathcal{N}_d(x) - n(x) - \mathcal{N}_a(x) + p(x)]/\epsilon^0,$$

Charge and voltage profile near a semiconductor p-n junction

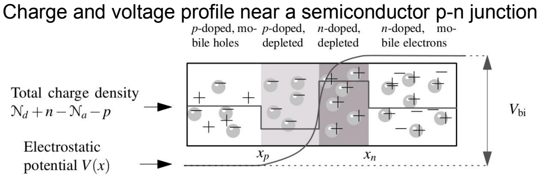


Simplified model:

$$\rho(x) = \begin{cases} 0 & \text{for } x < -\left|x_p\right| \\ -eN_a & \text{for } -\left|x_p\right| < x < 0 \\ eN_d & \text{for } 0 < x < x_n \\ 0 & \text{for } \quad x > x_n \end{cases}$$

$$V(x) = \begin{cases} V(-\infty) & \text{for } x < x_p \\ V(-\infty) + 2\pi e \frac{\mathcal{N}_a}{\epsilon^0}(x - x_p)^2 & \text{for } 0 > x > x_p \\ V(\infty) \quad - 2\pi e \frac{\mathcal{N}_d}{\epsilon^0}(x - x_n)^2 & \text{for } 0 < x < x_n \\ V(\infty) & \text{for } x > x_n. \end{cases}$$

Charge and voltage profile near a semiconductor p-n junction



Simplified model – continued:

$$V(-\infty) + 2\pi e \frac{\mathcal{N}_a}{\epsilon^0} x_p^2 = V(\infty) - 2\pi e \frac{\mathcal{N}_d}{\epsilon^0} x_n^2,$$

$$\mathcal{N}_d x_n = -\mathcal{N}_a x_p.$$

$$V_{bi} = V(\infty) - V(-\infty) = \frac{2\pi e}{\epsilon}\left(N_a x_p^2 + N_d x_n^2\right)$$

$$x_n = \sqrt{\frac{\epsilon^0 \mathcal{N}_a V_{\rm bi}}{2\pi e \mathcal{N}_d[\mathcal{N}_a + \mathcal{N}_d]}}$$

$$x_p = -\sqrt{\frac{\epsilon^0 \mathcal{N}_d V_{\rm bi}}{2\pi e \mathcal{N}_a[\mathcal{N}_a + \mathcal{N}_d]}},$$

Charge and voltage profile near a semiconductor p-n junction

Effects of adding an applied voltage to junction

Voltage across depletion zone:

$$\Delta V = V_{bi} - V_A$$

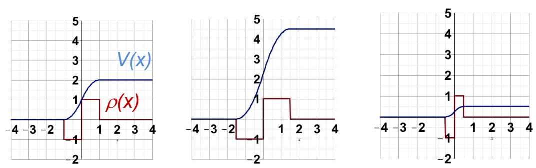
Depletion size in presence of applied voltage

$$x_n(V_A) = x_n(0)\left(1 - \frac{V_A}{V_{bi}}\right)^{1/2}$$

$$x_p(V_A) = x_p(0)\left(1 - \frac{V_A}{V_{bi}}\right)^{1/2}$$

Charge and voltage profile near a semiconductor p-n junction

$V_A=0$ $V_A<0$ $V_A>0$



Boltzmann equation treatment of current response to diode (relaxation time approximation)

$$\frac{\partial g}{\partial t} = -\frac{\partial}{\partial \vec{r}} \cdot \dot{\vec{r}} g - \frac{\partial}{\partial \vec{k}} \cdot \dot{\vec{k}} g + \frac{f-g}{\tau}.$$

$$n = \int [d\vec{k}]\, g_{\vec{r}\vec{k}},$$

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial \vec{r}} \cdot \langle \dot{\vec{r}} \rangle n + \frac{n^{(0)} - n}{\tau_n},$$

Boltzmann equation treatment of current response to diode
(relaxation time approximation)

$$\begin{aligned}
\langle \dot{\vec{r}} \rangle &= \frac{1}{n}\int [d\vec{k}]\, g_{\vec{r}\vec{k}} \vec{v}_{\vec{k}} \\
&= \frac{1}{n}\int [d\vec{k}] \left[f - \tau \vec{v}_{\vec{k}} \cdot \left\{ e\vec{E}\frac{\partial f}{\partial \mu} + \frac{\partial f}{\partial \vec{r}} \right\} \right] \vec{v}_{\vec{k}} \\
&\approx \frac{1}{n}\int [d\vec{k}] \left[-\tau \vec{v}_{\vec{k}} \cdot \left\{ e\vec{E}\beta g + \frac{\partial g}{\partial \vec{r}} \right\} \right] \vec{v}_{\vec{k}} \\
&= -\mu_n \vec{E} - \frac{\mathcal{D}_n}{n}\frac{\partial n}{\partial \vec{r}}
\end{aligned}$$

$$\mu_n = \frac{e}{3}\beta \left\langle \tau v_{\vec{k}}^2 \right\rangle$$ electron mobility

$$\mathcal{D}_n = \frac{1}{3}\left\langle \tau v_{\vec{k}}^2 \right\rangle = \frac{k_B T \mu_n}{e}.$$ electron diffusion coefficient; Einstein relation

Electron current:

$$\vec{j}_n = -en\left\langle \dot{\vec{r}} \right\rangle = e\mu_n \vec{E} + eD_n \nabla n$$

Boltzmann equation treatment of current response to diode
(relaxation time approximation)

Summary of equations:

$$\begin{aligned}
\vec{j}_n &= e\mu_n n\vec{E} + e\mathcal{D}_n \vec{\nabla} n \\
\vec{j}_p &= e\mu_p p\vec{E} - e\mathcal{D}_p \vec{\nabla} p,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial n}{\partial t} &= \frac{1}{e}\vec{\nabla}\cdot\vec{j}_n + \frac{n^{(0)} - n}{\tau_n} \\
\frac{\partial p}{\partial t} &= -\frac{1}{e}\vec{\nabla}\cdot\vec{j}_p + \frac{p^{(0)} - p}{\tau_p},
\end{aligned}$$

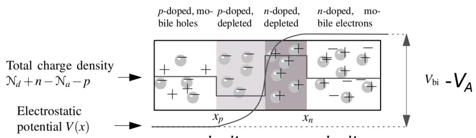
$$\vec{\nabla}\cdot\vec{E} = \frac{4\pi e(p - n + n_{\text{ions}})}{\epsilon^0}.$$

Boltzmann equation treatment of current response to diode
Approximate solution ignoring "minority" carriers and assuming spatially constant currents j_n and j_p

$$\begin{aligned}
n(x) &= \mathcal{N}_d e^{\beta e[V(x) - V(x_n)]} \left[1 + \frac{j_n}{e\mathcal{N}_d\mathcal{D}_n} \int_{x_n}^{x} dx'\, e^{-\beta e[V(x') - V(x_n)]} \right] \\
p(x) &= \mathcal{N}_a e^{-\beta e[V(x) - V(x_p)]} \left[1 - \frac{j_p}{e\mathcal{N}_a\mathcal{D}_p} \int_{x_p}^{x} dx'\, e^{\beta e[V(x') - V(x_p)]} \right]
\end{aligned}$$

For HW, you will show that the above solutions are consistent with the equations:

$$\begin{aligned}
\vec{j}_n &= e\mu_n n\vec{E} + e\mathcal{D}_n \vec{\nabla} n \\
\vec{j}_p &= e\mu_p p\vec{E} - e\mathcal{D}_p \vec{\nabla} p,
\end{aligned}$$



p majority
n minority

n majority
p minority

When $V_A = 0$: $p \approx N_a \quad n \approx \frac{n_i^2}{N_a}$

When $V_A = 0$: $n \approx N_d \quad p \approx \frac{n_i^2}{N_d}$

Estimation of minority carriers

$$
\begin{aligned}
n(x) &= \mathcal{N}_d e^{\beta e [V(x) - V(x_n)]} \\
p(x) &= \mathcal{N}_a e^{-\beta e [V(x) - V(x_p)]} \\
\Rightarrow n(x_p) &= \mathcal{N}_d e^{\beta e [V_A - V_{bi}]} = \frac{n_i^2}{\mathcal{N}_a} e^{\beta e V_A} \\
p(x_n) &= \mathcal{N}_a e^{\beta e [V_A - V_{bi}]} = \frac{n_i^2}{\mathcal{N}_d} e^{\beta e V_A}.
\end{aligned}
$$

Boltzmann equation treatment of current response to diode
Approximate steady state solutions in presence of applied voltage V_A

$$
\begin{aligned}
\frac{\partial n}{\partial t} &= \frac{1}{e} \vec{\nabla} \cdot \vec{j}_n + \frac{n^{(0)} - n}{\tau_n} = 0 \\
\vec{j}_n &= e\mu_n n \vec{E} + e \mathcal{D}_n \vec{\nabla} n
\end{aligned}
$$

≈ 0

$$
\begin{aligned}
0 &= \mathcal{D}_p \frac{d^2 p}{dx^2} - \frac{p - p^{(0)}}{\tau_p} \\
0 &= \mathcal{D}_n \frac{d^2 n}{dx^2} - \frac{n - n^{(0)}}{\tau_n},
\end{aligned}
$$

Boltzmann equation treatment of current response to diode
Approximate steady state solutions in presence of applied voltage V_A

Solution to diffusion equations:

$$
\begin{aligned}
p - p^{(0)} &= [p(x_n) - p^{(0)}] e^{-(x - x_n)/L_p} \quad \text{for } x > x_n \\
n - n^{(0)} &= [n(x_p) - n^{(0)}] e^{(x - x_p)/L_n} \quad \text{for } x < x_p
\end{aligned}
$$

$$L_n = \sqrt{\mathcal{D}_n \tau_n} \quad \text{and} \quad L_p = \sqrt{\mathcal{D}_p \tau_p}$$

$$p(x) = \frac{n_i^2}{N_d} + \left[p(x_n) - \frac{n_i^2}{N_d} \right] e^{-(x - x_n)/L_p} \quad \text{for } x > x_n$$

$$n(x) = \frac{n_i^2}{N_a} + \left[n(x_p) - \frac{n_i^2}{N_a} \right] e^{-(x - x_p)/L_n} \quad \text{for } x < x_p$$

Boltzmann equation treatment of current response to diode

Approximate steady state solutions in presence of applied voltage V_A

Current through device:

$$j = -ej_n(x_p) + ej_p(x_n)$$
$$= \frac{eD_n}{L_n}\left[n(x_p) - \frac{n_i^2}{N_a}\right] + \frac{eD_p}{L_p}\left[p(x_n) - \frac{n_i^2}{N_d}\right]$$

$$j_n(x_p) = -D_n \frac{dn(x_p)}{dx} = -\frac{D_n}{L_n}\left[n(x_p) - \frac{n_i^2}{N_a}\right] = -\frac{D_n}{L_n}\left[\frac{n_i^2}{N_a}e^{\beta eV_A} - \frac{n_i^2}{N_a}\right]$$

$$j_p(x_n) = D_p \frac{dp(x_n)}{dx} = \frac{D_p}{L_p}\left[p(x_n) - \frac{n_i^2}{N_d}\right] = \frac{D_p}{L_p}\left[\frac{n_i^2}{N_d}e^{\beta eV_A} - \frac{n_i^2}{N_d}\right]$$

$$\Rightarrow j = en_i^2\left(e^{\beta eV_A} - 1\right)\left[\frac{D_n}{L_nN_a} + \frac{D_p}{L_pN_d}\right]$$

Behavior of diode: