PHY 712 Electrodynamics
9-9:50 AM MWF Olin 103

Plan for Lecture 31:

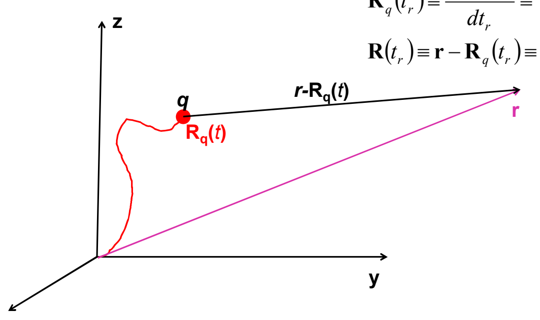
Finish reading Chap. 14 –

Radiation from charged particles

1. **Review of synchrotron radiation**
2. **Free electron laser**
3. **Thompson and Compton scattering**

	Fri: 03/17/2017		*APS Meeting - no class*		
23	Mon: 03/20/2017	Chap. 9	Sources of Radiation	#17	03/24/2017
24	Wed: 03/22/2017	Chap. 9 & 10	Radiation and Scattering		
25	Fri: 03/24/2017	Chap. 9 & 10	Radiation and Scattering	#18	03/27/2017
26	Mon: 03/27/2017	Chap. 11	Special relativity	#19	03/31/2017
27	Wed: 03/29/2017	Chap. 11	Special relativity		
28	Fri: 03/31/2017	Chap. 11	Special relativity	#20	04/3/2017
29	Mon: 04/03/2017	Chap. 14	Radiation from moving charges	#21	04/5/2017
30	Wed: 04/05/2017	Chap. 14	Radiation from moving charges	#22	04/7/2017
31	Fri: 04/07/2017	Chap. 14	Radiation from moving charges	#23	04/10/2017
32	Mon: 04/10/2017				
33	Wed: 04/12/2017				
	Fri: 04/14/2017		*Good Friday Holiday -- no class*		
34	Mon: 04/17/2017				
35	Wed: 04/19/2017				
36	Fri: 04/21/2017				
	Mon: 04/24/2017		Presentations I		
	Wed: 04/26/2017		Presentations II		

Radiation from a moving charged particle



Variables (notation) :

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R}$$

Liénard-Wiechert fields (cgs Gaussian units):

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c})^3}\left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right)\left(1 - \frac{v^2}{c^2}\right) + \left(\mathbf{R}\times\left\{\left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right)\times\frac{\dot{\mathbf{v}}}{c^2}\right\}\right)\right]. \quad (19)$$

$$\mathbf{B}(\mathbf{r},t) = \frac{q}{c}\left[\frac{-\mathbf{R}\times\mathbf{v}}{(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c})^3}\left(1 - \frac{v^2}{c^2} + \frac{\dot{\mathbf{v}}\cdot\mathbf{R}}{c^2}\right) - \frac{\mathbf{R}\times\dot{\mathbf{v}}/c}{(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c})^2}\right]. \quad (20)$$

In this case, the electric and magnetic fields are related according to

$$\mathbf{B}(\mathbf{r},t) = \frac{\mathbf{R}\times\mathbf{E}(\mathbf{r},t)}{R}. \quad (21)$$

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v} \quad \mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R} \quad \dot{\mathbf{v}} \equiv \frac{d^2\mathbf{R}_q(t_r)}{dt_r^{\,2}}$$

Electric and magnetic fields far from source:

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left\{\mathbf{R}\times\left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right)\times\frac{\dot{\mathbf{v}}}{c^2}\right]\right\}$$

$$\mathbf{B}(\mathbf{r},t) = \frac{\mathbf{R}\times\mathbf{E}(\mathbf{r},t)}{R}$$

Let $\hat{\mathbf{R}} \equiv \frac{\mathbf{R}}{R} \qquad \boldsymbol{\beta} \equiv \frac{\mathbf{v}}{c} \qquad \dot{\boldsymbol{\beta}} \equiv \frac{\dot{\mathbf{v}}}{c}$

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{cR(1 - \boldsymbol{\beta}\cdot\hat{\mathbf{R}})^3}\left\{\hat{\mathbf{R}}\times\left[(\hat{\mathbf{R}} - \boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]\right\}$$

$$\mathbf{B}(\mathbf{r},t) = \hat{\mathbf{R}}\times\mathbf{E}(\mathbf{r},t)$$

Poynting vector:

$$\mathbf{S}(\mathbf{r},t) = \frac{c}{4\pi}(\mathbf{E}\times\mathbf{B})$$

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{cR(1 - \boldsymbol{\beta}\cdot\hat{\mathbf{R}})^3}\left\{\hat{\mathbf{R}}\times\left[(\hat{\mathbf{R}} - \boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]\right\}$$

$$\mathbf{B}(\mathbf{r},t) = \hat{\mathbf{R}}\times\mathbf{E}(\mathbf{r},t)$$

$$\mathbf{S}(\mathbf{r},t) = \frac{c}{4\pi}\hat{\mathbf{R}}\left|\mathbf{E}(\mathbf{r},t)\right|^2 = \frac{q^2}{4\pi cR^2}\hat{\mathbf{R}}\frac{\left|\hat{\mathbf{R}}\times\left[(\hat{\mathbf{R}} - \boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]\right|^2}{(1 - \boldsymbol{\beta}\cdot\hat{\mathbf{R}})^6}$$

$$\frac{dP}{d\Omega} = \mathbf{S}\cdot\hat{\mathbf{R}}R^2 = \frac{q^2}{4\pi c}\frac{\left|\hat{\mathbf{R}}\times\left[(\hat{\mathbf{R}} - \boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]\right|^2}{(1 - \boldsymbol{\beta}\cdot\hat{\mathbf{R}})^6}$$

Spectral composition of electromagnetic radiation

Time integrated power per solid angle:

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} dt \frac{dP(t)}{d\Omega} = \int_{-\infty}^{\infty} dt |\mathbf{a}(t)|^2 = \int_{-\infty}^{\infty} d\omega |\tilde{\mathbf{a}}(\omega)|^2$$

Fourier amplitude:

$$\tilde{\mathbf{a}}(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\, \mathbf{a}(t) e^{i\omega t} \qquad \mathbf{a}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega\, \tilde{\mathbf{a}}(\omega) e^{-i\omega t}$$

Note that: $\tilde{\mathbf{a}}(\omega) = \tilde{\mathbf{a}}^*(-\omega)$

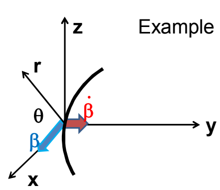
$$\Rightarrow \frac{dW}{d\Omega} = \int_0^{\infty} d\omega \left(|\tilde{\mathbf{a}}(\omega)|^2 + |\tilde{\mathbf{a}}(-\omega)|^2 \right) \equiv \int_0^{\infty} d\omega \frac{\partial^2 I}{\partial\Omega \partial\omega}$$

$$\frac{\partial^2 I}{\partial\Omega\partial\omega} \equiv 2|\tilde{\mathbf{a}}(\omega)|^2$$

Spectral composition of electromagnetic radiation -- continued

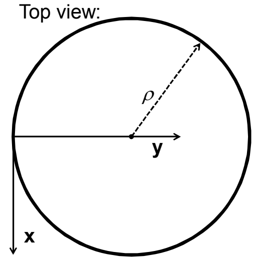
The spectral intensity therefore depends on the following integral:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} dt_r \left[\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \boldsymbol{\beta}(t_r) \right) \right] e^{i\omega\left(t_r - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)/c\right)} \right|^2$$



Example for circular motion:

Top view:

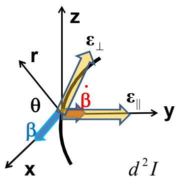


$$\mathbf{R}_q(t_r) = \rho\hat{\mathbf{x}}\sin(vt_r/\rho) + \rho\hat{\mathbf{y}}(1 - \cos(vt_r/\rho))$$

$$\boldsymbol{\beta}(t_r) = \beta(\hat{\mathbf{x}}\cos(vt_r/\rho) + \hat{\mathbf{y}}\sin(vt_r/\rho))$$

For convenience, choose:

$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$



$$\boldsymbol{\varepsilon}_{\parallel} = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_{\perp} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$$

$$\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\beta}) =$$

$$\beta\left(-\boldsymbol{\varepsilon}_{\parallel}\sin(vt_r/\rho) + \boldsymbol{\varepsilon}_{\perp}\sin\theta\cos(vt_r/\rho)\right)$$

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} \hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \beta) e^{i\omega(t - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)} dt\right|^2$$

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2\omega^2\beta^2}{4\pi^2 c}\left\{|C_{\parallel}(\omega)|^2 + |C_{\perp}(\omega)|^2\right\}$$

$$C_{\parallel}(\omega) = \int_{-\infty}^{\infty} dt \sin(vt/\rho) e^{i\omega(t - \frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$C_{\perp}(\omega) = \int_{-\infty}^{\infty} dt \sin\theta\cos(vt/\rho) e^{i\omega(t - \frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

Synchrotron radiation geometry – using modified Bessel functions

$$K_{1/3}(\xi) = \sqrt{3}\int_0^{\infty} dx \cos\left[\tfrac{3}{2}\xi\left(x + \tfrac{1}{3}x^3\right)\right] \qquad K_{2/3}(\xi) = \sqrt{3}\int_0^{\infty} dx\, x \sin\left[\tfrac{3}{2}\xi\left(x + \tfrac{1}{3}x^3\right)\right]$$

Exponential factor

$$\omega\left(t_r - \frac{\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)}{c}\right) = \omega\left(t_r - \frac{\rho}{c}\cos\theta\sin(vt_r/\rho)\right)$$

In the limit of $t_r \approx 0,\ \theta \approx 0,\ v \approx c\left(1 - \frac{1}{2\gamma^2}\right)$

$$\omega\left(t_r - \frac{\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)}{c}\right) \approx \frac{\omega t_r}{2\gamma^2}\left(1 + \gamma^2\theta^2\right) + \frac{\omega c^2 t_r^3}{6\rho^2} = \frac{3}{2}\xi\left(x + \frac{1}{3}x^3\right)$$

where $\xi = \frac{\omega\rho}{3c\gamma^3}\left(1 + \gamma^2\theta^2\right)^{3/2}$ and $x = \frac{c\gamma t_r}{\rho\left(1 + \gamma^2\theta^2\right)^{1/2}}$

Spectral form of synchrotron radiation in this case:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{3q^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2\left(1 + \gamma^2\theta^2\right)^2\left\{\left[K_{2/3}\left(\frac{\omega}{2\omega_c}\left(1 + \gamma^2\theta^2\right)^{3/2}\right)\right]^2 + \frac{\gamma^2\theta^2}{1 + \gamma^2\theta^2}\left[K_{1/3}\left(\frac{\omega}{2\omega_c}\left(1 + \gamma^2\theta^2\right)^{3/2}\right)\right]^2\right\}$$

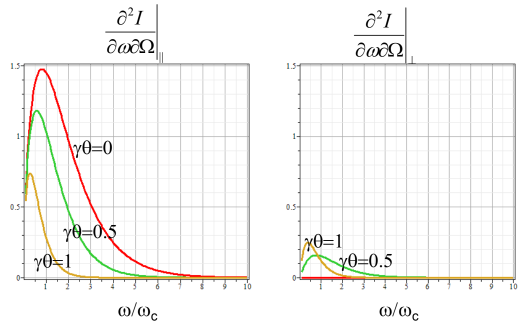
$$\omega_c \equiv \frac{3c\gamma^3}{2\rho}$$

At $\theta = 0$:

note that for $\omega << \omega_c \quad \Rightarrow \frac{\partial^2 I}{\partial\omega\partial\Omega} \approx \frac{q^2}{\pi^2 c}\left(\Gamma\left(\tfrac{2}{3}\right)\right)^2\left(\frac{3\omega^2\rho^2}{4c^2}\right)^{1/3}$

and for $\omega >> \omega_c \quad \Rightarrow \frac{\partial^2 I}{\partial\omega\partial\Omega} \approx \frac{3q^2}{4\pi c}\gamma^2\left(\frac{\omega}{\omega_c}\right)^2 e^{-\omega/\omega_c}$

Plots of the intensity function for synchrotron radiation



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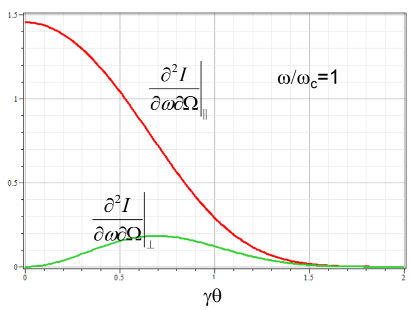
Plots of the intensity function for synchrotron radiation



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Synchrotron facilities in USA

http://www.lightsources.org/regions

Advanced Light Source	USA	http://www-als.lbl.gov/
Advanced Photon Source	USA	http://www.aps.anl.gov
Center for Advanced Microstructures and Devices	USA	http://www.camd.lsu.edu/
Cornell High Energy Synchrotron Source	USA	http://www.chess.cornell.edu/
National Synchrotron Light Source II	USA	http://www.bnl.gov/ps/
Stanford Synchrotron Radiation Lightsource	USA	http://www-ssrl.slac.stanford.edu
Synchrotron Ultraviolet Radiation Facility	USA	http://physics.nist.gov/MajResFac/SURF/SURF/index.html

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Free electron laser
Reference:

Classical Theory of Free-Electron Lasers

A text for students and researchers

Eric B Szarmes

ISBN 978-1-6270-5573-4 (ebook)
ISBN 978-1-6270-5572-7 (print)
ISBN 978-1-6270-5680-9 (mobi)

DOI 10.1088/978-1-6270-5573-4

1.1 The free-electron laser

A free-electron laser (FEL) is a laser source that produces spatially and temporally coherent optical radiation by stimulated emission, where in place of an atomic or molecular medium to provide amplification the gain medium is comprised of a beam of relativistic electrons traveling in a vacuum through a periodic magnetic field. The basic components common to all FELs are a relativistic electron beam, a periodic magnetic structure (an undulator or wiggler magnet of spatial period λ_w), and an optical resonator providing feedback and amplification. (X-ray FELs such as the Linac Coherent Light Source at Stanford omit the optical resonator by necessity and achieve the required gain on a single pass.) The features that make FELs particularly useful as research devices are the unique combination of continuous and broadband tunability, high peak and average power, and spatial and temporal coherence.

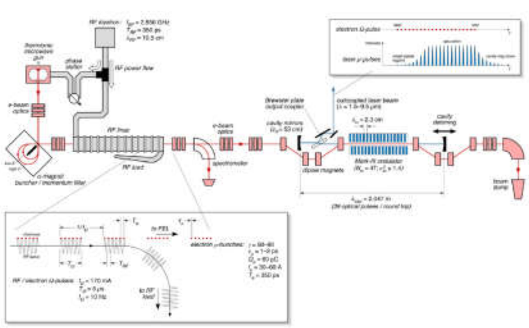


Figure 1.1. The MkIII RF-linac FEL.

Electron emission in periodic magnet

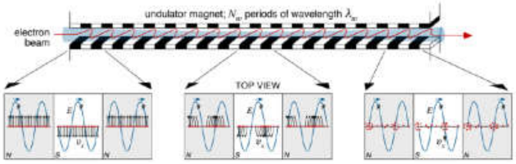


Figure 1.3. Conceptual illustration of the bunching mechanism in an FEL.

Because of Doppler shift, effective

wavelength is: $\lambda = \dfrac{\lambda_w}{2\gamma^2}$

frequency is: $\omega = 2\gamma^2 k_w c$

Electron trajectory near magnetic in undulator viewed in the electron rest frame (ERF)

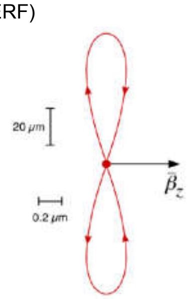
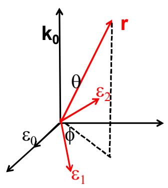


Figure 4.1. Electron motion in the ERF in a plane-polarized undulator for $\bar{K}^2 = 1.2$; $\gamma = 80$; $k_w = 2.73\,\text{cm}^{-1}$. The maximum transverse and longitudinal displacements are $\pm 71\,\mu\text{m}$ and $\pm 0.17\,\mu\text{m}$ respectively.

Free-Electron Laser

Name	Country	Website
Institute for Terahertz Science and Technology	USA	http://www.itst.ucsb.edu/
Jefferson Lab FEL	USA	https://www.jlab.org/free-electron-laser
Linac Coherent Light Source	USA	http://lcls.slac.stanford.edu

Some details of scattering of electromagnetic waves incident on a particle of charge q and mass m_q



$$\mathbf{E}(\mathbf{r},t) = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega t}\right)$$

Thompson scattering – non relativistic approximation

Power radiated in direction $\hat{\mathbf{r}}$ by charged particle with acceleration $\dot{\mathbf{v}}$:

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\left|\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\dot{\mathbf{v}}\right)\right|^2$$

Suppose that the acceleration $\dot{\mathbf{v}}$ of a particle (charge q and mass m_q) is caused by an electric field: $\mathbf{E}(\mathbf{r},t) = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega t}\right)$

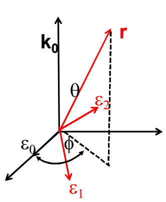
$$\dot{\mathbf{v}} = \frac{q}{m_q}\Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega t}\right)$$

Time averaged power: $\left\langle\frac{dP}{d\Omega}\right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0\right)\right|^2$

Thompson scattering – non relativistic approximation -- continued

Time averaged power: $\left\langle\frac{dP}{d\Omega}\right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0\right)\right|^2$

$$\hat{\mathbf{r}} = \sin\theta\left(\cos\phi\ \hat{\mathbf{x}} + \sin\phi\ \hat{\mathbf{y}}\right) + \cos\theta\hat{\mathbf{z}}$$



Polarization of incident light: $\varepsilon_0 = \hat{\mathbf{x}}$

Polarization of scattered light:

$$\varepsilon_1 = \cos\theta(\hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi) - \hat{\mathbf{z}}\sin\theta$$

$$\varepsilon_2 = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$$

Thompson scattering – non relativistic approximation -- continued

Time averaged power with polarization ε^*:

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 |\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0|^2$$

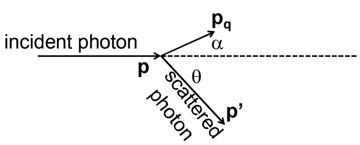
Scattered light may be polarized parallel to incident field or polarized with an angle θ so that the time and polarization averaged cross section is given by:

$$\left\langle |\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi = \left\langle |\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi + \left\langle |\boldsymbol{\varepsilon}_2 \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi = \frac{1}{2}\cos^2\theta + \frac{1}{2}$$

$$\text{Averaged cross section: } \left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2}\right)^2 \frac{1}{2}\left(1+\cos^2\theta\right)$$

This formula is appropriate in the X-ray scattering of electrons or soft γ-ray scattering of protons

Thompson scattering – relativistic and quantum modifications



Conservation of momentum and energy:

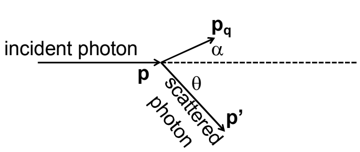
$$p = p'\cos\theta + p_q\cos\alpha \qquad pc = \hbar\omega$$

$$0 = p'\sin\theta - p_q\sin\alpha \qquad p'c = \hbar\omega'$$

$$\hbar\omega + m_q c^2 = \hbar\omega' + \sqrt{p_q^2 c^2 + \left(m_q c^2\right)^2}$$

$$\frac{p'}{p} = \frac{1}{1+\frac{\hbar\omega}{m_q c^2}\left(1-\cos\theta\right)}$$

Thompson scattering – relativistic and quantum modifications



Relativistic and quantum modifications to averaged cross section:

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2}\right)^2 \left(\frac{p'}{p}\right)^2 \frac{1}{2}\left(1+\cos^2\theta\right)$$

$$\frac{p'}{p} = \frac{1}{1+\frac{\hbar\omega}{m_q c^2}\left(1-\cos\theta\right)}$$