PHY 712 Electrodynamics
9-9:50 AM MWF Olin 103

Plan for Lecture 32:

Start reading Chap. 15 –

Radiation from collisions of charged particles

1. **Overview**
2. **X-ray tube**
3. **Radiation from Rutherford scattering**
4. **Other collision models**

	Fri: 03/17/2017		*APS Meeting - no class*		
23	Mon: 03/20/2017	Chap. 9	Sources of Radiation	#17	03/24/2017
24	Wed: 03/22/2017	Chap. 9 & 10	Radiation and Scattering		
25	Fri: 03/24/2017	Chap. 9 & 10	Radiation and Scattering	#18	03/27/2017
26	Mon: 03/27/2017	Chap. 11	Special relativity	#19	03/31/2017
27	Wed: 03/29/2017	Chap. 11	Special relativity		
28	Fri: 03/31/2017	Chap. 11	Special relativity	#20	04/3/2017
29	Mon: 04/03/2017	Chap. 14	Radiation from moving charges	#21	04/5/2017
30	Wed: 04/05/2017	Chap. 14	Radiation from moving charges	#22	04/7/2017
31	Fri: 04/07/2017	Chap. 14	Radiation from moving charges	#23	04/10/2017
32	Mon: 04/10/2017	Chap. 15	Radiation from collisions		
33	Wed: 04/12/2017	Chap. 13	Cherenkov radiation		
	Fri: 04/14/2017		*Good Friday Holiday -- no class*		
34	Mon: 04/17/2017				
35	Wed: 04/19/2017				
36	Fri: 04/21/2017				
	Mon: 04/24/2017		Presentations I		
	Wed: 04/26/2017		Presentations II		

PHY 712 Presentation Schedule

Monday April 24, 2017

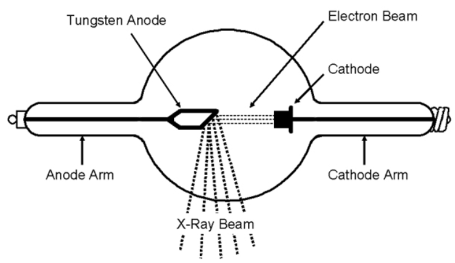
Time	Name	Topic/Title
9:00-9:25 AM	Ali Daraei	Light scattering of fibrin fibers
9:25-9:50 AM		

Wednesday April 26, 2017

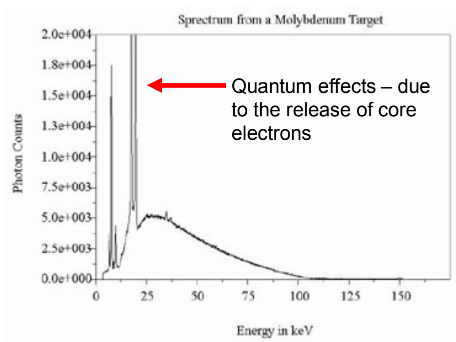
Time	Name	Topic/Title
9:00-9:25 AM	TJ Colvin	Smooth Particle Mesh Ewald
9:25-9:50 AM		

Generation of X-rays in a Coolidge tube

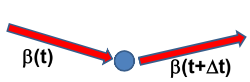
https://www.orau.org/ptp/collection/xraytubescoolidge/coolidgeinformation.htm

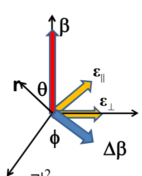


http://www.ndt-ed.org/EducationResources/CommunityCollege/Radiography/Physics/xrays.htm



Radiation during collisions





Intensity:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\int dt\; e^{i\omega\left(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right)}\frac{d}{dt}\left[\frac{\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}}\right]\right|^2$$

Note that $\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right)=\left(\boldsymbol{\varepsilon}_{\|}\cdot\boldsymbol{\beta}\right)\boldsymbol{\varepsilon}_{\|}+\left(\boldsymbol{\varepsilon}_{\perp}\cdot\boldsymbol{\beta}\right)\boldsymbol{\varepsilon}_{\perp}$

For a collision of duration τ emitting radiation with polarization $\boldsymbol{\varepsilon}$ and frequency $\omega\rightarrow 0$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)}-\frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)}\right)\right|^2$$

Radiation during collisions -- continued

For a collision of duration τ emitting radiation with polarization $\boldsymbol{\varepsilon}$ and frequency $\omega \to 0$:

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)} - \frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)}\right)\right|^2$$

Non-relativistic limit:

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot(\Delta\boldsymbol{\beta})\right|^2 \qquad \Delta\boldsymbol{\beta} \equiv \boldsymbol{\beta}(t+\tau) - \boldsymbol{\beta}(t)$$

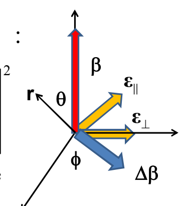
Relativistic collision with small $|\Delta\boldsymbol{\beta}| \equiv \boldsymbol{\beta}(t+\tau) - \boldsymbol{\beta}(t)$:

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})}{(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta})^2}\right)\right|^2$$

Radiation during collisions -- continued

Relativistic collision with small $|\Delta\boldsymbol{\beta}|$:

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})}{(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta})^2}\right)\right|^2$$



Also assume $\Delta\boldsymbol{\beta}$ is perpendicular to $\mathbf{r} - \boldsymbol{\beta}$ plane

Expressions (averaging over φ) for $\|$ or $\perp$ polarization:

$$\frac{d^2I_{\|}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2\frac{(\beta-\cos\theta)^2}{(1-\beta\cos\theta)^4}$$ polarization in ***r*** and β plane

$$\frac{d^2I_{\perp}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2\frac{1}{(1-\beta\cos\theta)^2}$$ polarization perpendicular to ***r*** and β plane

Some details:

$$\hat{\mathbf{r}} = \sin\theta\hat{\mathbf{x}} + \cos\theta\hat{\mathbf{z}}$$

$$\boldsymbol{\varepsilon}_{\perp} = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_{\|} = -\cos\theta\hat{\mathbf{x}} + \sin\theta\hat{\mathbf{z}}$$

$$\boldsymbol{\beta} = \beta\hat{\mathbf{z}}$$

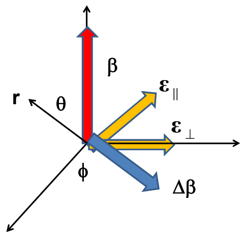
$$\Delta\boldsymbol{\beta} = \Delta\beta(\cos\phi\hat{\mathbf{x}} + \sin\phi\hat{\mathbf{y}})$$

$$\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta}) = \Delta\boldsymbol{\beta}(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}) + \boldsymbol{\beta}(\hat{\mathbf{r}}\cdot\Delta\boldsymbol{\beta})$$

$$\boldsymbol{\varepsilon}_{\perp}\cdot(\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})) = \Delta\beta\sin\phi(1-\beta\cos\theta)$$

$$\boldsymbol{\varepsilon}_{\|}\cdot(\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})) = \Delta\beta\cos\phi(\beta-\cos\theta)$$

Some details:



$$\boldsymbol{\beta} = \beta \hat{\mathbf{z}} \qquad \hat{\mathbf{r}} = \sin\theta \hat{\mathbf{x}} + \cos\theta \hat{\mathbf{z}}$$

$$\boldsymbol{\varepsilon}_{\|} = -\cos\theta \hat{\mathbf{x}} + \sin\theta \hat{\mathbf{z}} \qquad \boldsymbol{\varepsilon}_{\perp} = \hat{\mathbf{y}}$$

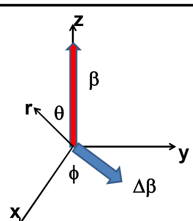
$$\Delta\boldsymbol{\beta} = \Delta\beta \left(\cos\phi \hat{\mathbf{x}} + \sin\phi \hat{\mathbf{y}} \right)$$

Radiation during collisions -- continued

Intensity expressions:

$$\frac{d^2 I_{\|}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c} |\Delta\boldsymbol{\beta}|^2 \frac{(\beta - \cos\theta)^2}{(1-\beta\cos\theta)^4}$$

$$\frac{d^2 I_{\perp}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c} |\Delta\boldsymbol{\beta}|^2 \frac{1}{(1-\beta\cos\theta)^2}$$

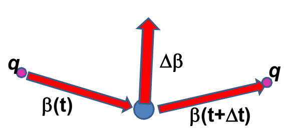


Relativistic collision at low ω and with small $|\Delta\boldsymbol{\beta}|$ and $\Delta\boldsymbol{\beta}$ perpendicular to plane of $\hat{\mathbf{r}}$ and $\boldsymbol{\beta}$, as a function of θ where $\hat{\mathbf{r}} \cdot \boldsymbol{\beta} = \beta\cos\theta$;

Integrating over solid angle:

$$\frac{dI}{d\omega} = \int d\Omega \left(\frac{d^2 I_{\|}}{d\omega d\Omega} + \frac{d^2 I_{\perp}}{d\omega d\Omega} \right) = \frac{2}{3\pi} \frac{q^2}{c} \gamma^2 \, |\Delta\boldsymbol{\beta}|^2$$

Estimation of Δβ



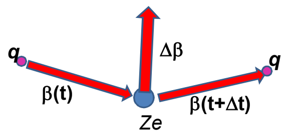
Momentum transfer:

$$Qc \equiv \left| \mathbf{p}(t+\tau) - \mathbf{p}(t) \right| c \approx \gamma M c^2 |\Delta\boldsymbol{\beta}|$$

mass of particle having charge *q*

$$\frac{dI}{d\omega} = \frac{2}{3\pi} \frac{q^2}{c} \gamma^2 |\Delta\boldsymbol{\beta}|^2 \approx \frac{2}{3\pi} \frac{q^2}{M^2 c^3} Q^2$$

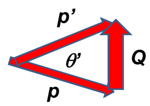
Estimation of $\Delta\beta$ -- for the case of Rutherford scattering



Assume that target nucleus (charge *Ze*) has mass >>M;

Rutherford scattering cross-section:

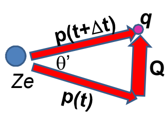
$$\frac{d\sigma}{d\Omega} = \left(\frac{2Zeq}{pv}\right)^2 \frac{1}{\left(2\sin\left(\theta'/2\right)\right)^4}$$



Assuming elastic scattering:

$$Q^2 = \left(2p\sin\left(\theta'/2\right)\right)^2 = 2p^2\left(1-\cos\theta'\right)$$

Case of Rutherford scattering -- continued



Rutherford scattering cross-section:

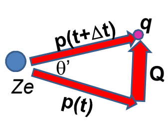
$$\frac{d\sigma}{d\Omega} = \left(\frac{2Zeq}{pv}\right)^2 \frac{1}{\left(2\sin(\theta'/2)\right)^4}$$

$$\frac{d\sigma}{dQ} = \int d\phi' \frac{d\sigma}{d\Omega}\frac{d\Omega}{dQ}$$

$$Q^2 = \left(2p\sin\left(\theta'/2\right)\right)^2 = 2p^2\left(1-\cos\theta'\right)$$

$$\Rightarrow \quad \frac{d\sigma}{dQ} = 8\pi\left(\frac{Zeq}{\beta c}\right)^2 \frac{1}{Q^3}$$

Case of Rutherford scattering -- continued



Differential radiation cross section:

$$\frac{d^2\chi}{d\omega dQ} = \frac{dI}{d\omega}\frac{d\sigma}{dQ} = \left(\frac{2}{3\pi}\frac{q^2}{M^2c^3}Q^2\right)\left(8\pi\left(\frac{Zeq}{\beta c}\right)^2\frac{1}{Q^3}\right)$$

$$= \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\frac{1}{Q}$$

Differential radiation cross section - - continued

Integrating over momentum transfer

$$\frac{d\chi}{d\omega} = \int_{Q_{\min}}^{Q_{\max}} dQ \frac{d^2\chi}{d\omega dQ} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2 \frac{1}{\beta^2}\ln\left(\frac{Q_{\max}}{Q_{\min}}\right)$$

Comment on frequency dependence --

Original expression for radiation intensity :

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\int dt\, e^{i\omega(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)} \frac{d}{dt}\left[\frac{\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta})}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}}\right]\right|^2$$

In the previous derivations, we have assumed that $\omega(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c) << 1$.

$$\omega(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c) = \omega\left(t-\hat{\mathbf{r}}\cdot\int_0^t dt'\boldsymbol{\beta}(t')\right) \approx \omega\tau(1-\hat{\mathbf{r}}\cdot\langle\boldsymbol{\beta}\rangle)$$

Differential radiation cross section - - continued

Radiation cross section in terms of momentum transfer

$$\frac{d\chi}{d\omega} = \int_{Q_{\min}}^{Q_{\max}} dQ \frac{d^2\chi}{d\omega dQ} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2 \frac{1}{\beta^2}\ln\left(\frac{Q_{\max}}{Q_{\min}}\right)$$

Note that: $Q^2 = 2p^2(1-\cos\theta') \quad \Rightarrow Q_{\max} = 2p$

In general, $Q_{\min}$ is determined by the collision time condition $\omega\tau < 1 \quad \Rightarrow Q_{\min} \approx \frac{2Zeq\omega}{v^2}$

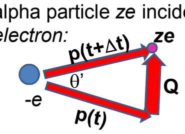
Radiation cross section for classical non - relativistic process

$$\frac{d\chi}{d\omega} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2 \frac{1}{\beta^2}\ln\left(\frac{\lambda M v^3}{Zeq\omega}\right)$$

λ= "fudge factor" of order unity

Electromagnetic effects in energy loss processes (see Chap. 13 of Jackson)

Again consider Rutherford scattering – now of a nucleus (or alpha particle *ze* incident on an electron *–e in rest frame of electron:*



Rutherford scattering cross-section:

$$\frac{d\sigma}{d\Omega} = \left(\frac{ze^2}{2pv}\right)^2 \frac{1}{(\sin(\theta'/2))^4}$$

$$\frac{d\sigma}{dQ^2} = \int d\varphi' \frac{d\sigma}{d\Omega}\frac{d\Omega}{dQ^2}$$

$$Q^2 = (2p\sin(\theta'/2))^2 = 2p^2(1-\cos\theta')$$

$$\Rightarrow \quad \frac{d\sigma}{dQ^2} = 4\pi\left(\frac{ze^2}{\beta c Q^2}\right)^2$$

Energy loss continued

Let T represent energy loss due to electron of mass m:

$$T = Q^2 / 2m$$

$$\frac{d\sigma}{dT} = \frac{2\pi z^2 e^4}{mc^2 \beta^2 T^2}$$

Estimate of energy loss per unit distance
in the presence of NZ electrons per unit volume

$$\frac{dE}{dx} \approx NZ \int_{\varepsilon}^{T_{max}} dT\,T \frac{d\sigma}{dT}$$

minimum energy transfer

$$= 2\pi NZ \frac{z^2 e^4}{mc^2 \beta^2} \ln\left(\frac{2\gamma^2 \beta^2 mc^2}{\varepsilon}\right) + \text{(quantum effects)}$$

Energy loss continued

Refining this result, Bethe and Fermi noticed that the analysis lacked consideration of the effects of electromagnetic fields. Representing the colliding electrons in terms of a dielectric function $\varepsilon(\omega)$ and the energetic particle of charge *ze* in terms of the charge and current density:

In Fourier space:

$$\left[k^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)\right]\Phi(\mathbf{k},\omega) = \frac{4\pi}{\varepsilon(\omega)}\rho(\mathbf{k},\omega)$$

$$\left[k^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)\right]\mathbf{A}(\mathbf{k},\omega) = \frac{4\pi}{c}\mathbf{J}(\mathbf{k},\omega)$$

$$\rho(\mathbf{k},\omega) = \frac{ze}{2\pi}\delta(\omega - \mathbf{v}\cdot\mathbf{k})$$

$$\mathbf{J}(\mathbf{k},\omega) = \mathbf{v}\rho(\mathbf{k},\omega)$$

Energy loss continued

$$\Phi(\mathbf{k},\omega) = \frac{2ze}{\varepsilon(\omega)} \frac{\delta(\omega - \mathbf{v}\cdot\mathbf{k})}{k^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)}$$

$$\mathbf{A}(\mathbf{k},\omega) = \varepsilon(\omega)\frac{\mathbf{v}}{c}\Phi(\mathbf{k},\omega)$$

The energy loss will be calculated from the work on the electron by the field:

$$\Delta E = -e\int_{-\infty}^{\infty} dt\ \mathbf{v}\cdot\mathbf{E}(t) = 2e\Re\left(\int_0^{\infty} d\omega\ i\omega\mathbf{r}(\omega)\cdot\mathbf{E}^*(\omega)\right)$$

The resultant loss estimate is

$$\frac{dE}{dx} \approx \frac{z^2 e^2 \omega_P^2}{2c^2} \ln\left(\frac{2mc^2\varepsilon}{\hbar^2\omega_P^2}\right) \quad \text{where } \omega_P^2 \equiv \frac{4\pi NZe^2}{m}$$