PHY 712 Electrodynamics
9-9:50 AM MWF Olin 103

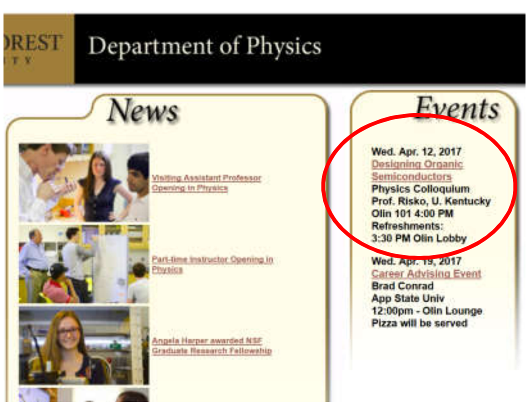
Plan for Lecture 33:

Special Topics in Electrodynamics:

Cherenkov radiation

References: Jackson Chapter 13.4
Zangwill Chapter 23.7
Smith Chapter 6.4

	Fri: 03/17/2017		APS Meeting - no class		
23	Mon: 03/20/2017	Chap. 9	Sources of Radiation	#17	03/24/2017
24	Wed: 03/22/2017	Chap. 9 & 10	Radiation and Scattering		
25	Fri: 03/24/2017	Chap. 9 & 10	Radiation and Scattering	#18	03/27/2017
26	Mon: 03/27/2017	Chap. 11	Special relativity	#19	03/31/2017
27	Wed: 03/29/2017	Chap. 11	Special relativity		
28	Fri: 03/31/2017	Chap. 11	Special relativity	#20	04/3/2017
29	Mon: 04/03/2017	Chap. 14	Radiation from moving charges	#21	04/5/2017
30	Wed: 04/05/2017	Chap. 14	Radiation from moving charges	#22	04/7/2017
31	Fri: 04/07/2017	Chap. 14	Radiation from moving charges	#23	04/10/2017
32	Mon: 04/10/2017	Chap. 15	Radiation from collisions		
33	Wed: 04/12/2017	Chap. 13	Cherenkov radiation		
	Fri: 04/14/2017		*Good Friday Holiday -- no class*		
34	Mon: 04/17/2017				
35	Wed: 04/19/2017				
36	Fri: 04/21/2017				
	Mon: 04/24/2017		Presentations I		
	Wed: 04/26/2017		Presentations II		



Mon. April 17, 2017 -- David Harrison, WFU (Ph. D. Thesis; Mentor: T. Thonhauser) Improving Existing and Discovering New Hydrogen Storage Materials Using Computational Materials Modeling Note: Public talk will begin at 12:30 PM in ZSR 204.

Mon. April 17, 2017 -- Ryan Godwin, WFU (Ph. D. Thesis; Mentor: F. Salsbury) Binding NEMO: Adventures in Molecular Dynamics Note: Public talk will begin at 3:00 PM in Olin 101.

Wed. April 19, 2017 -- Honors presentations Part I --

Fri. April 21, 2017 -- Larry Rush, WFU (MS. Thesis; Mentor: N. Holzwarth) Note: Public talk will begin at 12:30 PM in Scales 009.

Mon. April 24, 2017 -- Xiaohua (Nina) Liu, WFU (Ph. D. Thesis; Mentor: D. Kim-Shapiro) "Effects of Red Blood Cells on Nitric Oxide Bioactivity" Note: Public talk will begin at 10:00 AM in ZSR 204.

Wed. Apr. 26, 2017 -- Honors presentations Part II --

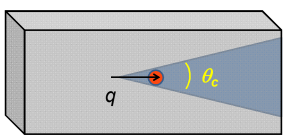
Thur. April 27, 2017 -- Crystal Bolden, WFU (Ph. D. Thesis; Mentor: D. Kim-Shapiro) "Interaction between RSNO and H_2S:The formation, stability, and NO-donating capacity of SSNO" and the effects of SSNO" on platelet activation" Note: Public talk will begin at 9:00 AM in ??

04/12/2017 PHY 712 Spring 2017 -- Lecture 33 4

References for notes: Glenn S. Smith, *An Introduction to Electromagnetic Radiation* (Cambridge UP, 1997), Andrew Zangwill, Modern Electrodynamics (Cambridge UP, 2013)

Cherenkov radiation

Discovered ~1930; bluish light emitted by energetic charged particles traveling within dielectric materials



04/12/2017 PHY 712 Spring 2017 -- Lecture 33 5

Maxwell's potential equations within a material having permittivity and permeability (Lorentz gauge; cgs Gaussian units)

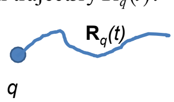
$$\nabla^2\Phi - \mu\varepsilon\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{4\pi}{\varepsilon}\rho$$

$$\nabla^2\mathbf{A} - \mu\varepsilon\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi\mu}{c}\mathbf{J}$$

Source: charged particle moving on trajectory $\mathbf{R}_q(t)$:

$$\rho(\mathbf{r},t) = q\delta(\mathbf{r}-\mathbf{R}_q(t))$$

$$\mathbf{J}(\mathbf{r},t) = q\dot{\mathbf{R}}_q(t)\delta(\mathbf{r}-\mathbf{R}_q(t))$$



04/12/2017 PHY 712 Spring 2017 -- Lecture 33 6

Liénard-Wiechert potential solutions:

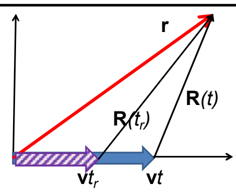
$$\Phi(\mathbf{r},t)=\frac{q}{\varepsilon}\frac{1}{R(t_r)-\boldsymbol{\beta}_n\cdot\mathbf{R}(t_r)}$$

$$\mathbf{A}(\mathbf{r},t)=q\mu\frac{\boldsymbol{\beta}_n}{R(t_r)-\boldsymbol{\beta}_n\cdot\mathbf{R}(t_r)}$$

$$\mathbf{R}(t_r)\equiv\mathbf{r}-\mathbf{R}_q(t_r)$$

$$\boldsymbol{\beta}_n(t_r)\equiv\frac{\dot{\mathbf{R}}_q(t_r)}{c_n}\qquad c_n\equiv\sqrt{\mu\varepsilon}\,c\equiv\frac{c}{n}$$

$$t_r=t-\frac{R(t_r)}{c_n}$$



Some algebra

$$\mathbf{R}(t)=\mathbf{r}-\mathbf{v}t$$

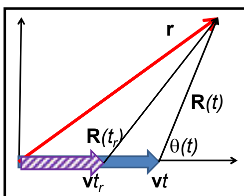
$$\mathbf{R}(t_r)=\mathbf{r}-\mathbf{v}t_r=\mathbf{R}(t)+\mathbf{v}(t-t_r)$$

$$(t-t_r)c_n=R(t_r)=|\mathbf{R}(t)+\mathbf{v}(t-t_r)|$$

Quadratic equation for $(t-t_r)c_n$:

$$((t-t_r)c_n)^2=R^2(t)+2\mathbf{R}(t)\cdot\boldsymbol{\beta}_n(t-t_r)c_n+\beta_n^{\,2}((t-t_r)c_n)^2$$

$$(t-t_r)c_n=\frac{-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n\pm\sqrt{(\mathbf{R}(t)\cdot\boldsymbol{\beta}_n)^2-(\beta_n^{\,2}-1)R^2(t)}}{\beta_n^{\,2}-1}$$



$$\mathbf{R}(t_r)=\mathbf{r}-\mathbf{v}t_r=\mathbf{R}(t)+\mathbf{v}(t-t_r)$$

$$(t-t_r)c_n=R(t_r)$$

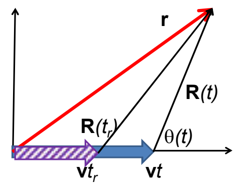
$$R(t_r)-\mathbf{R}(t_r)\cdot\boldsymbol{\beta}_n=(t-t_r)c_n(1-\beta_n^2)-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n$$

$$R(t_r)=\frac{-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n\pm\sqrt{(\mathbf{R}(t)\cdot\boldsymbol{\beta}_n)^2-(\beta_n^{\,2}-1)R^2(t)}}{\beta_n^{\,2}-1}$$

$$R(t_r)=\frac{R(t)}{\beta_n^{\,2}-1}\left(-\beta_n\cos\theta\pm\sqrt{1-\beta_n^{\,2}\sin^2\theta}\right)$$

$$R(t_r)-\mathbf{R}(t_r)\cdot\boldsymbol{\beta}_n=\mp R(t)\sqrt{1-\beta_n^{\,2}\sin^2\theta}$$

Liénard-Wiechert potentials:



$$\Phi(\mathbf{r},t) = \pm\frac{q}{\varepsilon}\frac{1}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}$$

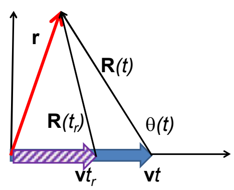
$$\mathbf{A}(\mathbf{r},t) = \pm q\mu\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}$$

For $\beta_n > 1$, range of θ is limited:

$$R(t_r) = \frac{R(t)}{\beta_n^2-1}\left(-\beta_n\cos\theta \pm \sqrt{1-\beta_n^2\sin^2\theta}\right) \geq 0$$

$$\Rightarrow \theta \leq \sin^{-1}\left(\frac{1}{\beta_n}\right)$$

Physical fields for $\beta_n > 1$



$$\theta \leq \sin^{-1}\left(\frac{1}{\beta_n}\right)$$

Define $\cos\theta_C \equiv \sqrt{1-\frac{1}{\beta_n^2}}$

$$\Rightarrow \cos\theta \leq \cos\theta_C$$

$$\Phi(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{1}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{A}(\mathbf{r},t) = 2q\mu\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

Physical fields for $\beta > 1$

$$\Phi(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{1}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{A}(\mathbf{r},t) = 2q\mu\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{E}(\mathbf{r},t) = -\nabla\Phi - \frac{1}{c_n}\frac{\partial\mathbf{A}}{\partial t} \qquad \mathbf{B}(\mathbf{r},t) = \nabla\times\mathbf{A}$$

$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}}{(R(t))^2\sqrt{1-\beta_n^2\sin^2\theta}} \times$$

$$\left(\frac{\beta_n^2-1}{1-\beta_n^2\sin^2\theta}\Theta(\cos\theta_C - \cos\theta(t)) + \sqrt{\beta_n^2-1}\;\delta(\cos\theta_C - \cos\theta(t))\right)$$

$$\mathbf{B}(\mathbf{r},t) = -\beta_n\sin\theta\left(\hat{\theta}\times\mathbf{E}(\mathbf{r},t)\right)$$

Intermediate steps:

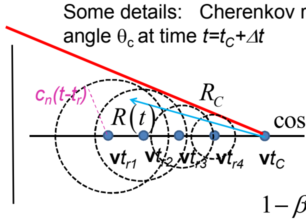
$$\frac{d\theta}{dt}=\frac{v\sin\theta}{R} \qquad \frac{dR}{dt}=-v\cos\theta$$

Using instantaneous polar coordinates: $\nabla \equiv \hat{\mathbf{R}}\frac{\partial}{\partial R}+\hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial}{\partial\theta}$

$$\nabla\Theta(\cos\theta_C-\cos\theta(t))=\delta(\cos\theta_C-\cos\theta(t))\frac{\sin\theta(t)}{R(t)}\hat{\boldsymbol{\theta}}$$

$$\frac{\partial\Theta(\cos\theta_C-\cos\theta(t))}{\partial t}=\delta(\cos\theta_C-\cos\theta(t))\frac{v\sin^2\theta(t)}{R(t)}$$

Some details: Cherenkov radiation observed near the angle θ_c at time $t=t_C+\Delta t$



$$\cos\theta_C-\cos\theta(t) \approx \frac{c_n\Delta t}{\beta_n R_C}$$

$$1-\beta_n\sin^2\theta(t)\approx\frac{2c_n\Delta t\sqrt{\beta_n^2-1}}{R_C}$$

When the dust clears ....

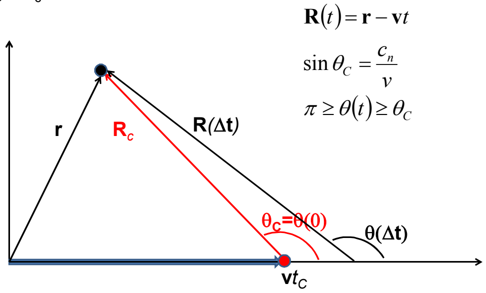
$$\frac{d^2I}{d\omega d\ell}\propto\left(1-\frac{c_n^2}{v^2}\right)\omega$$

Some details: Cherenkov radiation observed near the angle θ_c

$$\mathbf{R}(t)=\mathbf{r}-\mathbf{v}t$$

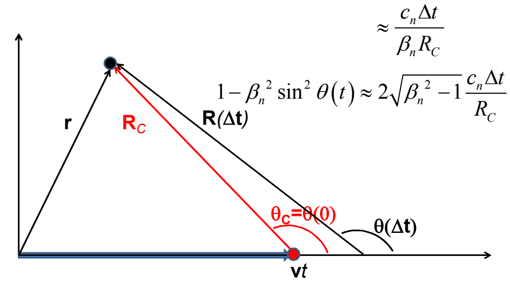
$$\sin\theta_C=\frac{c_n}{v}$$

$$\pi\geq\theta(t)\geq\theta_C$$



Cherenkov radiation observed near the angle θ_c -- continued

$$\cos\theta_C - \cos(\theta_C + \Delta\theta) \approx \sin\theta_C\ \Delta\theta$$
$$\approx \frac{c_n \Delta t}{\beta_n R_C}$$
$$1 - \beta_n^2 \sin^2\theta(t) \approx 2\sqrt{\beta_n^2 - 1}\,\frac{c_n \Delta t}{R_C}$$



Cherenkov radiation observed near the angle θ_c -- continued

$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}}{(R(t))^2\sqrt{1-\beta_n^2\sin^2\theta}} \times$$
$$\left(\frac{\beta_n^2 - 1}{1-\beta_n^2\sin^2\theta}\Theta(\cos\theta_C - \cos\theta(t)) + \sqrt{\beta_n^2-1}\ \delta(\cos\theta_C - \cos\theta(t))\right)$$
$$\mathbf{B}(\mathbf{r},t) = -\beta_n \sin\theta\left(\hat{\theta}\times\mathbf{E}(\mathbf{r},t)\right)$$

Estimates at $t = t_C + \Delta t$

$$\mathbf{E}(\mathbf{r},t) \approx -\frac{2q}{\varepsilon}\hat{\mathbf{R}}_C\frac{(\beta_n^2-1)^{1/4}}{(2c_n^3 R_C)^{1/2}}\left[(\Delta t)^{-1/2}\delta(\Delta t) - \frac{1}{2}(\Delta t)^{-3/2}\Theta(\Delta t)\right]$$
$$\mathbf{B}(\mathbf{r},t) = -\left(\hat{\boldsymbol{\theta}}(t_C)\times\mathbf{E}(\mathbf{r},t)\right)$$

Cherenkov radiation observed near the angle θ_c -- continued

Spectral analysis:

$$\tilde{\mathbf{E}}(\omega) = -\frac{2q}{\varepsilon}\hat{\mathbf{R}}_C\frac{(\beta_n^2-1)^{1/4}}{(2c_n^3R_C)^{1/2}}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dt\left[t^{-1/2}\delta(t) - \frac{1}{2}t^{-3/2}\Theta(t)\right]e^{i\omega t}$$
$$= -i\omega\frac{2q}{\varepsilon}\hat{\mathbf{R}}_C\frac{(\beta_n^2-1)^{1/4}}{(2c_n^3R_C)^{1/2}}\frac{1}{\sqrt{2\pi}}\int_0^{\infty}dt\ t^{-1/2}e^{i\omega t}$$
$$= \frac{q}{\varepsilon}\hat{\mathbf{R}}_C\frac{(\beta_n^2-1)^{1/4}}{(2c_n^3R_C)^{1/2}}(1-i)\sqrt{\omega}$$

Spectral intensity: $$\frac{d^2I}{d\Omega d\omega} \propto \left|\tilde{\mathbf{E}}(\omega)\right|^2 = \frac{q^2}{\varepsilon^2}\frac{(\beta_n^2-1)^{1/2}}{c_n^3 R_C}\omega$$

Cherenkov radiation emitted by the core of the Reed Research Reactor located at Reed College in Portland, Oregon, U.S. *Cherenkov radiation*. Photograph. *Encyclopædia Britannica Online*. Web. 12 Apr. 2013.
http://www.britannica.com/EBchecked/media/174732