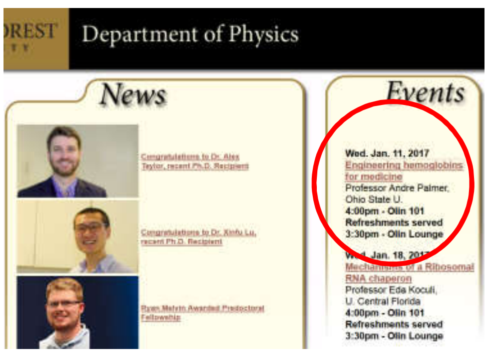
PHY 745 Group Theory
11-11:50 AM MWF Olin 102

Plan for Lecture 1:

Reading: Chapters 1 in DDJ (Dresselhaus, Dresselhaus, and Jorio)

1. **Course structure and expectations**
2. **Definition of a group**
3. **Some examples**



http://users.wfu.edu/natalie/s17phy745/

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Instructor: Natalie Holzwarth	Phone:758-5510	Office:300 OPL	e-mail:natalie@wfu.edu

- General information
- Syllabus and homework assignments
- Lecture notes

Last modified: Saturday, 07-Jan-2017 17:52:07 EST

Spring 2017 Schedule
for N. A. W. Holzwarth

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00-9:00	Office Hours	Physics Research	Office Hours	Physics Research	Office Hours
9:00-10:00	Electrodynamics PHY712		Electrodynamics PHY712		Electrodynamics PHY712
10:00-11:00	Office Hours		Office Hours		Office Hours
11:00-12:00	Group Theory PHY745		Group Theory PHY745		Group Theory PHY745
12:00-1:00	Office Hours		Office Hours		Office Hours
1:00-2:30	Condensed Matter Theory Journal Club		Physics Research		Physics Research
2:30-3:30	Physics Research				
3:30-5:00			Physics Colloquium		

Travel dates:

- **Mar. 12 - 17, 2017** -- March 2017 APS Meeting in New Orleans, LA

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General Information

This course is a one semester survey of Group Theory as it is used in physics. The textbook is , **Group Theory by M. S. Dresselhaus, G. Dresselhaus, and A. Jorio (Springer 2008)** with supplemental materials taken from additional texts to extend the ideas to continuous groups.

It is likely that your grade for the course will depend upon the following factors:

Problem sets*	45%
Presentation	10%
Exams	45%

*The schedule notes the "due" date for each assignment. Homeworks may be turned in 1 lecture past their due date without grade penalty. After that, the homework grade will be reduced by 10% for each succeeding late date. According to the honor system, all work submitted for grading purposes should represent the student's own best efforts. This means that students who work together on homework assignments should all contribute roughly equally and independently verify all derivations and results.

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Course schedule for Spring 2017

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	DDJ Reading	Topic	HW	Due date
1	Wed: 01/11/2017	Chap. 1	Definition and properties of groups	#1	01/18/2017
2	Fri: 01/13/2017	Chap. 1	Subgroups and classes		01/18/2017
	Mon: 01/16/2017		*MLK Holiday - no class*		
3	Wed: 01/18/2017				
4	Fri: 01/20/2017				
5	Mon: 01/23/2017				
6	Wed: 01/25/2017				

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Group theory
An abstract algebraic construction in mathematics

Definition of a group:

A group is a collection of "elements" – $A, B, C, \ldots$ and a "multiplication" process. The abstract multiplication $(\cdot)$ pairs two group elements, and associates the "result" with a third element. (For example $(A \cdot B = C)$.) The elements and the multiplication process must have the following properties.

1. The collection of elements is closed under multiplication. That is, if elements A and B are in the group and $A \cdot B = C$, element C must be in the group.
2. One of the members of the group is a "unit element" (E). That is, for any element A of the group, $A \cdot E = E \cdot A = A$.
3. For each element A of the group, there is another element A^{-1} which is its "inverse". That is $A \cdot A^{-1} = A^{-1} \cdot A = E$.
4. The multiplication process is "associative". That is for sequential mulplication of group elements A, B, and C, $(A \cdot B) \cdot C = A \cdot (B \cdot C)$.

Some definitions:

Order of the group → number of elements (members) in the group (positive integer for finite group, ∞ for infinite group)

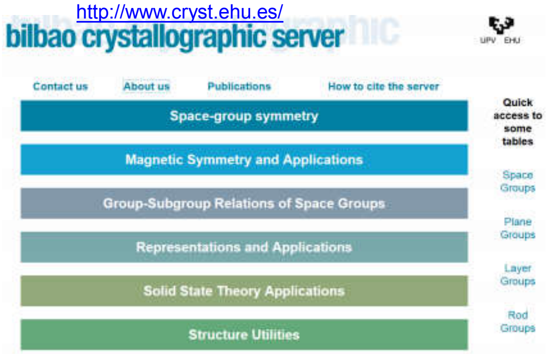
Subgroup → collection of elements within a group which by themselves form a group

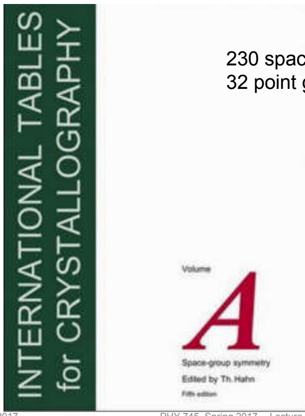
What does this have to do with physics?

- Provides valuable analysis tools for physical objects we know – crystals, molecules, etc.
- Provides framework for understanding physical processes we don't yet understand – particle physics

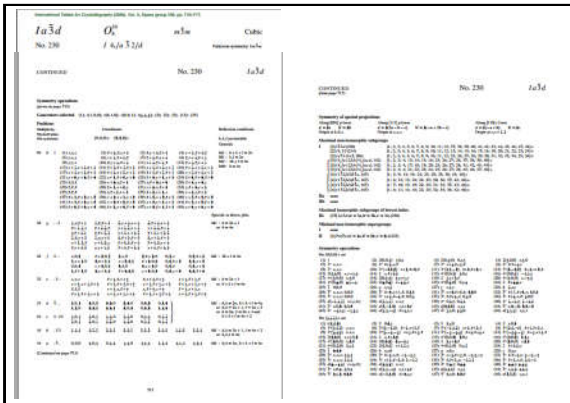
Examples of group theory applied to crystals

http://www.cryst.ehu.es/





230 space groups
32 point groups



Some details from the International Tables of Crystallography

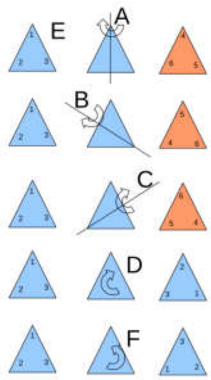
	Wyckoff letter	Symmetry	Fractional coordinates of sites			
16	f	$.3m$	x,x,x $x,x,\bar{x}$	$\bar{x},\bar{x},x$ $\bar{x},\bar{x},\bar{x}$	$\bar{x},x,\bar{x}$ $x,\bar{x},x$	$x,\bar{x},\bar{x}$ $\bar{x},x,x$
12	e	$4m.m$	$x,0,0$	$\bar{x},0,0$	$0,x,0$	$0,\bar{x},0$
12	d	$\bar{4}m.2$	$\frac{1}{4},0,\frac{1}{2}$	$\frac{3}{4},0,\frac{1}{2}$	$\frac{1}{2},\frac{1}{4},0$	$\frac{1}{2},\frac{3}{4},0$
8	c	$.\bar{3}m$	$\frac{1}{4},\frac{1}{4},\frac{1}{4}$	$\frac{3}{4},\frac{3}{4},\frac{1}{4}$	$\frac{3}{4},\frac{1}{4},\frac{3}{4}$	$\frac{1}{4},\frac{3}{4},\frac{3}{4}$
6	b	$4/mm.m$	$0,\frac{1}{2},\frac{1}{2}$	$\frac{1}{2},0,\frac{1}{2}$	$\frac{1}{2},\frac{1}{2},0$	
2	a	$m\bar{3}m$	$0,0,0$			

Example of a 6-member group *E,A,B,C,D,F,G*

Group multiplication table

Group of order 6

	E	A	B	C	D	F
E	E	A	B	C	D	F
A	A	E	D	F	B	C
B	B	F	E	D	C	A
C	C	D	F	E	A	B
D	D	C	A	B	F	E
F	F	B	C	A	E	D



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	E	A	B	C	D	F
E	E	A	B	C	D	F
A	A	E	D	F	B	C
B	B	F	E	D	C	A
C	C	D	F	E	A	B
D	D	C	A	B	F	E
F	F	B	C	A	E	D

Check on group properties:

1. Closed; multiplication table uniquely generates group members.
2. Unit element included.
3. Each element has inverse.
4. Multiplication process is associative.

Subgroup: members of larger group which have the property of a group

In this case:

Subgroup g_1: E

Subgroup g_2: (E, A); Subgroup g_3: (E, B); Subgroup g_4: (E, C)

Subgroup g_5: (E, D, F)

Note that the orders of the subgroups are 1, 2, 3 which are divisors of the order of the group (6).

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Other details

Coset: Given a subgroup $\mathbf{g_i}$ of a group $\mathbf{g}$ a right coset can be formed by multiply an element of $\mathbf{g}$ with each element of $\mathbf{g_i}$

Example -- consider the right cosets of $\mathbf{g_2} = (E, A)$

$\mathbf{g_2}E \rightarrow E, A$

$\mathbf{g_2}A \rightarrow A, E$

$\mathbf{g_2}B \rightarrow B, D$

$\mathbf{g_2}C \rightarrow C, F$

$\mathbf{g_2}D \rightarrow D, B$

$\mathbf{g_2}F \rightarrow F, C$

Note that each pair of right cosets are either identical or completely distinct.

Distinct cosets:

(E, A)

(B, F)

(C, D)

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Notions of subgroups and their cosets result in the theorem: *The order of a subgroup is a divisor of the order of the group.*

Definition:

An element $B \equiv XAX^{-1}$ is defined as conjugate to element A, where X is any element of the group.

Definition:

A class is composed of members of a group which are generated by the conjugate construction:

$$\mathcal{C} = X_i^{-1} Y X_i \quad \text{where } Y \text{ is a fixed group element and } X_i \text{ are all of the elements of the group.}$$

For our example:

	E	A	B	C	D	F
E	E	A	B	C	D	F
A	A	E	D	F	B	C
B	B	F	E	D	C	A
C	C	D	F	E	A	B
D	D	C	A	B	F	E
F	F	B	C	A	E	D

$A^{-1} = A$

$B^{-1} = B$

$C^{-1} = C$

$D^{-1} = F$

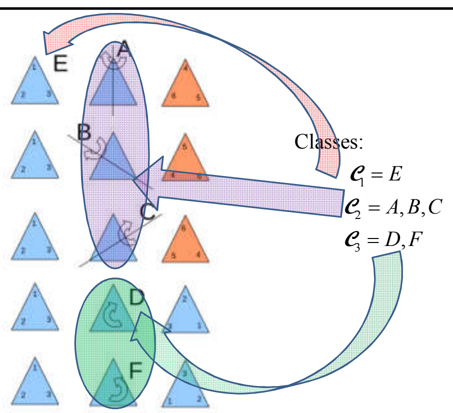
$F^{-1} = D$

Classes:

$\mathcal{C}_1 = E$

$\mathcal{C}_2 = A, B, C$

$\mathcal{C}_3 = D, F$



Other examples –

1. Groups of order 1: *E* (only possibility)
2. Groups of order 2: *E, A* with $A^2=E$ (only possibility)
3. Groups of order 3: *E, A, B* with $A^2=B$ and $A^3=E$
4. Groups of order 4:

Possibility #1

	E	A	B	C
E	E	A	B	C
A	A	B	C	E
B	B	C	E	A
C	C	E	A	B

Possibility #2

	E	A	B	C
E	E	A	B	C
A	A	E	C	B
B	B	C	E	A
C	C	B	A	E

5. Groups of prime order are always cyclic and Abelian.

Special groups and terminologies

A group is called **Abelian** if for every pair of elements *A,B*; *AB=BA.*

A group is called **cyclic** if all the elements can be formed according to $X, X^2, X^3, \ldots X^n = E$
n is called the order of *X* (the period).

Recall for n=4 example:

	E	A	B	C
E	E	A	B	C
A	A	B	C	E
B	B	C	E	A
C	C	E	A	B

➔

	E	A	A^2	A^3
E	E	A	A^2	A^3
A	A	A^2	A^3	E
A^2	A^2	A^3	E	A
A^3	A^3	E	A	A^2