March 12, 2018

PHY 712 – Problem Set # 13

Start reading Chapter 8 of **Jackson**.

1. Suppose that an electromagnetic wave of pure (real) frequency ω is traveling along the z-axis of a wave guide having a square cross section with side dimension a composed of a medium having a real permittivity constant ϵ and a real permeability constant μ. Suppose that the wave is known to have the form:

$$\mathbf{E}(\mathbf{r},t) = \Re\left\{H_0 \mathrm{e}^{ikz-i\omega t}(i\mu\omega)\frac{a}{\pi}\sin\left(\frac{\pi x}{a}\right)\hat{\mathbf{y}}\right\}$$

$$\mathbf{H}(\mathbf{r},t) = \Re\left\{H_0 \mathrm{e}^{ikz-i\omega t}\left[-ik\frac{a}{\pi}\sin\left(\frac{\pi x}{a}\right)\hat{\mathbf{x}} + \cos\left(\frac{\pi x}{a}\right)\hat{\mathbf{z}}\right]\right\}.$$

   Here H_0 denotes a real amplitude, and the parameter k is assumed to be real and equal to

$$k \equiv \sqrt{\mu\epsilon\omega^2 - \left(\frac{\pi}{a}\right)^2},$$

   for $\mu\epsilon\omega^2 > \left(\frac{\pi}{a}\right)^2$.

   (a) Show that this wave satisfies the sourceless Maxwell's equations.

   (b) Find the form of the time-averaged Poynting vector

$$\langle \mathbf{S} \rangle_{avg} \equiv \frac{1}{2}\Re\left\{\mathbf{E}(\mathbf{r},t) \times \mathbf{H}^*(\mathbf{r},t)\right\}$$

   for this electromagnetic wave.