

**PHY 712 Electrodynamics
9-9:50 AM MWF Olin 105**

Plan for Lecture 12:

Start reading Chapter 5

A. Magnetostatics

B. Vector potential

C. Example: current loop

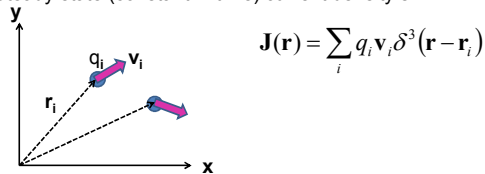
Course schedule for Spring 2018

(Preliminary schedule -- subject to frequent adjustment.)

| Lecture date | JDJ Reading | Topic | HW | Due date |
|--------------------|------------------|--|----|------------|
| Wed: 01/17/2018 | No class | Snow | | |
| 1 Fri: 01/19/2018 | Chap. 1 & Appen. | Introduction, units and Poisson equation | #1 | 01/26/2018 |
| 2 Mon: 01/22/2018 | Chap. 1 | Electrostatic energy calculations | #2 | 01/26/2018 |
| 3 Wed: 01/24/2018 | Chap. 1 | Poisson's equation and Green's theorem | #3 | 01/26/2018 |
| 4 Thu: 01/25/2018 | Chap. 1 & 2 | Poisson's equation in 2 and 3 dimensions | | |
| 5 Fri: 01/26/2018 | Chap. 1 & 2 | Brief introduction to numerical methods | #4 | 01/29/2018 |
| 6 Mon: 01/29/2018 | Chap. 2 | Method of image charges | #5 | 01/31/2018 |
| 7 Wed: 01/31/2018 | Chap. 2 & 3 | Cylindrical and spherical geometries | #6 | 02/02/2018 |
| 8 Fri: 02/02/2018 | Chap. 3 & 4 | Multipole analysis | #7 | 02/07/2018 |
| 9 Mon: 02/05/2018 | Chap. 4 | Dipoles and Dielectrics | #8 | 02/09/2018 |
| 10 Wed: 02/07/2018 | Chap. 4 | Dipoles and Dielectrics | | |
| 11 Fri: 02/09/2018 | Chap. 1-4 | Review | | |
| 12 Mon: 02/12/2018 | Chap. 5 | Magnetostatics | #9 | 02/16/2018 |
| 13 Wed: 02/14/2018 | | | | |
| 14 Fri: 02/16/2018 | | | | |
| 15 Mon: 02/19/2018 | | | | |

Magnetostatics

Magnetic flux density or magnetic induction field **B**
Steady state (constant in time) current density **J**



$$\mathbf{J}(\mathbf{r}) = \sum_i q_i \mathbf{v}_i \delta^3(\mathbf{r} - \mathbf{r}_i)$$

Note that "statics" implies that $\nabla \cdot \mathbf{J} = 0$.

This follows from the continuity equation :

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$$

Comparison of electrostatics and magnetostatics

Electrostatic field due to charge density $\rho(\mathbf{r})$:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int d^3r' \rho(\mathbf{r}') \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3}$$

Magnetostatic field due to current density $\mathbf{J}(\mathbf{r})$:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3r' \mathbf{J}(\mathbf{r}') \times \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3}$$

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Alternative forms magnetostatic equations

Magnetostatic field due to current density $\mathbf{J}(\mathbf{r})$:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3r' \mathbf{J}(\mathbf{r}') \times \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} = \frac{\mu_0}{4\pi} \int d^3r' \left(\nabla \frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) \times \mathbf{J}(\mathbf{r}')$$

Note that: $\nabla \frac{1}{|\mathbf{r} - \mathbf{r}'|} = -\frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3}$

Also note that: $\nabla \times (s(\mathbf{r}) \mathbf{V}(\mathbf{r})) = \nabla s(\mathbf{r}) \times \mathbf{V}(\mathbf{r}) + s(\mathbf{r}) \nabla \times \mathbf{V}(\mathbf{r})$

$$\Rightarrow \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \nabla \times \int d^3r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \quad \text{In this case } \mathbf{V}(\mathbf{r}) \equiv \mathbf{J}(\mathbf{r}') \text{ so that } \nabla \times \mathbf{V}(\mathbf{r}) = 0$$

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Alternative forms magnetostatic equations -- continued

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \nabla \times \int d^3r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

$\Rightarrow \nabla \cdot \mathbf{B}(\mathbf{r}) = 0$ No magnetic monopoles

$\Rightarrow \nabla \times \mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{J}(\mathbf{r})$ Ampere's law

"Proof" of Ampere's law for magnetostatic system :

$$\nabla \times \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \nabla \times \nabla \times \int d^3r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

Note that : $\nabla \times \nabla \times \mathbf{V} = \nabla(\nabla \cdot \mathbf{V}) - \nabla^2 \mathbf{V}$

Recall that : $\nabla^2 \frac{1}{|\mathbf{r} - \mathbf{r}'|} = -4\pi \delta^3(\mathbf{r} - \mathbf{r}')$ and $\nabla \cdot \mathbf{J}(\mathbf{r}) = 0$

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Differential forms of magnetostatic equations:

$$\begin{aligned} \Rightarrow \nabla \cdot \mathbf{B}(\mathbf{r}) &= 0 && \text{No magnetic monopoles} \\ \Rightarrow \nabla \times \mathbf{B}(\mathbf{r}) &= \mu_0 \mathbf{J}(\mathbf{r}) && \text{Ampere's law} \end{aligned}$$

Magnetostatic vector potential

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \nabla \times \int d^3r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

$$\Rightarrow \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + \nabla s(\mathbf{r})$$

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Non uniqueness of the magnetostatic vector potential

$$\begin{aligned} \text{Note that : } \mathbf{B}(\mathbf{r}) &= \nabla \times \mathbf{A}(\mathbf{r}) = \nabla \times \mathbf{A}'(\mathbf{r}) \\ \text{if } \mathbf{A}'(\mathbf{r}) &= \mathbf{A}(\mathbf{r}) + \nabla s(\mathbf{r}) \end{aligned}$$

Example : for $\mathbf{B}(\mathbf{r}) = B_0 \hat{z}$

$$\mathbf{A}(\mathbf{r}) = \frac{1}{2} B_0 (x\hat{y} - y\hat{x})$$

$$\text{or } \mathbf{A}(\mathbf{r}) = B_0 x\hat{y}$$

$$\text{or } \mathbf{A}(\mathbf{r}) = -B_0 y\hat{x}$$

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Differential form of Ampere's law in terms of vector potential:

$$\nabla \times \mathbf{B}(\mathbf{r}) = \nabla \times \nabla \times \mathbf{A}(\mathbf{r}) = \mu_0 \mathbf{J}(\mathbf{r})$$

$$\Rightarrow \nabla(\nabla \cdot \mathbf{A}(\mathbf{r})) - \nabla^2 \mathbf{A}(\mathbf{r}) = \mu_0 \mathbf{J}(\mathbf{r})$$

$$\text{If } \nabla \cdot \mathbf{A}(\mathbf{r}) = 0 \text{ (Coulomb gauge)} \Rightarrow \nabla^2 \mathbf{A}(\mathbf{r}) = -\mu_0 \mathbf{J}(\mathbf{r})$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

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Magnetostatics example: current loop

$$\mathbf{J}(\mathbf{r}') = \frac{I}{a} \sin \theta' \delta(\cos \theta') \delta(r' - a) (-\sin \phi' \hat{\mathbf{x}} + \cos \phi' \hat{\mathbf{y}})$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3 r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

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Magnetostatics example: current loop -- continued

$$\mathbf{J}(\mathbf{r}') = \frac{I}{a} \sin \theta' \delta(\cos \theta') \delta(r' - a) (-\sin \phi' \hat{\mathbf{x}} + \cos \phi' \hat{\mathbf{y}})$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3 r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi a} \int r'^2 dr' d \cos \theta' d \phi' \frac{\sin \theta' \delta(\cos \theta') \delta(r' - a) (-\sin \phi' \hat{\mathbf{x}} + \cos \phi' \hat{\mathbf{y}})}{(r^2 + r'^2 - 2r r' (\cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\phi - \phi')))^{1/2}}$$

Completing integration over r' and θ' :

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I a^2}{4\pi a} \int_0^{2\pi} d\phi' \frac{(-\sin \phi' \hat{\mathbf{x}} + \cos \phi' \hat{\mathbf{y}})}{(r^2 + a^2 - 2ra(\sin \theta \cos(\phi - \phi')))^{1/2}}$$

Let $\phi - \phi' \equiv \phi$
 $\sin \phi' = \sin(\phi - \phi) = \sin \phi \cos \phi - \cos \phi \sin \phi$
 $\cos \phi' = \cos(\phi - \phi) = \cos \phi \cos \phi + \sin \phi \sin \phi$
 Remaining non-trivial terms

$$\mathbf{A}(\mathbf{r}) = -\frac{\mu_0 I a}{4\pi} (\sin \phi \hat{\mathbf{x}} - \cos \phi \hat{\mathbf{y}}) \int_0^{2\pi} d\phi' \frac{\cos \phi'}{(r^2 + a^2 - 2ra(\sin \theta \cos \phi'))^{1/2}}$$

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Magnetostatics example: current loop -- continued

$$\mathbf{A}(\mathbf{r}) = -\frac{\mu_0 I a}{4\pi} (\sin \phi \hat{\mathbf{x}} - \cos \phi \hat{\mathbf{y}}) \int_0^{2\pi} d\phi' \frac{\cos \phi'}{(r^2 + a^2 - 2ra(\sin \theta \cos \phi'))^{1/2}}$$

Elliptic integrals:

$$K(m) = \int_0^{\pi/2} \frac{du}{(1 - m \sin^2 u)^{1/2}}$$

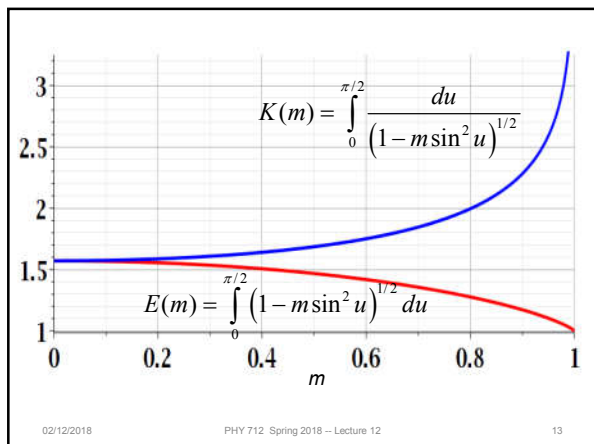
$$E(m) = \int_0^{\pi/2} (1 - m \sin^2 u)^{1/2} du$$

$$\mathbf{A}(\mathbf{r}) = -\frac{\mu_0}{4\pi} 4Ia \frac{(\sin \phi \hat{\mathbf{x}} - \cos \phi \hat{\mathbf{y}})}{(r^2 + a^2 + 2ra \sin \theta)^{1/2}} \left[\frac{(2 - k^2)K(k) - 2E(k)}{k^2} \right]$$

where: $k^2 \equiv \frac{4ar \sin \theta}{r^2 + a^2 + 2ra \sin \theta}$

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$$

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Magnetostatics example: current loop -- continued

$$\mathbf{A}(\mathbf{r}) = -\frac{\mu_0 4Ia}{4\pi} \frac{(\sin \phi \hat{\mathbf{x}} - \cos \phi \hat{\mathbf{y}})}{(r^2 + a^2 + 2ra \sin \theta)^{1/2}} \left[\frac{(2 - k^2)K(k) - 2E(k)}{k^2} \right]$$

where: $k^2 \equiv \frac{4ar \sin \theta}{r^2 + a^2 + 2ra \sin \theta} = \frac{4ax}{x^2 + z^2 + a^2 + 2ax}$
 For $\phi = 0$: $x = r \sin \theta$, $y = 0$, $z = r \cos \theta$

$$A_y(x, z) = \frac{\mu_0 4Ia}{4\pi} \frac{1}{(x^2 + z^2 + a^2 + 2ax)^{1/2}} \left[\frac{(2 - k^2)K(k) - 2E(k)}{k^2} \right]$$

where: $k^2 \equiv \frac{4ax}{x^2 + z^2 + a^2 + 2ax}$

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Magnetostatics example: current loop -- continued

$$\mathbf{A}(\mathbf{r}) = -\frac{\mu_0 4Ia}{4\pi} \frac{(\sin \phi \hat{\mathbf{x}} - \cos \phi \hat{\mathbf{y}})}{(r^2 + a^2 + 2ra \sin \theta)^{1/2}} \left[\frac{(2 - k^2)K(k) - 2E(k)}{k^2} \right]$$

$$= \hat{\phi} \frac{\mu_0}{4\pi} \frac{4Ia}{(r^2 + a^2 + 2ra \sin \theta)^{1/2}} \left[\frac{(2 - k^2)K(k) - 2E(k)}{k^2} \right]$$

where: $k^2 \equiv \frac{4ar \sin \theta}{r^2 + a^2 + 2ra \sin \theta}$

$$\mathbf{B} = \nabla \times \mathbf{A} = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (r \sin \theta A_\phi(r, \theta)) \hat{\mathbf{r}} - \frac{1}{r} \frac{\partial (r A_\phi(r, \theta))}{\partial r} \hat{\theta}$$

Evaluation for special cases

For $k^2 \rightarrow 0$:

$$\frac{(2 - k^2)K(k) - 2E(k)}{k^2} \approx \frac{\pi}{16} k^2$$

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Evaluation for special case $k^2 \rightarrow 0$

$$\mathbf{A}(\mathbf{r}) = \hat{\phi} \frac{\mu_0 I a^2}{4} \frac{r \sin \theta}{(r^2 + a^2 + 2ra \sin \theta)^{3/2}}$$

$$\mathbf{B} = \nabla \times \mathbf{A} = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (r \sin \theta A_\phi(r, \theta)) \hat{\mathbf{r}} - \frac{1}{r} \frac{\partial (r A_\phi(r, \theta))}{\partial r} \hat{\boldsymbol{\theta}}$$

$$= \frac{\mu_0 I a^2}{4} \frac{(\hat{\mathbf{r}} \cos \theta (2a^2 + 2r^2 + ar \sin \theta) - \hat{\boldsymbol{\theta}} \sin \theta (2a^2 - r^2 + ar \sin \theta))}{(r^2 + a^2 + 2ra \sin \theta)^{5/2}}$$

where $k^2 \equiv \frac{4ar \sin \theta}{r^2 + a^2 + 2ra \sin \theta} \rightarrow 0$

For $\theta = 0$: $r = z$

$$\mathbf{B} = \nabla \times \mathbf{A} = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (r \sin \theta A_\phi(r, \theta)) \hat{\mathbf{r}} - \frac{1}{r} \frac{\partial (r A_\phi(r, \theta))}{\partial r} \hat{\boldsymbol{\theta}}$$

$$= \hat{\mathbf{z}} \frac{\mu_0 I a^2}{2} \frac{1}{(z^2 + a^2)^{3/2}}$$

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Evaluation for special case $k^2 \rightarrow 0$

$$\mathbf{A}(\mathbf{r}) = \hat{\phi} \frac{\mu_0 I a^2}{4} \frac{r \sin \theta}{(r^2 + a^2 + 2ra \sin \theta)^{3/2}}$$

$$\mathbf{B} = \nabla \times \mathbf{A} = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (r \sin \theta A_\phi(r, \theta)) \hat{\mathbf{r}} - \frac{1}{r} \frac{\partial (r A_\phi(r, \theta))}{\partial r} \hat{\boldsymbol{\theta}}$$

$$= \frac{\mu_0 I a^2}{4} \frac{(\hat{\mathbf{r}} \cos \theta (2a^2 + 2r^2 + ar \sin \theta) - \hat{\boldsymbol{\theta}} \sin \theta (2a^2 - r^2 + ar \sin \theta))}{(r^2 + a^2 + 2ra \sin \theta)^{5/2}}$$

where $k^2 \equiv \frac{4ar \sin \theta}{r^2 + a^2 + 2ra \sin \theta} \rightarrow 0$

For $r \rightarrow \infty$

$$\mathbf{B} = \nabla \times \mathbf{A} = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (r \sin \theta A_\phi(r, \theta)) \hat{\mathbf{r}} - \frac{1}{r} \frac{\partial (r A_\phi(r, \theta))}{\partial r} \hat{\boldsymbol{\theta}}$$

$$= \frac{\mu_0 I a^2}{4r^3} (\hat{\mathbf{r}} (2 \cos \theta) + \hat{\boldsymbol{\theta}} \sin \theta)$$

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Magnetostatics example: current loop -- continued

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} 4Ia \frac{(\sin \phi \hat{\mathbf{x}} - \cos \phi \hat{\mathbf{y}})}{(r^2 + a^2 + 2ra \sin \theta)^{3/2}} \left[\frac{(2 - k^2)K(k) - 2E(k)}{k^2} \right]$$

where: $k \equiv \frac{4ar \sin \theta}{r^2 + a^2 + 2ra \sin \theta}$

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$$

Note that for spherical polar coordinates: $\hat{\phi} = \sin \theta \hat{\mathbf{x}} - \cos \theta \hat{\mathbf{y}}$

$$\mathbf{A}(\mathbf{r}) = A_\phi(\mathbf{r}) \hat{\phi}$$

where $A_\phi(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{4Ia}{(r^2 + a^2 + 2ra \sin \theta)^{3/2}} \left[\frac{(2 - k^2)K(k) - 2E(k)}{k^2} \right]$

$$\mathbf{B}(\mathbf{r}) = \frac{1}{r \sin \theta} \frac{\partial (\sin \theta A_\phi(\mathbf{r}))}{\partial \theta} \hat{\mathbf{r}} - \frac{1}{r} \frac{\partial (r A_\phi(\mathbf{r}))}{\partial r} \hat{\boldsymbol{\theta}}$$

For $r \rightarrow \infty$:

$$\mathbf{B}(\mathbf{r}) \approx \frac{\mu_0}{4\pi} \frac{I \pi a^2}{r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}})$$

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Other examples of current density sources:

Quantum mechanical expression for current density
for a particle of mass M and charge e and of probability amplitude $\Psi(\mathbf{r})$:

$$\mathbf{J}(\mathbf{r}) = -\frac{e\hbar}{2Mi} (\Psi^*(\mathbf{r})\nabla\Psi(\mathbf{r}) - \Psi(\mathbf{r})\nabla\Psi^*(\mathbf{r}))$$

For an electron in a spherical potential (such as in an atom):

$$\Psi(\mathbf{r}) \equiv \Psi_{nlm_l}(\mathbf{r}) = R_{nl}(r)Y_{lm_l}(\hat{\mathbf{r}})$$

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{2Mi} |R_{nl}(r)|^2 \frac{1}{r \sin \theta} \left(Y_{lm_l}^*(\hat{\mathbf{r}}) \frac{\partial Y_{lm_l}(\hat{\mathbf{r}})}{\partial \phi} - Y_{lm_l}(\hat{\mathbf{r}}) \frac{\partial Y_{lm_l}^*(\hat{\mathbf{r}})}{\partial \phi} \right) \hat{\phi}$$

$$= \frac{e\hbar}{M} \frac{m_l}{r \sin \theta} |\Psi_{nlm_l}(\mathbf{r})|^2 \hat{\phi}$$

Note that: $\hat{\phi} = -\sin \theta \hat{x} + \cos \theta \hat{y} = \frac{\hat{z} \times \mathbf{r}}{r \sin \theta}$

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{M} \frac{m_l}{r^2 \sin^2 \theta} |\Psi_{nlm_l}(\mathbf{r})|^2 (\hat{z} \times \mathbf{r})$$

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Magnetic vector potential for this case:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

$$\mathbf{J}(\mathbf{r}') = \frac{e\hbar}{M} \frac{m_l}{r'^2 \sin^2 \theta'} |\Psi_{nlm_l}(\mathbf{r}')|^2 (\hat{z} \times \mathbf{r}')$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{e\hbar m_l}{M} \int d^3r' \frac{(\hat{z} \times \mathbf{r}') |\Psi_{nlm_l}(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'| r'^2 \sin^2 \theta'}$$

For example: electron in the $nlm_l = 211$ state of H:

$$|\Psi_{211}(\mathbf{r}')|^2 = \frac{1}{64\pi a^3} \left(\frac{r'}{a}\right)^2 e^{-r'/a} \sin^2 \theta'$$

$$\mathbf{A}(\mathbf{r}) = -\frac{\mu_0}{8\pi} \frac{e\hbar}{M} \frac{(\hat{z} \times \mathbf{r})}{r^3} \left[1 - e^{-r/a} \left(1 + \frac{r}{a} + \frac{r^2}{2a^2} + \frac{r^3}{8a^3} \right) \right]$$

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