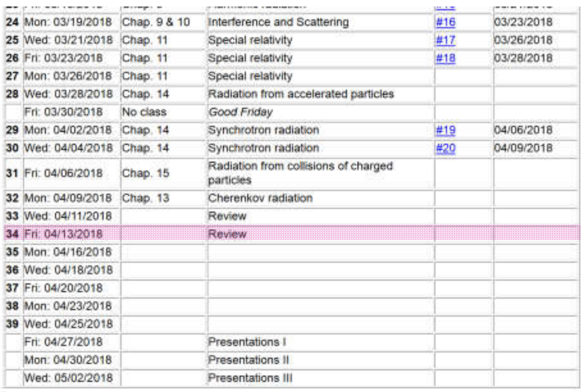
PHY 712 Electrodynamics
9-9:50 AM MWF Olin 105

Plan for Lecture 34:

Review – radiating systems

- Solution of Maxwell's equations with sources
- Time periodic sources
- Examples



24	Mon: 03/19/2018	Chap. 9 & 10	Interference and Scattering	#16	03/23/2018
25	Wed: 03/21/2018	Chap. 11	Special relativity	#17	03/26/2018
26	Fri: 03/23/2018	Chap. 11	Special relativity	#18	03/28/2018
27	Mon: 03/26/2018	Chap. 11	Special relativity		
28	Wed: 03/28/2018	Chap. 14	Radiation from accelerated particles		
	Fri: 03/30/2018	No class	*Good Friday*		
29	Mon: 04/02/2018	Chap. 14	Synchrotron radiation	#19	04/06/2018
30	Wed: 04/04/2018	Chap. 14	Synchrotron radiation	#20	04/09/2018
31	Fri: 04/06/2018	Chap. 15	Radiation from collisions of charged particles		
32	Mon: 04/09/2018	Chap. 13	Cherenkov radiation		
33	Wed: 04/11/2018		Review		
34	Fri: 04/13/2018		Review		
35	Mon: 04/16/2018				
36	Wed: 04/18/2018				
37	Fri: 04/20/2018				
38	Mon: 04/23/2018				
39	Wed: 04/25/2018				
	Fri: 04/27/2018		Presentations I		
	Mon: 04/30/2018		Presentations II		
	Wed: 05/02/2018		Presentations III		

General review --

Maxwell's equations

SI units; microscopic or vacuum form $(\mathbf{P}=0;\ \mathbf{M}=0;$ $\mu=\mu_0;\ \varepsilon=\varepsilon_0)$:

Coulomb's law: $\nabla\cdot\mathbf{E}=\rho/\varepsilon_0$

Ampere-Maxwell's law: $\nabla\times\mathbf{B}-\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}=\mu_0\mathbf{J}$

Faraday's law: $\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$

No magnetic monopoles: $\nabla\cdot\mathbf{B}=0$

$$\Rightarrow c^2=\frac{1}{\varepsilon_0\mu_0}$$

Formulation of Maxwell's equations in terms of vector and scalar potentials

$$\nabla \cdot \mathbf{B} = 0 \qquad \Rightarrow \quad \mathbf{B} = \nabla \times \mathbf{A}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \Rightarrow \nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0$$

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla \Phi$$

$$\text{or} \quad \mathbf{E} = -\nabla \Phi - \frac{\partial \mathbf{A}}{\partial t}$$

Formulation of Maxwell's equations in terms of vector and scalar potentials -- continued

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 :$$

$$-\nabla^2 \Phi - \frac{\partial (\nabla \cdot \mathbf{A})}{\partial t} = \rho / \varepsilon_0$$

$$\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$$

$$\nabla \times (\nabla \times \mathbf{A}) + \frac{1}{c^2} \left(\frac{\partial (\nabla \Phi)}{\partial t} + \frac{\partial^2 \mathbf{A}}{\partial t^2} \right) = \mu_0 \mathbf{J}$$

Formulation of Maxwell's equations in terms of vector and scalar potentials -- continued

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla \Phi$$

Lorentz gauge form -- require $\nabla \cdot \mathbf{A}_L + \frac{1}{c^2} \frac{\partial \Phi_L}{\partial t} = 0$

$$-\nabla^2 \Phi_L + \frac{1}{c^2} \frac{\partial^2 \Phi_L}{\partial t^2} = \rho / \varepsilon_0$$

$$-\nabla^2 \mathbf{A}_L + \frac{1}{c^2} \frac{\partial^2 \mathbf{A}_L}{\partial t^2} = \mu_0 \mathbf{J}$$

Solution of Maxwell's equations in the Lorentz gauge -- continued

Let Ψ represent Φ, A_x, A_y, A_z Let f represent ρ, J_x, J_y, J_z

$$\nabla^2\Psi(\mathbf{r},t)-\frac{1}{c^2}\frac{\partial^2\Psi(\mathbf{r},t)}{\partial t^2}=-4\pi f(\mathbf{r},t)$$

Green's function :

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)G(\mathbf{r},t;\mathbf{r}',t')=-4\pi\delta^3(\mathbf{r}-\mathbf{r}')\delta(t-t')$$

Formal solution for field $\Psi(\mathbf{r},t)$:

$$\Psi(\mathbf{r},t)=\Psi_{f=0}(\mathbf{r},t)+\int d^3r'\int dt' G(\mathbf{r},t;\mathbf{r}',t')f(\mathbf{r}',t')$$

Solution of Maxwell's equations in the Lorentz gauge -- continued

Determination of the form for the Green's function :

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)G(\mathbf{r},t;\mathbf{r}',t')=-4\pi\delta^3(\mathbf{r}-\mathbf{r}')\delta(t-t')$$

For the case of isotropic boundary values at infinity :

$$G(\mathbf{r},t;\mathbf{r}',t')=\frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\left(t'-\left(t-\frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right)$$

Formal solution for field $\Psi(\mathbf{r},t)$:

$$\Psi(\mathbf{r},t)=\Psi_{f=0}(\mathbf{r},t)+$$
$$\int d^3r'\int dt'\frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\left(t'-\left(t-\frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right)f(\mathbf{r}',t')$$

Formal solution for field $\Psi(\mathbf{r},t)$:

$$\Psi(\mathbf{r},t)=\Psi_{f=0}(\mathbf{r},t)+$$
$$\int d^3r'\int dt'\frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\left(t'-\left(t-\frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right)f(\mathbf{r}',t')$$

Two types of systems:

$\Rightarrow$ Time harmonic source: $f(\mathbf{r},t)=\tilde{f}(r,\omega)e^{-i\omega t}$

$\Rightarrow$ spherical harmonic expansion;
spherical Bessel functions

$\Rightarrow$ Charged particle with known trajectory: $\rho(\mathbf{r},t)=q\delta(\mathbf{r}-\mathbf{R}_q(t))$

➔**Liénard-Wiechert potentials**

Summary --

Lorentz gauge form -- require: $\nabla\cdot\mathbf{A}_L+\frac{1}{c^2}\frac{\partial\Phi_L}{\partial t}=0$

$$-\nabla^2\Phi_L+\frac{1}{c^2}\frac{\partial^2\Phi_L}{\partial t^2}=\rho/\varepsilon_0$$

$$-\nabla^2\mathbf{A}_L+\frac{1}{c^2}\frac{\partial^2\mathbf{A}_L}{\partial t^2}=\mu_0\mathbf{J}$$

General equation form:

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi=-4\pi f$$

$$\Psi(\mathbf{r},t)=\begin{cases}\Phi(\mathbf{r},t)\\ A_x(\mathbf{r},t)\\ A_y(\mathbf{r},t)\\ A_z(\mathbf{r},t)\end{cases}\qquad f(\mathbf{r},t)=\begin{cases}\rho(\mathbf{r},t)/(4\pi\varepsilon_0)\\ \mu_0 J_x(\mathbf{r},t)/(4\pi)\\ \mu_0 J_y(\mathbf{r},t)/(4\pi)\\ \mu_0 J_z(\mathbf{r},t)/(4\pi)\end{cases}$$

Solution of Maxwell's equations in the Lorentz gauge -- continued

$$G(\mathbf{r},t;\mathbf{r}',t')=\frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\left(t'-\left(t-|\mathbf{r}-\mathbf{r}'|/c\right)\right)$$

Solution for field $\Psi(\mathbf{r},t)$:

$$\Psi(\mathbf{r},t)=\Psi_{f=0}(\mathbf{r},t)+\int d^3r'\int dt'\frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\left(t'-\left(t-\frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right)f(\mathbf{r}',t')$$

Electromagnetic waves from time harmonic sources

Charge density: $\rho(\mathbf{r},t)=\Re\left(\tilde{\rho}(\mathbf{r},\omega)e^{-i\omega t}\right)$

Current density: $\mathbf{J}(\mathbf{r},t)=\Re\left(\tilde{\mathbf{J}}(\mathbf{r},\omega)e^{-i\omega t}\right)$

Note that the continuity condition:

$$\frac{\partial\rho(\mathbf{r},t)}{\partial t}+\nabla\cdot\mathbf{J}(\mathbf{r},t)=0\ \Rightarrow -i\omega\tilde{\rho}(\mathbf{r},\omega)+\nabla\cdot\tilde{\mathbf{J}}(\mathbf{r},\omega)=0$$

General source: $f(\mathbf{r},t)=\Re\left(\tilde{f}(\mathbf{r},\omega)e^{-i\omega t}\right)$

For $\tilde{f}(\mathbf{r},\omega)=\frac{1}{4\pi\varepsilon_0}\tilde{\rho}(\mathbf{r},\omega)$

or $\tilde{f}(\mathbf{r},\omega)=\frac{\mu_0}{4\pi}\tilde{J}_i(\mathbf{r},\omega)$

Electromagnetic waves from time harmonic sources – continued:

$$\Psi(\mathbf{r},t)=\Psi_{f=0}(\mathbf{r},t)+$$

$$\int d^3r'\int dt'\frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\left(t'-\left(t-\frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right)f(\mathbf{r}',t')$$

$$\tilde{\Psi}(\mathbf{r},\omega)e^{-i\omega t}=\tilde{\Psi}_{f=0}(\mathbf{r},\omega)e^{-i\omega t}+$$

$$\int d^3r'\int dt'\frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\left(t'-\left(t-\frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right)\tilde{f}(\mathbf{r}',\omega)e^{-i\omega t'}$$

$$=\tilde{\Psi}_{f=0}(\mathbf{r},\omega)e^{-i\omega t}+\int d^3r'\frac{e^{i\frac{\omega}{c}|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{f}(\mathbf{r}',\omega)e^{-i\omega t}$$

Radiation from time harmonic sources --

Charge density: $\rho(\mathbf{r},t)=\Re\left(\tilde{\rho}(\mathbf{r},\omega)e^{-i\omega t}\right)$

Current density: $\mathbf{J}(\mathbf{r},t)=\Re\left(\tilde{\mathbf{J}}(\mathbf{r},\omega)e^{-i\omega t}\right)$

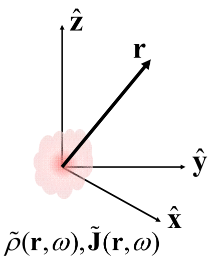
Continuity condition: $\dfrac{\partial\rho(\mathbf{r},t)}{\partial t}+\nabla\cdot\mathbf{J}(\mathbf{r},t)=0$

$$\Rightarrow -i\omega\tilde{\rho}(\mathbf{r},\omega)+\nabla\cdot\tilde{\mathbf{J}}(\mathbf{r},\omega)=0$$

Potential fields from time harmonic sources:

$$\tilde{\Phi}(\mathbf{r},\omega)=\frac{1}{4\pi\varepsilon_0}\int d^3r'\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\rho}(\mathbf{r}',\omega)$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\frac{\mu_0}{4\pi}\int d^3r'\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$



$$\tilde{\Phi}(\mathbf{r},\omega)=\frac{1}{4\pi\varepsilon_0}\int d^3r'\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\rho}(\mathbf{r}',\omega)$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\frac{\mu_0}{4\pi}\int d^3r'\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

Electromagnetic waves from time harmonic sources – continued:

Useful expansion :

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}=ik\sum_{lm} j_l(kr_<)h_l(kr_>)Y_{lm}(\hat{\mathbf{r}})Y^*_{lm}(\hat{\mathbf{r}}')$$

Spherical Bessel function : $j_l(kr)$

Spherical Hankel function : $h_l(kr)=j_l(kr)+in_l(kr)$

$$\widetilde{\Phi}(\mathbf{r},\omega)=\widetilde{\Phi}_0(\mathbf{r},\omega)+\sum_{lm}\widetilde{\phi}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

$$\widetilde{\phi}_{lm}(r,\omega)=\frac{ik}{\varepsilon_0}\int d^3r'\,\widetilde{\rho}(\mathbf{r}',\omega)j_l(kr_<)h_l(kr_>)Y^*_{lm}(\hat{\mathbf{r}}')$$

Electromagnetic waves from time harmonic sources – continued:

Useful expansion :

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}=ik\sum_{lm} j_l(kr_<)h_l(kr_>)Y_{lm}(\hat{\mathbf{r}})Y^*_{lm}(\hat{\mathbf{r}}')$$

Spherical Bessel function : $j_l(kr)$

Spherical Hankel function : $h_l(kr)=j_l(kr)+in_l(kr)$

$$\widetilde{\mathbf{A}}(\mathbf{r},\omega)=\widetilde{\mathbf{A}}_0(\mathbf{r},\omega)+\sum_{lm}\widetilde{\mathbf{a}}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

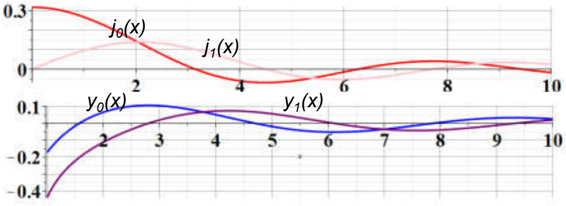
$$\widetilde{\mathbf{a}}_{lm}(r,\omega)=ik\mu_0\int d^3r'\,\widetilde{\mathbf{J}}(\mathbf{r}',\omega)j_l(kr_<)h_l(kr_>)Y^*_{lm}(\hat{\mathbf{r}}')$$

Forms of spherical Bessel and Hankel functions:

$$j_0(x)=\frac{\sin(x)}{x} \qquad h_0(x)=\frac{e^{ix}}{ix}$$

$$j_1(x)=\frac{\sin(x)}{x^2}-\frac{\cos(x)}{x} \qquad h_1(x)=-\left(1+\frac{i}{x}\right)\frac{e^{ix}}{x}$$

$$j_2(x)=\left(\frac{3}{x^3}-\frac{1}{x}\right)\sin(x)-\frac{3\cos(x)}{x^2} \qquad h_2(x)=i\left(1+\frac{3i}{x}-\frac{3}{x^2}\right)\frac{e^{ix}}{x}$$



Electromagnetic waves from time harmonic sources – continued:

$$\tilde{\Phi}(\mathbf{r},\omega)=\tilde{\Phi}_0(\mathbf{r},\omega)+\sum_{lm}\tilde{\phi}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

$$\tilde{\phi}_{lm}(r,\omega)=\frac{ik}{\varepsilon_0}\int d^3r'\,\tilde{\rho}(\mathbf{r}',\omega)j_l(kr_<)h_l(kr_>)Y^*{}_{lm}(\hat{\mathbf{r}}')$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\tilde{\mathbf{A}}_0(\mathbf{r},\omega)+\sum_{lm}\tilde{\mathbf{a}}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

$$\tilde{\mathbf{a}}_{lm}(r,\omega)=ik\mu_0\int d^3r'\,\tilde{\mathbf{J}}(\mathbf{r}',\omega)j_l(kr_<)h_l(kr_>)Y^*{}_{lm}(\hat{\mathbf{r}}')$$

For r >> (extent of source)

$$\tilde{\phi}_{lm}(r,\omega)\approx\frac{ik}{\varepsilon_0}h_l(kr)\int d^3r'\,\tilde{\rho}(\mathbf{r}',\omega)j_l(kr')Y^*{}_{lm}(\hat{\mathbf{r}}')$$

$$\tilde{\mathbf{a}}_{lm}(r,\omega)\approx ik\mu_0h_l(kr)\int d^3r'\,\tilde{\mathbf{J}}(\mathbf{r}',\omega)j_l(kr')Y^*{}_{lm}(\hat{\mathbf{r}}')$$

Electromagnetic waves from time harmonic sources – continued -- some details:

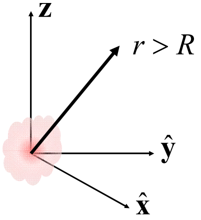
$$\tilde{\varphi}_{lm}(r,\omega)=\frac{ik}{\varepsilon_0}\int d^3r'\,\tilde{\rho}(\mathbf{r}',\omega)\,j_l(kr_<)h_l(kr_>)Y^*{}_{lm}(\hat{\mathbf{r}}')$$

$$=\frac{ik}{\varepsilon_0}\left(h_l(kr)\int_0^r r'^2\,dr'\rho_{lm}(\mathbf{r}',\omega)j_l(kr')+j_l(kr)\int_r^\infty r'^2\,dr'\rho_{lm}(\mathbf{r}',\omega)h_l(kr')\right)$$

where $\rho_{lm}(\mathbf{r}',\omega)\equiv\int d\Omega'\rho_{lm}(\mathbf{r}',\omega)Y^*_{lm}(\hat{\mathbf{r}}')$

note that for $r>R$, where $\rho_{lm}(\mathbf{r},\omega)\approx 0$,

$$\tilde{\varphi}_{lm}(r,\omega)\approx\frac{ik}{\varepsilon_0}h_l(kr)\int_0^\infty r'^2\,dr'\rho_{lm}(\mathbf{r}',\omega)j_l(kr')$$



Similar relationships can be written for $\tilde{\mathbf{a}}_{lm}(r,\omega)$ and $\tilde{\mathbf{J}}(\mathbf{r}',\omega)$.

Electromagnetic waves from time harmonic sources – continued:

For r >> (extent of source)

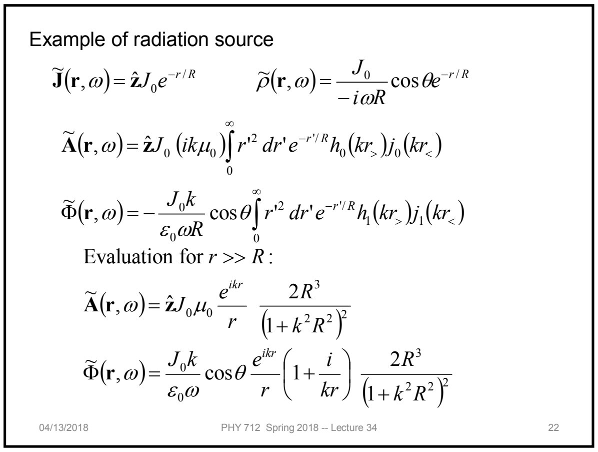
$$\tilde{\phi}_{lm}(r,\omega)\approx\frac{ik}{\varepsilon_0}h_l(kr)\int d^3r'\,\tilde{\rho}(\mathbf{r}',\omega)j_l(kr')Y^*{}_{lm}(\hat{\mathbf{r}}')$$

$$\tilde{\mathbf{a}}_{lm}(r,\omega)\approx ik\mu_0h_l(kr)\int d^3r'\,\tilde{\mathbf{J}}(\mathbf{r}',\omega)j_l(kr')Y^*{}_{lm}(\hat{\mathbf{r}}')$$

Note that $\tilde{\rho}(\mathbf{r}',\omega)$ and $\tilde{\mathbf{J}}(\mathbf{r}',\omega)$ are connected via the continuity condition: $-i\omega\,\tilde{\rho}(\mathbf{r},\omega)+\nabla\cdot\tilde{\mathbf{J}}(\mathbf{r},\omega)=0$

Example of radiation source

$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0e^{-r/R}\qquad\tilde{\rho}(\mathbf{r},\omega)=\frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$



Example of radiation source

$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0e^{-r/R} \qquad \tilde{\rho}(\mathbf{r},\omega)=\frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$

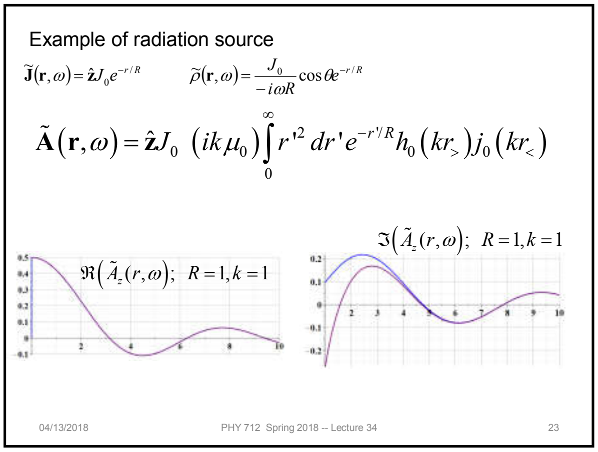
$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0\,(ik\mu_0)\int_0^\infty r'^2\,dr'\,e^{-r'/R}h_0(kr_>)j_0(kr_<)$$

$$\tilde{\Phi}(\mathbf{r},\omega)=-\frac{J_0k}{\varepsilon_0\omega R}\cos\theta\int_0^\infty r'^2\,dr'\,e^{-r'/R}h_1(kr_>)j_1(kr_<)$$

Evaluation for $r >> R$:

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0\mu_0\frac{e^{ikr}}{r}\;\frac{2R^3}{(1+k^2R^2)^2}$$

$$\tilde{\Phi}(\mathbf{r},\omega)=\frac{J_0k}{\varepsilon_0\omega}\cos\theta\,\frac{e^{ikr}}{r}\left(1+\frac{i}{kr}\right)\;\frac{2R^3}{(1+k^2R^2)^2}$$



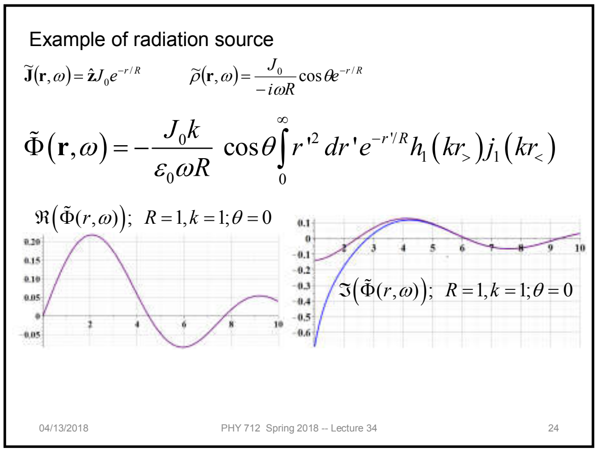
Example of radiation source

$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0e^{-r/R} \qquad \tilde{\rho}(\mathbf{r},\omega)=\frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0\;(ik\mu_0)\int_0^\infty r'^2\,dr'e^{-r'/R}h_0(kr_>)j_0(kr_<)$$

$\Re\left(\tilde{A}_z(r,\omega)\right);\;\; R=1,k=1$

$\Im\left(\tilde{A}_z(r,\omega)\right);\;\; R=1,k=1$



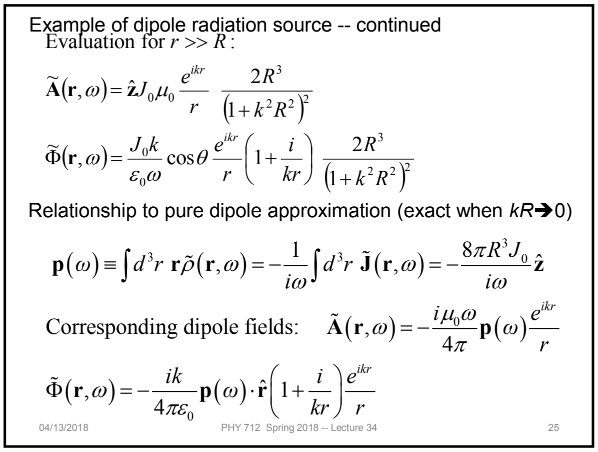
Example of radiation source

$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0e^{-r/R} \qquad \tilde{\rho}(\mathbf{r},\omega)=\frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$

$$\tilde{\Phi}(\mathbf{r},\omega)=-\frac{J_0k}{\varepsilon_0\omega R}\;\cos\theta\int_0^\infty r'^2\,dr'e^{-r'/R}h_1(kr_>)j_1(kr_<)$$

$\Re\left(\tilde{\Phi}(r,\omega)\right);\;\; R=1,k=1;\theta=0$

$\Im\left(\tilde{\Phi}(r,\omega)\right);\;\; R=1,k=1;\theta=0$



Example of dipole radiation source -- continued

Evaluation for $r >> R$:

$$\widetilde{\mathbf{A}}(\mathbf{r},\omega) = \hat{\mathbf{z}} J_0 \mu_0 \frac{e^{ikr}}{r} \; \frac{2R^3}{\left(1+k^2R^2\right)^2}$$

$$\widetilde{\Phi}(\mathbf{r},\omega) = \frac{J_0 k}{\varepsilon_0 \omega} \cos\theta \, \frac{e^{ikr}}{r} \left(1+\frac{i}{kr}\right) \; \frac{2R^3}{\left(1+k^2R^2\right)^2}$$

Relationship to pure dipole approximation (exact when *kR*→0)

$$\mathbf{p}(\omega) \equiv \int d^3r \; \mathbf{r}\tilde{\rho}(\mathbf{r},\omega) = -\frac{1}{i\omega}\int d^3r \; \tilde{\mathbf{J}}(\mathbf{r},\omega) = -\frac{8\pi R^3 J_0}{i\omega}\hat{\mathbf{z}}$$

Corresponding dipole fields: $\tilde{\mathbf{A}}(\mathbf{r},\omega) = -\frac{i\mu_0\omega}{4\pi}\mathbf{p}(\omega)\frac{e^{ikr}}{r}$

$$\tilde{\Phi}(\mathbf{r},\omega) = -\frac{ik}{4\pi\varepsilon_0}\mathbf{p}(\omega)\cdot\hat{\mathbf{r}}\left(1+\frac{i}{kr}\right)\frac{e^{ikr}}{r}$$

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