PHY 712 Electrodynamics
9-9:50 AM MWF Olin 105

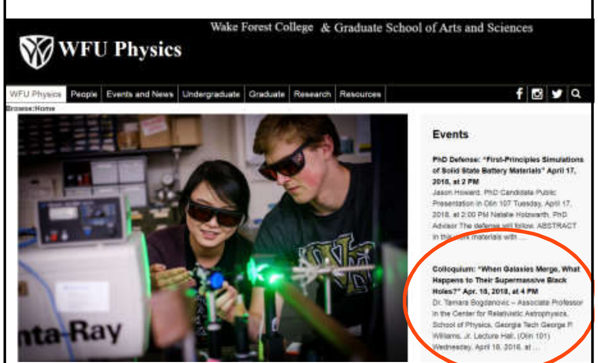
Plan for Lecture 36:

Special Topics in Electrodynamics:

Electromagnetic aspects of superconductivity

- London equations
- Brief mention of quantum mechanism
- Tunneling between two superconductors

23	Fri: 03/16/2018	Chap. 9	Harmonic radiation	#15	03/21/2018
24	Mon: 03/19/2018	Chap. 9 & 10	Interference and Scattering	#16	03/23/2018
25	Wed: 03/21/2018	Chap. 11	Special relativity	#17	03/26/2018
26	Fri: 03/23/2018	Chap. 11	Special relativity	#18	03/28/2018
27	Mon: 03/26/2018	Chap. 11	Special relativity		
28	Wed: 03/28/2018	Chap. 14	Radiation from accelerated particles		
	Fri: 03/30/2018	No class	*Good Friday*		
29	Mon: 04/02/2018	Chap. 14	Synchrotron radiation	#19	04/06/2018
30	Wed: 04/04/2018	Chap. 14	Synchrotron radiation	#20	04/09/2018
31	Fri: 04/06/2018	Chap. 15	Radiation from collisions of charged particles		
32	Mon: 04/09/2018	Chap. 13	Cherenkov radiation		
33	Wed: 04/11/2018		Review		
34	Fri: 04/13/2018		Review		
35	Mon: 04/16/2018		Special topic: Superconductivity		
36	Wed: 04/18/2018		Special topic: Superconductivity		
37	Fri: 04/20/2018		Special topic: Topics in optics		
38	Mon: 04/23/2018				
39	Wed: 04/25/2018				
	Fri: 04/27/2018		Presentations I		
	Mon: 04/30/2018		Presentations II		
	Wed: 05/02/2018		Presentations III		



Special topic: Electromagnetic properties of superconductors

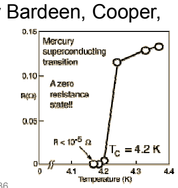
Ref:D. Teplitz, editor, Electromagnetism – paths to research, Plenum Press (1982); Chapter 1 written by Brian Schwartz and Sonia Frota-Pessoa

History:

1908 H. Kamerlingh Onnes successfully liquified He

1911 H. Kamerlingh Onnes discovered that Hg at 4.2 K has vanishing resistance

1957 Theory of superconductivity by Bardeen, Cooper, and Schrieffer



Behavior of superconducting material – exclusion of magnetic field according to the London model

Penetration length for superconductor: $\lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$

$B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

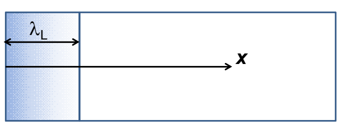
Vector potential for $\nabla \cdot \mathbf{A} = 0$:

$\mathbf{A} = \hat{\mathbf{y}} A_y(x) \qquad A_y(x) = -\lambda_L B_z(0) e^{-x/\lambda_L}$

Current density: $J_y(x) = \lambda_L \frac{ne^2}{mc} \mathrm{B}_z(0)\mathrm{e}^{-x/\lambda_L}$

$\Rightarrow \mathbf{J} + \frac{ne^2}{mc}\mathbf{A} = 0 \quad \text{or} \quad \frac{ne}{m}\left(m\mathbf{v} + \frac{e}{c}\mathbf{A}\right) = 0$

Typically, $\lambda_L \approx 10^{-7} m$



Behavior of magnetic field lines near superconductor

normal state: superconducting state:

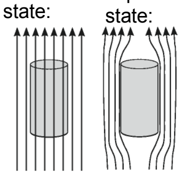
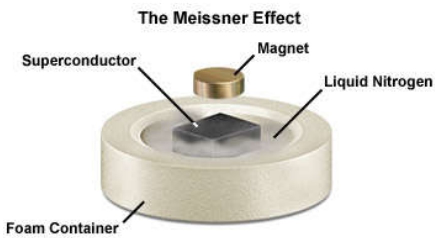


Figure 18.2 Exclusion of a weak external magnetic field from the interior of a superconductor.



PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. BARDEEN, L. N. COOPER,† AND J. R. SCHRIEFFER‡
Department of Physics, University of Illinois, Urbana, Illinois
(Received July 8, 1957)

$$G_S(0) - G_N(0) = -\frac{H_C^2}{8\pi} \approx -2N(E_F)(\hbar\omega)^2 e^{-2/(N(E_F)V)}$$

characteristic phonon energy

density of electron states at E_F

attraction potential between electron pairs

Temperature dependence of critical field

$$H_c(T) \approx H_c(0)\left(1 - \left(\frac{T}{T_c}\right)^2\right)$$

From PR **108**, 1175 (1957) Bardeen, Cooper, and Schrieffer, "Theory of Superconductivity"

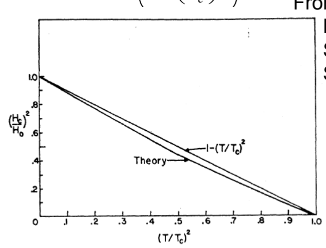


FIG. 2. Ratio of the critical field to its value at $T=0°K$ vs $(T/T_c)^2$. The upper curve is the $1-(T/T_c)^2$ law of the Gorter-Casimir theory and the lower curve is the law predicted by the theory in the weak-coupling limit. Experimental values generally lie between the two curves.

$$T_c \approx \frac{\hbar\omega}{k} e^{-2/(N(E_F)V)}$$

characteristic phonon energy

density of electron states at E_F

attraction potential between electron pairs

Type I superconductors:

$$H_c(T) = H_c(0)\left(1 - \frac{T^2}{T_c^2}\right)$$

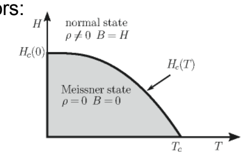


Figure 18.3 Schematic phase diagram illustrating normal and superconducting regions of a type-I superconductor.

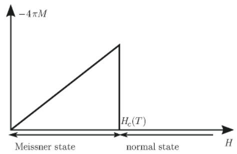


Figure 18.4 Magnetization versus applied field for type-I superconductors.

Type II superconductors

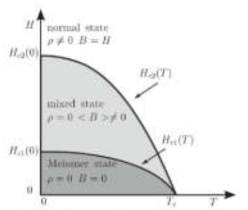


Figure 18.5 Schematic phase diagram illustrating normal, mixed and Meissner regions of a type-II superconductor (the vanishingly small resistivity of the mixed state occurs if flux lines are "pinned" by appropriate material defects); in the mixed state, $\langle B \rangle$ denotes the average magnetic field in the superconductor.

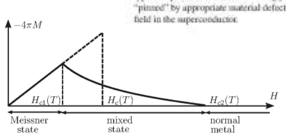


Figure 18.6 Magnetization versus applied field H for a type-II superconductor. The equivalent area construction of the thermodynamic field $H_c(T)$ is also illustrated.

Quantization of current flux associated with the superconducting state (Ref: Ashcroft and Mermin, ***Solid State Physics***)

From the London equations for the interior of the superconductor:

$$\left(m\mathbf{v} + \frac{e}{c}\mathbf{A}\right) = 0$$

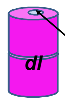
Now suppose that the current carrier is a pair of electrons characterized by a wavefunction of the form $\psi = |\psi| e^{i\phi}$

The quantum mechanical current associated with the electron pair is

$$\mathbf{j} = -\frac{e\hbar}{2mi}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right) - \frac{2e^2}{mc}\mathbf{A}|\psi|^2$$

$$= -\left(\frac{e\hbar}{m}\nabla\phi + \frac{2e^2}{mc}\mathbf{A}\right)|\psi|^2$$

Quantization of current flux associated with the superconducting state -- continued



Suppose a superconducting material has a cylindrical void. Evaluate the integral of the current in a closed path within the superconductor containing the void.

$$\oint \mathbf{j} \cdot d\mathbf{l} = 0 = -\oint \left(\frac{e\hbar}{m} \nabla \phi + \frac{2e^2}{mc} \mathbf{A} \right) |\psi|^2 \cdot d\mathbf{l}$$

$$\oint \mathbf{A} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \int \mathbf{B} \cdot d\mathbf{a} = \Phi \quad \text{magnetic flux}$$

$$\oint \nabla \phi \cdot d\mathbf{l} = 2\pi n \qquad \text{for some integer } n$$

$$\Rightarrow \text{Quantization of flux in the void: } |\Phi| = n\frac{hc}{2e} \equiv n\Phi_0$$

Such “vortex” fields can exist within type II superconductors.

Table 18.1 Critical temperature of some selected superconductors, and zero-temperature critical field. For elemental materials, the thermodynamic critical field $H_c(0)$ is given in gauss. For the compounds, which are type-II superconductors, the upper critical field $H_{c2}(0)$ is given in Tesla ($1\ T = 10^4$ G). The data for metallic elements and binary compounds of V and Nb are taken from G. Burns (1992). The data for MgB_2 and iron pnictide are taken from the references cited in the text, and refer to the two principal crystallographic axes. The data for the other compounds are taken from D. R. Harshman and A. P. Mills, Phys. Rev. B *45*, 10684 (1992)]. A more extensive list of data can be found in the mentioned references.

Metallic elements	$T_c(K)$	$H_c(0)$ (gauss)
Al	1.17	105
Sn	3.72	305
Pb	7.19	803
Hg	4.15	411
Nb	9.25	2060
V	5.40	1410
Binary compounds	$T_c(K)$	$H_{c2}(0)$ (Tesla)
V_3Ga	16.5	27
V_3Si	17.1	25
Nb_3Al	20.3	34
Nb_3Ge	23.3	38
MgB_2	40	≈ 5; ≈ 20
Other compounds	$T_c(K)$	$H_{c2}(0)$ (Tesla)
UPt_3 (heavy fermion)	0.53	2.1
$PbMo_6S_8$ (Chevrel phase)	12	55
$\kappa-[BEDT-TTF]_2Cu[NCS]_2$ (organic phase)	10.5	≈ 10
Rb_2CsC_{60} (fullerene)	31.3	≈ 30
$NdFeAsO_{0.7}F_{0.3}$ (iron pnictide)	47	≈ 30; ≈ 50
Cuprate oxides	$T_c(K)$	$H_{c2}(0)$ (Tesla)
$La_{2-x}Sr_xCuO_4$ ($x \approx 0.15$)	38	≈ 45
$YBa_2Cu_3O_7$	92	≈ 140
$Bi_2Sr_2CaCu_2O_8$	89	≈ 107
$Tl_2Ba_2Ca_2Cu_3O_{10}$	125	≈ 75

Crystal structure of one of the high temperature superconductors

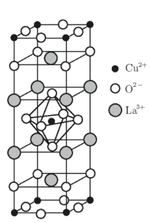


Figure 18.1 Crystal structure of the ceramic material La_2CuO_4. Appropriately doped, lanthanum-based cuprates opened the path to high-T_c superconductivity in 1986.

Some details of single vortex in type II superconductor

London equation without vortices:

$$\frac{4\pi}{c}\nabla\times\mathbf{J} = -\nabla^2\mathbf{B} = -\frac{1}{\lambda_L^2}\mathbf{B} \qquad \text{where} \quad \lambda_L^2 \equiv \frac{mc^2}{4\pi n e^2}$$

Equation for field with single quantum of vortex along z-axis:

$$\nabla^2\mathbf{B} - \frac{1}{\lambda_L^2}\mathbf{B} = -\frac{\Phi_0}{\lambda_L^2}\hat{\mathbf{z}}\delta(\mathbf{r}) \qquad \Phi_0 = \frac{hc}{2e} \qquad \mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$$

Solution: $\mathbf{B}(\mathbf{r}) = \hat{\mathbf{z}}\frac{\Phi_0}{2\pi\lambda_L^2}K_0\left(\frac{r}{\lambda_L}\right)$

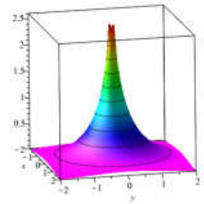
Check:

For $r > 0$ $\quad \left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{\lambda_L^2}\right)K_0\left(\frac{r}{\lambda_L}\right) = 0$

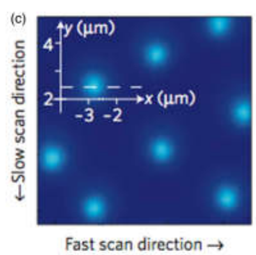
For $r \to 0$ $\quad 2\pi\int_0^r dr'\, r'\left(\frac{d^2}{dr'^2} + \frac{1}{r'}\frac{d}{dr'} - \frac{1}{\lambda_L^2}\right)K_0\left(\frac{r'}{\lambda_L}\right) = -2\pi$

Since $K_0(u) \underset{u\to 0}{\approx} -\ln u$

$\mathbf{B}(\mathbf{r}) = \hat{\mathbf{z}}\frac{\Phi_0}{2\pi\lambda_L^2}K_0\left(\frac{r}{\lambda_L}\right)$



Scanning probe images of vortices in YBCO at 22 K

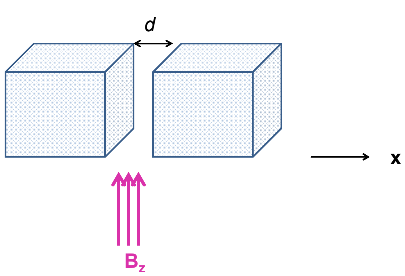


Fundamental studies of superconductors using scanning magnetic imaging

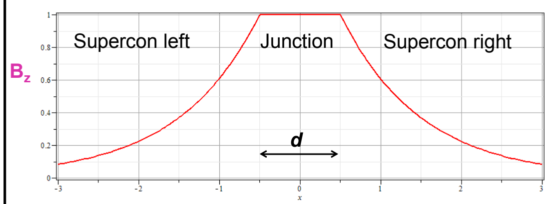
J R Kirtley

Mechanism for vortex detection --

Josephson junction -- tunneling current between two superconductors (Ref. Teplitz, ***Electromagnetism*** (1982))



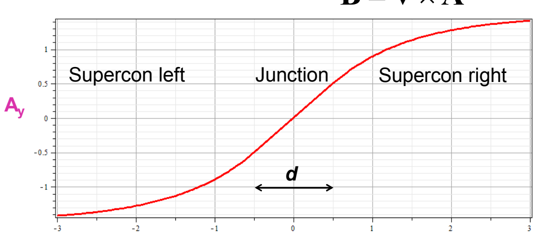
Josephson junction -- continued



$$B_z(x) = \begin{cases} B_0 e^{(x+d/2)/\lambda_L} & x < -d/2 \\ B_0 & -d/2 < x < d/2 \\ B_0 e^{(-x+d/2)/\lambda_L} & x > d/2 \end{cases}$$

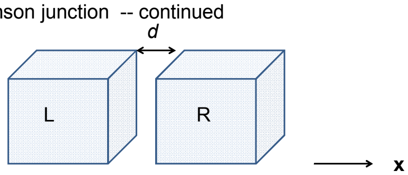
Josephson junction -- continued

$$\mathbf{B} = \nabla \times \mathbf{A}$$



$$A_y(x) = \begin{cases} B_0\left(\lambda_L e^{(x+d/2)/\lambda_L} - (\lambda_L + d/2)\right) & x < -d/2 \\ B_0 x & -d/2 < x < d/2 \\ B_0\left(-\lambda_L e^{(-x+d/2)/\lambda_L} + (\lambda_L + d/2)\right) & x > d/2 \end{cases}$$

Josephson junction -- continued



Quantum mechanical model of tunnelling current

Let $\Psi_L = \Psi_L{}^0 e^{i\phi_L}$ denote a wavefunction for a Cooper pair on left

Let $\Psi_R = \Psi_R{}^0 e^{i\phi_R}$ denote a wavefunction for a Cooper pair on right

$$-i\hbar \frac{\partial \Psi_L}{\partial t} = E_L \Psi_L + \varepsilon \Psi_R$$

$$-i\hbar \frac{\partial \Psi_R}{\partial t} = E_R \Psi_R + \varepsilon \Psi_L$$

Coupling parameter

Josephson junction -- continued

Solving for wavefunctions

$$\frac{1}{2}\frac{\partial|\Psi_L^{\,0}|^2}{\partial t} + i|\Psi_L^{\,0}|^2\frac{\partial\phi_L}{\partial t} = -\frac{i}{\hbar}\left(E_L|\Psi_L^{\,0}|^2 + \varepsilon\Psi_L^{\,0}\Psi_R^{\,0}e^{i(\phi_R-\phi_L)}\right)$$

$$\frac{1}{2}\frac{\partial|\Psi_R^{\,0}|^2}{\partial t} + i|\Psi_R^{\,0}|^2\frac{\partial\phi_R}{\partial t} = -\frac{i}{\hbar}\left(E_R|\Psi_R^{\,0}|^2 + \varepsilon\Psi_L^{\,0}\Psi_R^{\,0}e^{-i(\phi_R-\phi_L)}\right)$$

$$|\Psi_L^{\,0}|^2 \equiv n_L \qquad |\Psi_R^{\,0}|^2 \equiv n_R \qquad \phi_{LR} \equiv \phi_L - \phi_R$$

$$\frac{\partial n_L}{\partial t} = -\frac{\partial n_R}{\partial t} = -\frac{2\varepsilon}{\hbar}\sqrt{(n_L n_R)}\sin\phi_{LR}$$

$$\frac{\partial\phi_L}{\partial t} = -\frac{E_L}{\hbar} - \varepsilon\sqrt{\frac{n_R}{n_L}}\cos\phi_{LR}$$

$$\frac{\partial\phi_R}{\partial t} = -\frac{E_R}{\hbar} - \varepsilon\sqrt{\frac{n_L}{n_R}}\cos\phi_{LR}$$

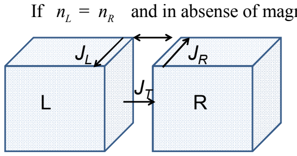
Note that

$$\frac{\partial\phi_{LR}}{\partial t} = -\frac{1}{\hbar}\left(E_L - E_R\right)$$

Josephson junction -- continued

Tunneling current: $J_T = 2e\frac{\partial n_L}{\partial t} = -\frac{4e\varepsilon}{\hbar}\sqrt{(n_L n_R)}\sin\phi_{LR}$

If $n_L = n_R$ and in absense of magnetic field, $\phi_{LR}(t) = \phi_{LR}(0) + \frac{E_R - E_L}{\hbar}t$



$$\Rightarrow J_L = \frac{2e}{2m}|\Psi_L^{\,0}|^2\left(\hbar\nabla\phi_L - \frac{2e}{c}\mathbf{A}\right)$$

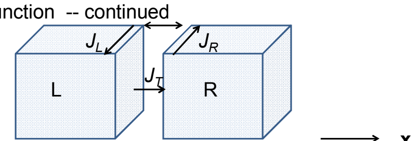
$$\Rightarrow J_R = \frac{2e}{2m}|\Psi_R^{\,0}|^2\left(\hbar\nabla\phi_R - \frac{2e}{c}\mathbf{A}\right)$$

→ **x**

Relationship between superconductor currents J_L and J_R and tunneling current. Within the superconductor, denote the generalized current operator acting on pair wavefunction $\Psi = \Psi^0 e^{i\phi}$

$$J = \frac{2e}{2}\left(\Psi^*(\hat{\mathbf{v}}\Psi) + \Psi(\hat{\mathbf{v}}\Psi)^*\right) \text{ with } \hat{\mathbf{v}} \equiv \frac{1}{2m}\left(-i\hbar\nabla - \frac{2e}{c}\mathbf{A}\right)$$

Josephson junction -- continued



Tunneling current: $J_T = 2e\frac{\partial n_L}{\partial t} = -\frac{4e\varepsilon}{\hbar}\sqrt{(n_L n_R)}\sin\phi_{LR}$

If $n_L = n_R = n$ and in absense of magnetic field, $\phi_{LR}(t) = \phi_{LR}(0) + \frac{E_R - E_L}{\hbar}t$

$\Rightarrow$ Constant Josephson tunneling current for $E_R - E_L = 0$

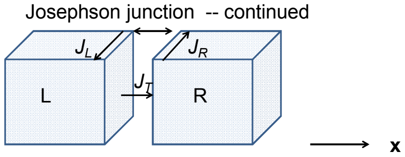
$$J_T = -\frac{4e\varepsilon}{\hbar}n\sin\phi_{LR}(0)$$

$\Rightarrow$ Oscillatory Josephson tunneling current for $E_R - E_L = 2eV$

$$J_T = -\frac{4e\varepsilon}{\hbar}n\sin\left(\phi_{LR}(0) + \frac{2eV}{\hbar}t\right)$$

Method for precise measurement of $e/\hbar$

Josephson junction -- continued



$$\Rightarrow J_L = \frac{2e}{2m}\left|\Psi_L^{\,0}\right|^2\left(\hbar\nabla\phi_L - \frac{2e}{c}\mathbf{A}\right) \equiv 2en_L\mathbf{v}_L$$

$$\Rightarrow J_R = \frac{2e}{2m}\left|\Psi_R^{\,0}\right|^2\left(\hbar\nabla\phi_R - \frac{2e}{c}\mathbf{A}\right) \equiv 2en_R\mathbf{v}_R$$

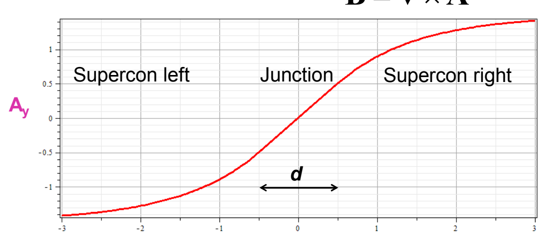
$$\nabla\phi_L = \frac{2m\mathbf{v}_L}{\hbar} + \frac{2e}{\hbar c}\mathbf{A} \qquad \nabla\phi_R = \frac{2m\mathbf{v}_R}{\hbar} + \frac{2e}{\hbar c}\mathbf{A}$$

Tunneling current: $J_T = 2e\frac{\partial n_L}{\partial t} = -\frac{4e\varepsilon}{\hbar}\sqrt{(n_L n_R)}\sin\phi_{LR}$

Need to evaluate ϕ_{LR} in presence of magnetic field

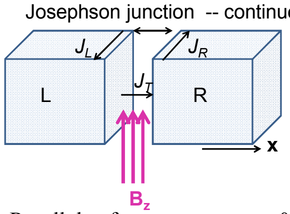
Josephson junction -- continued

$$\mathbf{B} = \nabla\times\mathbf{A}$$



$$A_y(x) = \begin{cases} B_0\left(\lambda_L e^{(x+d/2)/\lambda_L} - (\lambda_L + d/2)\right) & x < -d/2 \\ B_0 x & -d/2 < x < d/2 \\ B_0\left(-\lambda_L e^{(-x+d/2)/\lambda_L} + (\lambda_L + d/2)\right) & x > d/2 \end{cases}$$

Josephson junction -- continued



Tunneling current:

$$J_T = -\frac{4e\varepsilon}{\hbar}\sqrt{(n_L n_R)}\sin\phi_{LR}$$

$$\mathbf{v}_L = \left(\frac{\hbar}{2m}\nabla\phi_L - \frac{2e}{2mc}\mathbf{A}\right)$$

$$\mathbf{v}_R = \left(\frac{\hbar}{2m}\nabla\phi_R - \frac{2e}{2mc}\mathbf{A}\right)$$

Recall that for $x\to-\infty$ $\mathbf{v}_L\to 0$ and $\mathbf{A}\to-(\lambda_L + d/2)B_0\hat{\mathbf{y}}$

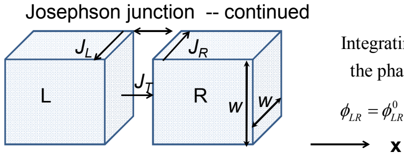
for $x\to\infty$ $\mathbf{v}_R\to 0$ and $\mathbf{A}\to(\lambda_L + d/2)B_0\hat{\mathbf{y}}$

Integrating the difference of the phase angles along y:

$$\phi_{LR} \equiv \phi_L\left(-\tfrac{d}{2}, y\right) - \phi_L\left(-\tfrac{d}{2}, 0\right) - \phi_R\left(\tfrac{d}{2}, y\right) + \phi_R\left(\tfrac{d}{2}, 0\right)$$

$$= \phi_{LR}^0 + \frac{2e}{\hbar c}B_0(2\lambda_L + d)y$$

Josephson junction -- continued



Integrating the difference of the phase angles along y:

$$\phi_{LR} = \phi^0_{LR} + \frac{2e}{\hbar c} B_0 (2\lambda_L + d) y$$

Tunneling current density: $J_T = \frac{4e\varepsilon}{\hbar} n_L \sin\phi_{LR} = J_{T0} \sin\left(\phi^0_{LR} + \frac{2e}{\hbar c} B_0 (2\lambda_L + d) y\right)$

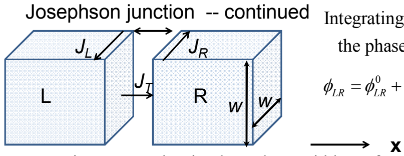
Integrating current density throughout width w of superconductors

$$I_T = w \int_{-w/2}^{w/2} J_T dy$$

$$= \frac{w J_{T0}}{\frac{2e}{\hbar c} B_0 (2\lambda_L + d)} \left(\cos\left(\phi^0_{LR} - \frac{ew}{\hbar c} B_0 (2\lambda_L + d)\right) - \cos\left(\phi^0_{LR} + \frac{ew}{\hbar c} B_0 (2\lambda_L + d)\right)\right)$$

Define: $\Phi = B_0 w (2\lambda_L + d)$ and $\Phi^0 = \frac{2\pi\hbar c}{2e}$ $\Rightarrow \frac{ew}{\hbar c} B_0 (2\lambda_L + d) = \frac{\pi\Phi}{\Phi^0}$

Josephson junction -- continued



Integrating the difference of the phase angles along y:

$$\phi_{LR} = \phi^0_{LR} + \frac{2e}{\hbar c} B_0 (2\lambda_L + d) y$$

Integrating current density throughout width w of superconductors

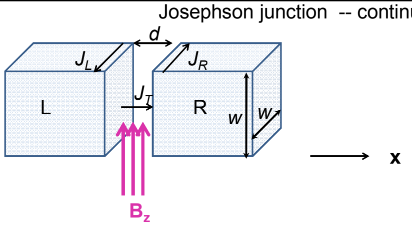
$$I_T = w \int_{-w/2}^{w/2} J_T dy =$$

$$= \frac{w J_{T0}}{\frac{2e}{\hbar c} B_0 (2\lambda_L + d)} \left(\cos\left(\phi^0_{LR} - \frac{ew}{\hbar c} B_0 (2\lambda_L + d)\right) - \cos\left(\phi^0_{LR} + \frac{ew}{\hbar c} B_0 (2\lambda_L + d)\right)\right)$$

$$= w^2 J_{T0} \sin(\phi^0_{LR}) \frac{\sin(\pi\Phi/\Phi^0)}{\pi\Phi/\Phi^0}$$

where $\Phi = B_0 w (2\lambda_L + d)$ and $\Phi^0 = \frac{2\pi\hbar c}{2e}$

Josephson junction -- continued



Tunneling current density: $J_T = \frac{4e\varepsilon}{\hbar} n_L \sin\phi_{LR}$

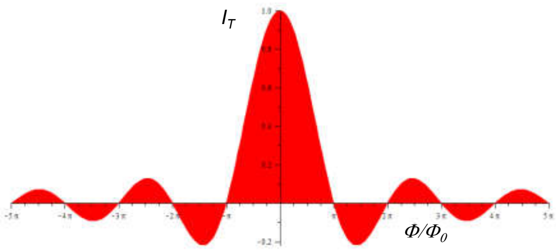
Integrating current density throughout width w of superconductors

$$I_T = w \int_{-w/2}^{w/2} J_T dy = w^2 J_{T0} \sin(\phi^0_{LR}) \frac{\sin(\pi\Phi/\Phi^0)}{\pi\Phi/\Phi^0}$$

where $\Phi = B_0 w (2\lambda_L + d)$ and $\Phi^0 = \frac{2\pi\hbar c}{2e}$

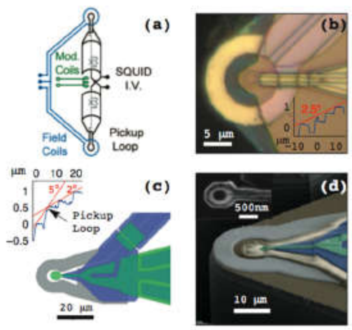
Josephson junction -- continued

SQUID =*superconducting quantum interference device*



Note: This very sensitive "SQUID" technology has been used in scanning probe techniques. See for example, J. R. Kirtley, Rep. Prog. Physics 73, 126501 (2010).

Scanning SQIUD microscopy
Ref. J. R. Kirtley, Rep. Prog. Phys. **73** 126501 (2010)



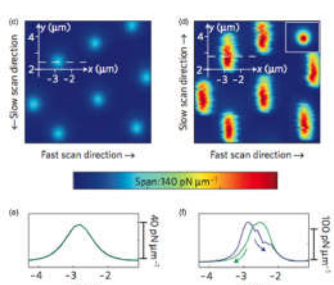


Figure 31. MFM imaging and manipulation of individual vortices in YBCO at $T = 22.3$ K. (*a*), (*b*) Schematic drawings of an MFM tip (triangle) that attracts a vortex (thick lines) in a sample with randomly distributed pinning sites (dots): at large heights (*a*) the force between tip and vortex is too weak to move the vortex; at small heights (*b*) the vortex moves right and then left as the tip rasters over it. (*c*) MFM image taken at $z = 420$ nm (maximum applied lateral force $F^{lat}_{max} = 6$ nN). (*d*) $z = 170$ nm ($F^{lat}_{max} = 12$ nN). Inset: scan taken at a comparable height at $T = 5.2$ K. (*e*) Line cut through the data in (*c*) along the dashed line, showing the signal from a stationary vortex (solid curve with peak to the left, blue online). Overlapping it is a line cut from the reverse scan (solid curve with peak to the right, green online). (*f*) Line cut through the data in (*d*) along the dashed line, showing a typical signal from a dragged vortex. Figure reprinted by permission from [312]. Copyright 2008 Macmillan Publishers Ltd.

NATURE | VOL 387 | 29 MAY 1997 pg. 481

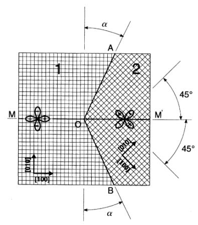


Figure 1 Schematic diagram illustrating the geometrical configuration of a tetracrystal (100) $SrTiO_3$ substrate, which effectively consists of two crystals rotated about the normal to the substrate plane by $\pi/4$ with respect to each other. For a given total misorientation angle $\theta = \theta_1 + \theta_2$, the pair tunnelling current is maximized[9] for a symmetric grain boundary $\theta_1 = \theta_2 = \pi/8$. We chose the angle $\alpha = \theta_1$ between the vertical ([010] in grain 1) and the grain boundaries (OA and OB) as 25°. Also shown are the polar plots of the assumed $d_{x^2-y^2}$ gap functions aligned with the crystallographic axes in the substrate.

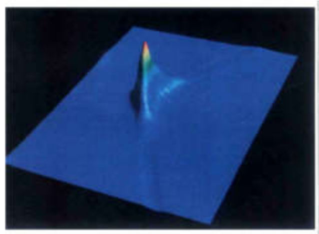


Figure 3 A three-dimensional presentation of the SSM image of the Josephson vortex trapped at the wedge tip in a Tl2201 blanket film deposited on a tetracrystal (100) $SrTiO_3$ substrate of the geometry shown in Fig. 1. The sample was cooled in a field of <1 mG and imaged at 4.2 K.