PHY 712 Electrodynamics
9-9:50 AM Olin 105

Plan for Lecture 19:

Complete reading of Chapter 7

1. **Comments on reflectivity of plane waves**
2. **Summary of complex response functions for electromagnetic fields**

6	Mon: 01/29/2018	Chap. 2	Method of image charges	#5	01/31/2018
7	Wed: 01/31/2018	Chap. 2 & 3	Cylindrical and spherical geometries	#6	02/02/2018
8	Fri: 02/02/2018	Chap. 3 & 4	Multipole analysis	#7	02/07/2018
9	Mon: 02/05/2018	Chap. 4	Dipoles and Dielectrics	#8	02/09/2018
10	Wed: 02/07/2018	Chap. 4	Dipoles and Dielectrics		
11	Fri: 02/09/2018	Chap. 1-4	Review		
12	Mon: 02/12/2018	Chap. 5	Magnetostatics	#9	02/16/2018
13	Wed: 02/14/2018	Chap. 5	Magnetic dipoles and hyperfine interaction	#10	02/19/2018
14	Fri: 02/16/2018	Chap. 5	Magnetic dipoles and dipolar fields	#11	02/21/2018
15	Mon: 02/19/2018	Chap. 6	Maxwell's Equations	#12	02/23/2018
16	Wed: 02/21/2018	Chap. 6	Electromagnetic energy and forces		
17	Fri: 02/23/2018	Chap. 7	Electromagnetic plane waves		
18	Mon: 02/26/2018	Chap. 7	Dielectric response of media		Exam
19	Wed: 02/28/2018	Chap. 7	Complex dielectrics		Exam
20	Fri: 03/02/2018	Chap. 1-7	Review		Exam due
	Mon: 03/05/2018	No class	*Spring Break*		
	Wed: 03/07/2018	No class	*Spring Break*		
	Fri: 03/09/2018	No class	*Spring Break*		
21	Mon: 03/12/2018				
22	Wed: 03/14/2018				
23	Fri: 03/16/2018				

Some comments on the Fresnel Equations

1. Different behaviors of *s* and *p* polarization
2. Brewster's angle
3. Total internal reflection

Review: Electromagnetic plane waves in isotropic medium with real permeability and permittivity: μ ε.

$$\mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\frac{\omega}{c}(n\hat{\mathbf{k}}\cdot\mathbf{r} - ct)}\right) \qquad n^2 = c^2\mu\varepsilon$$

$$\mathbf{B}(\mathbf{r},t) = \frac{n}{c}\hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r},t) = \sqrt{\mu\varepsilon}\,\hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r},t)$$

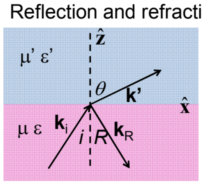
Poynting vector for plane electromagnetic waves:

$$\langle \mathbf{S} \rangle_{avg} = \frac{n|\mathbf{E}_0|^2}{2\mu c}\hat{\mathbf{k}} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|\mathbf{E}_0|^2\hat{\mathbf{k}}$$

Energy density for plane electromagnetic waves:

$$\langle u \rangle_{avg} = \frac{1}{2}\varepsilon|\mathbf{E}_0|^2$$

Review:
Reflection and refraction between two isotropic media

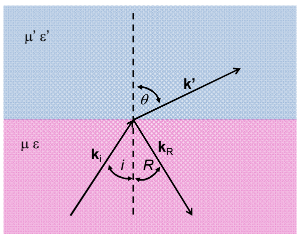


Reflectance, transmittance:

$$\mathrm{R} = \frac{\mathbf{S}_R\cdot\hat{\mathbf{z}}}{\mathbf{S}_i\cdot\hat{\mathbf{z}}} = \left|\frac{E_{0R}}{E_{0i}}\right|^2 \qquad \mathrm{T} = \frac{\mathbf{S}'\cdot\hat{\mathbf{z}}}{\mathbf{S}_i\cdot\hat{\mathbf{z}}} = \left|\frac{E'_0}{E_{0i}}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'}\frac{\cos\theta}{\cos i}$$

Note that $\mathrm{R} + \mathrm{T} = 1$

Reflection and refraction of plane electromagnetic waves at a plane interface between dielectrics (assumed to be lossless)



$$n' = \varepsilon' \mu'$$
$$n = \varepsilon\mu$$
$$i = R$$
$$n \sin i = n' \sin\theta$$
$$|\mathbf{k}_i| = |\mathbf{k}_R| = n\frac{\omega}{c}$$
$$|\mathbf{k}'| = n'\frac{\omega}{c}$$

For s-polarization (E in plane of incidence)

$$\frac{E_{0R}}{E_{0i}} = \frac{n\cos i - \frac{\mu}{\mu'} n'\cos\theta}{n\cos i + \frac{\mu}{\mu'} n'\cos\theta} \qquad \frac{E'_0}{E_{0i}} = \frac{2n\cos i}{n\cos i + \frac{\mu}{\mu'} n'\cos\theta}$$

$$\text{Note that: } n'\cos\theta = \sqrt{n'^2 - n^2\sin^2 i}$$

For p-polarization (E perpendicular to plane of incidence)

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'} n'\cos i - n\cos\theta}{\frac{\mu}{\mu'} n'\cos i + n\cos\theta} \qquad \frac{E'_0}{E_{0i}} = \frac{2n\cos i}{\frac{\mu}{\mu'} n'\cos i + n\cos\theta}$$

$$\text{Note that: } n'\cos\theta = \sqrt{n'^2 - n^2\sin^2 i}$$

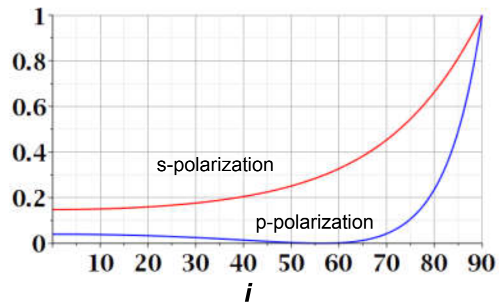
Reflectance for s-polarization

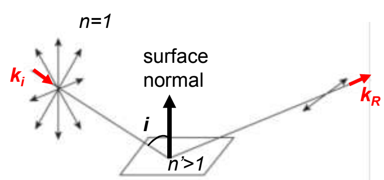
$$R_s = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{n\cos i - \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}{n\cos i + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}\right|^2$$

Reflectance for p-polarization

$$R_p = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'} n'\cos i - \frac{n}{n'}\sqrt{n'^2 - n^2\sin^2 i}}{\frac{\mu}{\mu'} n'\cos i + \frac{n}{n'}\sqrt{n'^2 - n^2\sin^2 i}}\right|^2$$

Example for $n = 1$ and $n' = 1.5$





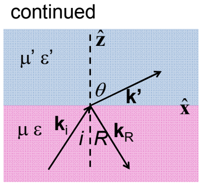
Polarization due to reflection from a refracting surface

Brewster's angle: for $i = i_B$, $R_p(i_B) = 0$

$$R_p = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'} n' \cos i - \frac{n}{n'}\sqrt{n'^2 - n^2 \sin^2 i}}{\frac{\mu}{\mu'} n' \cos i + \frac{n}{n'}\sqrt{n'^2 - n^2 \sin^2 i}}\right|^2$$

For $\mu' = \mu$, $i_B = \tan^{-1}\left(\frac{n'}{n}\right)$

Reflection and refraction between two isotropic media -- continued



For each wave:

$$\mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\frac{\omega}{c}(n\hat{\mathbf{k}}\cdot\mathbf{r} - ct)}\right) \qquad n^2 = c^2\mu\varepsilon$$

$$\mathbf{B}(\mathbf{r},t) = \frac{n}{c}\hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r},t) = \sqrt{\mu\varepsilon}\,\hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r},t)$$

Matching condition at interface:

$$n' \cos\theta = \sqrt{n'^2 - n^2 \sin^2 i}$$

Total internal reflection:

If $n > n'$, for $i > i_0 \equiv \sin^{-1}\left(\frac{n'}{n}\right)$,

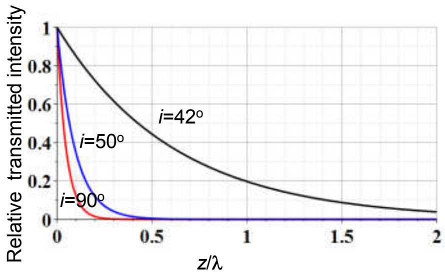
refracted field no longer propagates in medium $\mu' \varepsilon'$

$$n' \cos\theta = i\sqrt{n^2 \sin^2 i - n'^2} = i\, n \sqrt{\frac{\sin^2 i}{\sin^2 i_0} - 1}$$

$$\mathbf{E}'(\mathbf{r},t) = e^{-\left(\frac{n\omega}{c}\sqrt{\frac{\sin^2 i}{\sin^2 i_0} - 1}\right)z}\, \Re\left(\mathbf{E}'_0\, e^{i\frac{\omega}{c}(n\hat{\mathbf{k}}_\parallel\cdot\mathbf{r} - ct)}\right)$$

Example of total internal reflection

$n'=1$ and $n=1.5$ ➔ $i_0 = \sin^{-1}(1/1.5)=41.81°$



Transmitted illumination confined within a few wavelengths of the surface.

TIRF (total internal reflection fluorescence)

www.nikon.com/products/microscope-solutions/bioscience.../nikon_note_10_lr.pdf

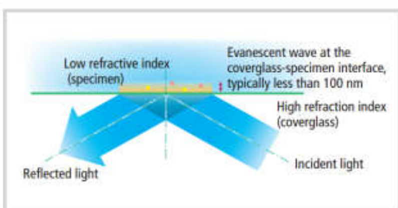


Figure 1: Creation of an evanescent wave at the coverglass-specimen interface

Design of TIRF device using laser and high power lens

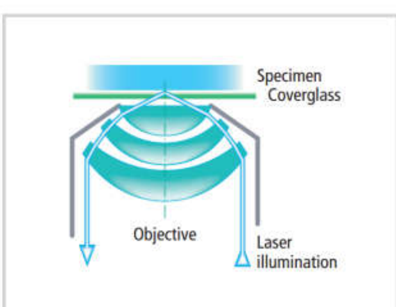


Figure 2: Through-the-lens laser TIRF.

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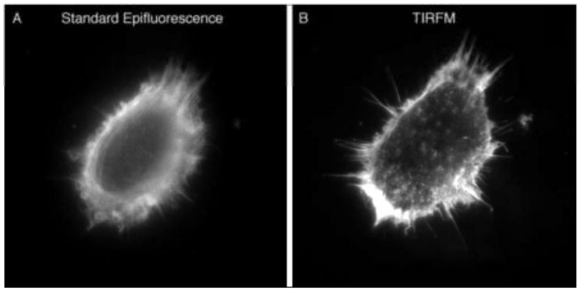
Curr Protoc Cytom. 2009 Oct; 0 12: Unit12.18.
doi: 10.1002/0471142956.cy1218s50

Figure 1



Special case: normal incidence (i=0, θ=0)

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'}n' - n}{\frac{\mu}{\mu'}n' + n} \qquad \frac{E'_0}{E_{0i}} = \frac{2n}{\frac{\mu}{\mu'}n' + n}$$

Reflectance, transmittance:

$$\mathrm{R} = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n' - n}{\frac{\mu}{\mu'}n' + n}\right|^2$$

$$\mathrm{T} = \left|\frac{E'_0}{E_{0i}}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'} = \left|\frac{2n}{\frac{\mu}{\mu'}n' + n}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'}$$

Extension to complex refractive index $n = n_R + i\, n_I$

Suppose $\mu = \mu'$, n = real, $n' = n'_R + i n'_I$

Reflectance at normal incidence:

$$\mathrm{R} = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n' - n}{\frac{\mu}{\mu'}n' + n}\right|^2 = \frac{(n'_R - n)^2 + (n'_I)^2}{(n'_R + n)^2 + (n'_I)^2}$$

Note that for $n'_I >> |n'_R \pm n|$:

$$\mathrm{R} \approx 1$$

Origin of imaginary contributions to permittivity --
Review: Drude model dielectric function:

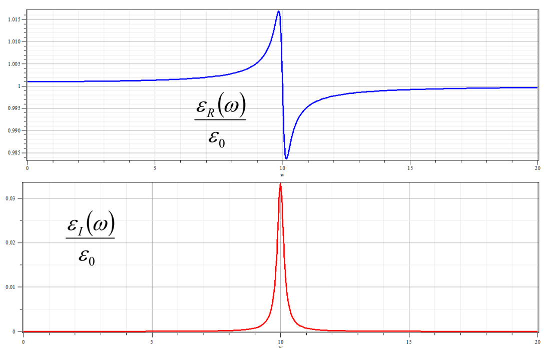
$$\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i} \frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$$

$$= \frac{\varepsilon_R(\omega)}{\varepsilon_0} + i\frac{\varepsilon_I(\omega)}{\varepsilon_0}$$

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i} \frac{\omega_i^2 - \omega^2}{\left(\omega_i^2 - \omega^2\right)^2 + \omega^2\gamma_i^2}$$

$$\frac{\varepsilon_I(\omega)}{\varepsilon_0} = N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i} \frac{\omega\gamma_i}{\left(\omega_i^2 - \omega^2\right)^2 + \omega^2\gamma_i^2}$$

Drude model dielectric function:



Drude model dielectric function – some analytic properties:

$$\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i} \frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$$

For $\omega >> \omega_i$

$$\frac{\varepsilon(\omega)}{\varepsilon_0} \approx 1 - \frac{1}{\omega^2}\left(N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i}\right)$$

$$\equiv 1 - \frac{\omega_P^2}{\omega^2}$$

Analysis for Drude model dielectric function – continued --

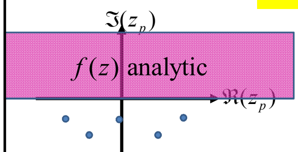
Analytic properties:

$$f(z)=\frac{\varepsilon(z)}{\varepsilon_0}-1=N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{1}{\omega_i^2-z^2-iz\gamma_i}$$

$f(z)$ has poles z_P at $\omega_i^2-z_P{}^2-iz_P\gamma_i=0$

$$z_P=-i\frac{\gamma_i}{2}\pm\sqrt{\omega_i^2-\left(\frac{\gamma_i}{2}\right)^2}$$

Note that $\Im(z_P)\leq 0 \;\Rightarrow f(z)$ is analytic for $\Im(z_P)>0$



Because of these analytic properties, Cauchy's integral theorem results in:

Kramers-Kronig transform – for dielectric function:

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0}-1=\frac{1}{\pi}P\int_{-\infty}^{\infty}d\omega'\frac{\varepsilon_I(\omega')}{\varepsilon_0}\frac{1}{\omega'-\omega}$$

$$\frac{\varepsilon_I(\omega)}{\varepsilon_0}=-\frac{1}{\pi}P\int_{-\infty}^{\infty}d\omega'\left(\frac{\varepsilon_R(\omega')}{\varepsilon_0}-1\right)\frac{1}{\omega'-\omega}$$

with $\varepsilon_R(-\omega)=\varepsilon_R(\omega);\quad \varepsilon_I(-\omega)=-\varepsilon_I(\omega)$

Further comments on analytic behavior of dielectric function

"Causal" relationship between **E** and **D** fields:

$$\mathbf{D}(\mathbf{r},t)=\varepsilon_0\left\{\mathbf{E}(\mathbf{r},t)+\int_0^{\infty}d\tau\, G(\tau)\mathbf{E}(\mathbf{r},t-\tau)\right\}$$

$$\frac{\varepsilon(\omega)}{\varepsilon_0}-1=\int_0^{\infty}d\tau\, G(\tau)e^{i\omega\tau}$$

Further comments on analytic behavior of dielectric function

"Causal" relationship between **E** and **D** fields:

$$\mathbf{D}(\mathbf{r},t) = \varepsilon_0 \left\{ \mathbf{E}(\mathbf{r},t) + \int_0^\infty d\tau \; G(\tau)\mathbf{E}(\mathbf{r},t-\tau) \right\}$$

$$G(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(\frac{\varepsilon(\omega)}{\varepsilon_0} - 1 \right) e^{-i\omega\tau} d\omega \qquad \frac{\varepsilon(\omega)}{\varepsilon_0} - 1 = \int_0^\infty d\tau \; G(\tau) e^{i\omega\tau}$$

For $\dfrac{\varepsilon(\omega)}{\varepsilon_0} - 1 = \dfrac{N}{\varepsilon_0}\sum_i f_i \dfrac{q_i^2}{m_i} \dfrac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$

$$G(\tau) = \frac{N}{\varepsilon_0}\sum_i f_i \frac{q_i^2}{m_i} e^{-\gamma_i\tau/2} \frac{\sin(\nu_i\tau)}{\nu_i}\Theta(\tau)$$

where $\nu_i \equiv \sqrt{\omega_i^2 - \gamma_i^2/4}$