PHY 712 Electrodynamics
9-9:50 AM Olin 105

Plan for Lecture 16:

Read Chapter 7

1. **Plane polarized electromagnetic waves**
2. **Reflectance and transmittance of electromagnetic waves – extension to anisotropy and complexity**

11	Fri: 02/08/2019	Chap. 5	Magnetostatics	#8	02/11/2019
12	Mon: 02/11/2019	Chap. 5	Magnetic dipoles and hyperfine interaction	#9	02/13/2019
13	Wed: 02/13/2019	Chap. 5	Magnetic dipoles and dipolar fields	#10	02/15/2019
14	Fri: 02/15/2019	Chap. 6	Maxwell's Equations	#11	02/18/2019
15	Mon: 02/18/2019	Chap. 6	Electromagnetic energy and forces	#12	02/20/2019
16	Wed: 02/20/2019	Chap. 7	Electromagnetic plane waves	#13	02/22/2019
17	Fri: 02/22/2019				
18	Mon: 02/25/2019				
19	Wed: 02/27/2019				
20	Fri: 03/01/2019				
	Mon: 03/04/2019	No class	*APS March Meeting*	Take Home Exam	
	Wed: 03/06/2019	No class	*APS March Meeting*	Take Home Exam	
	Fri: 03/08/2019	No class	*APS March Meeting*	Take Home Exam	
	Mon: 03/11/2019	No class	*Spring Break*		
	Wed: 03/13/2019	No class	*Spring Break*		
	Fri: 03/15/2019	No class	*Spring Break*		
21	Mon: 03/18/2019				

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Colloquium: "Plastic Nanoscale Optoelectronics Enabled Via Adhesion-Lithography" – Wednesday, February 20, 2019, at 4:00 PM

Thomas Anthopoulos, PhD,
Professor of Material Science and Engineering at King Abdullah University of Science and Technology
George P. Williams, Jr. Lecture Hall, (Olin 101)
Wednesday, February 20, 2019, at 4:00 PM

There will be a reception with refreshments at 3:30 PM in the lounge. All interested persons are cordially invited to attend.

Maxwell's equations

For linear isotropic media and no sources: $\mathbf{D}=\varepsilon\mathbf{E}; \quad \mathbf{B}=\mu\mathbf{H}$

Coulomb's law: $\nabla\cdot\mathbf{E}=0$

Ampere-Maxwell's law: $\nabla\times\mathbf{B}-\mu\varepsilon\frac{\partial\mathbf{E}}{\partial t}=0$

Faraday's law: $\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$

No magnetic monopoles: $\nabla\cdot\mathbf{B}=0$

Analysis of Maxwell's equations without sources -- continued:

Coulomb's law: $\nabla\cdot\mathbf{E}=0$

Ampere-Maxwell's law: $\nabla\times\mathbf{B}-\mu\varepsilon\frac{\partial\mathbf{E}}{\partial t}=0$

Faraday's law: $\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$

No magnetic monopoles: $\nabla\cdot\mathbf{B}=0$

$$\nabla\times\left(\nabla\times\mathbf{B}-\mu\varepsilon\frac{\partial\mathbf{E}}{\partial t}\right)=-\nabla^2\mathbf{B}-\mu\varepsilon\frac{\partial(\nabla\times\mathbf{E})}{\partial t}$$

$$=-\nabla^2\mathbf{B}+\mu\varepsilon\frac{\partial^2\mathbf{B}}{\partial t^2}=0$$

$$\nabla\times\left(\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}\right)=-\nabla^2\mathbf{E}+\frac{\partial(\nabla\times\mathbf{B})}{\partial t}$$

$$=-\nabla^2\mathbf{E}+\mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2}=0$$

Analysis of Maxwell's equations without sources -- continued:

Both E and B fields are solutions to a wave equation:

$$\nabla^2\mathbf{B}-\frac{1}{v^2}\frac{\partial^2\mathbf{B}}{\partial t^2}=0$$

$$\nabla^2\mathbf{E}-\frac{1}{v^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=0$$

where $v^2\equiv c^2\frac{\mu_0\varepsilon_0}{\mu\varepsilon}\equiv\frac{c^2}{n^2}$

Plane wave solutions to wave equation:

$$\mathbf{B}(\mathbf{r},t)=\Re\left(\mathbf{B}_0e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right) \qquad \mathbf{E}(\mathbf{r},t)=\Re\left(\mathbf{E}_0e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)$$

Analysis of Maxwell's equations without sources -- continued:

Plane wave solutions to wave equation:

$$\mathbf{B}(\mathbf{r},t)=\Re\left(\mathbf{B}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right) \qquad \mathbf{E}(\mathbf{r},t)=\Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)$$

$$|\mathbf{k}|^2=\left(\frac{\omega}{v}\right)^2=\left(\frac{n\omega}{c}\right)^2 \qquad \text{where } n\equiv\sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$$

Note: ε, μ, n, k can all be complex; for the moment we will assume that they are all real (no dissipation).

Note that $\mathbf{E}_0$ and $\mathbf{B}_0$ are not independent;

$$\mathbf{k}=n\frac{\omega}{c}\hat{\mathbf{k}}$$

from Faraday's law: $\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$

$$\Rightarrow \mathbf{B}_0=\frac{\mathbf{k}\times\mathbf{E}_0}{\omega}=\frac{n\hat{\mathbf{k}}\times\mathbf{E}_0}{c}$$

also note: $\hat{\mathbf{k}}\cdot\mathbf{E}_0=0$ and $\hat{\mathbf{k}}\cdot\mathbf{B}_0=0$

Analysis of Maxwell's equations without sources -- continued:

Summary of plane electromagnetic waves:

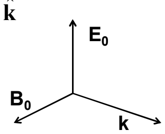
$$\mathbf{B}(\mathbf{r},t)=\Re\left(\frac{n\hat{\mathbf{k}}\times\mathbf{E}_0}{c}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right) \qquad \mathbf{E}(\mathbf{r},t)=\Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)$$

$$|\mathbf{k}|^2=\left(\frac{\omega}{v}\right)^2=\left(\frac{n\omega}{c}\right)^2 \quad \text{where } n\equiv\sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} \quad \text{and } \hat{\mathbf{k}}\cdot\mathbf{E}_0=0$$

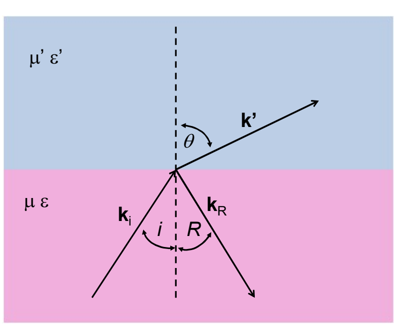
Poynting vector and energy density:

$$\langle\mathbf{S}\rangle_{avg}=\frac{n|\mathbf{E}_0|^2}{2\mu c}\hat{\mathbf{k}}=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|\mathbf{E}_0|^2\,\hat{\mathbf{k}}$$

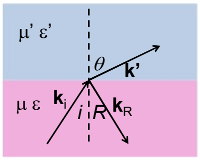
$$\langle u\rangle_{avg}=\frac{1}{2}\varepsilon|\mathbf{E}_0|^2$$



Reflection and refraction of plane electromagnetic waves at a plane interface between dielectrics (assumed to be lossless)



Reflection and refraction -- continued



In medium $\mu'\varepsilon'$:

$$\mathbf{E}'(\mathbf{r},t) = \Re\left(\mathbf{E}'_0\, e^{i\frac{\omega}{c}\left(n'\hat{\mathbf{k}}'\cdot\mathbf{r}-ct\right)}\right)$$

$$\mathbf{B}'(\mathbf{r},t) = \frac{n'}{c}\hat{\mathbf{k}}'\times\mathbf{E}'(\mathbf{r},t) = \sqrt{\mu'\varepsilon'}\hat{\mathbf{k}}'\times\mathbf{E}(\mathbf{r},t)$$

In medium $\mu\varepsilon$:

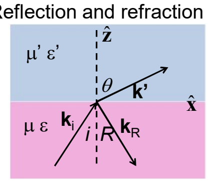
$$\mathbf{E}_i(\mathbf{r},t) = \Re\left(\mathbf{E}_{0i}\, e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_i\cdot\mathbf{r}-ct\right)}\right)$$

$$\mathbf{B}_i(\mathbf{r},t) = \frac{n}{c}\hat{\mathbf{k}}_i\times\mathbf{E}_i(\mathbf{r},t) = \sqrt{\mu\varepsilon}\hat{\mathbf{k}}_i\times\mathbf{E}_i(\mathbf{r},t)$$

$$\mathbf{E}_R(\mathbf{r},t) = \Re\left(\mathbf{E}_{0R}\, e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_R\cdot\mathbf{r}-ct\right)}\right)$$

$$\mathbf{B}_R(\mathbf{r},t) = \frac{n}{c}\hat{\mathbf{k}}_R\times\mathbf{E}_R(\mathbf{r},t) = \sqrt{\mu\varepsilon}\hat{\mathbf{k}}_R\times\mathbf{E}_R(\mathbf{r},t)$$

Reflection and refraction -- continued



Snell's law – matching phase factors at boundary plane *z=0*.

$$\left. e^{i\frac{\omega}{c}\left(n'\hat{\mathbf{k}}'\cdot\mathbf{r}-ct\right)}\right|_{z=0} = \left. e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_i\cdot\mathbf{r}-ct\right)}\right|_{z=0} = \left. e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_R\cdot\mathbf{r}-ct\right)}\right|_{z=0}$$

matching plane: $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + 0\hat{\mathbf{z}}$

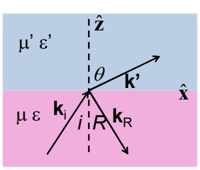
$$\hat{\mathbf{k}}'\cdot\mathbf{r} = x\sin\theta$$

$$\hat{\mathbf{k}}_i\cdot\mathbf{r} = x\sin i = \hat{\mathbf{k}}_R\cdot\mathbf{r} = x\sin R \quad \Rightarrow i = R$$

$$n'\hat{\mathbf{k}}'\cdot\mathbf{r} = n\hat{\mathbf{k}}_i\cdot\mathbf{r} \quad \Rightarrow n'x\sin\theta = nx\sin i$$

Snell's law: $n'\sin\theta = n\sin i$

Reflection and refraction -- continued



Continuity equations at boundary with no sources:

$$\nabla\cdot\mathbf{D} = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

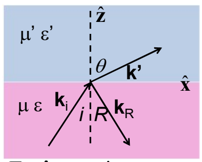
$$\nabla\times\mathbf{H} - \frac{\partial\mathbf{D}}{\partial t} = 0 \qquad \nabla\times\mathbf{E} + \frac{\partial\mathbf{B}}{\partial t} = 0$$

Matching field amplitudes at boundary plane:

$\mathbf{D}\cdot\hat{\mathbf{z}},\ \mathbf{B}\cdot\hat{\mathbf{z}}$ continuous

$\mathbf{H}\times\hat{\mathbf{z}}, \mathbf{E}\times\hat{\mathbf{z}}$ continuous

Reflection and refraction -- continued



Matching field amplitudes at boundary plane:

$\mathbf{D}\cdot\hat{\mathbf{z}}$ continuous:

$\varepsilon\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\cdot\hat{\mathbf{z}}=\varepsilon'\mathbf{E}'_0\cdot\hat{\mathbf{z}}$

$\mathbf{B}\cdot\hat{\mathbf{z}}$ continuous:

$n\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\cdot\hat{\mathbf{z}}=$

$n'\hat{\mathbf{k}}'\times\mathbf{E}'_0\cdot\hat{\mathbf{z}}$

$\mathbf{E}\times\hat{\mathbf{z}}$ continuous:

$\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\mathbf{E}'_0\times\hat{\mathbf{z}}$

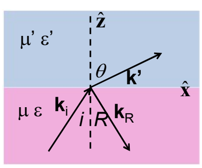
$\mathbf{H}\times\hat{\mathbf{z}}$ continuous:

$$\frac{n}{\mu}\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\frac{n'}{\mu'}\hat{\mathbf{k}}'\times\mathbf{E}'_0\times\hat{\mathbf{z}}$$

Known: $\mathbf{E}_{0i},\hat{\mathbf{k}}_i$

Unknown: $\mathbf{E}'_0,\mathbf{E}_{0R},\hat{\mathbf{k}}'$

Reflection and refraction -- continued



s-polarization – **E** field "polarized" perpendicular to plane of incidence

$\mathbf{E}\times\hat{\mathbf{z}}$ continuous:

$\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\mathbf{E}'_0\times\hat{\mathbf{z}}$

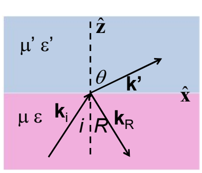
$\mathbf{H}\times\hat{\mathbf{z}}$ continuous:

$$\frac{n}{\mu}\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\frac{n'}{\mu'}\hat{\mathbf{k}}'\times\mathbf{E}'_0\times\hat{\mathbf{z}}$$

$$\frac{E_{0R}}{E_{0i}}=\frac{n\cos i-\frac{\mu}{\mu'}n'\cos\theta}{n\cos i+\frac{\mu}{\mu'}n'\cos\theta}\qquad\frac{E'_0}{E_{0i}}=\frac{2n\cos i}{n\cos i+\frac{\mu}{\mu'}n'\cos\theta}$$

$$\text{Note that: }\; n'\cos\theta=\sqrt{n'^2-n^2\sin^2 i}$$

Reflection and refraction -- continued



p-polarization – **E** field "polarized" parallel to plane of incidence

$\mathbf{D}\cdot\hat{\mathbf{z}}$ continuous:

$\varepsilon\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\cdot\hat{\mathbf{z}}=\varepsilon'\mathbf{E}'_0\cdot\hat{\mathbf{z}}$

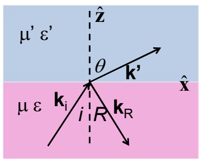
$\mathbf{H}\times\hat{\mathbf{z}}$ continuous:

$$\frac{n}{\mu}\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\frac{n'}{\mu'}\hat{\mathbf{k}}'\times\mathbf{E}'_0\times\hat{\mathbf{z}}$$

$$\frac{E_{0R}}{E_{0i}}=\frac{\frac{\mu}{\mu'}n'\cos i-n\cos\theta}{\frac{\mu}{\mu'}n'\cos i+n\cos\theta}\qquad\frac{E'_0}{E_{0i}}=\frac{2n\cos i}{\frac{\mu}{\mu'}n'\cos i+n\cos\theta}$$

$$\text{Note that: }\; n'\cos\theta=\sqrt{n'^2-n^2\sin^2 i}$$

Reflection and refraction -- continued



Reflectance, transmittance:

$$\mathrm{R} = \frac{\mathbf{S}_R \cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}} = \left|\frac{E_{0R}}{E_{0i}}\right|^2 \qquad \mathrm{T} = \frac{\mathbf{S}' \cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}} = \left|\frac{E'_0}{E_{0i}}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'}\frac{\cos\theta}{\cos i}$$

Note that $\mathrm{R} + \mathrm{T} = 1$

For s-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{n\cos i - \frac{\mu}{\mu'}n'\cos\theta}{n\cos i + \frac{\mu}{\mu'}n'\cos\theta} \qquad \frac{E'_0}{E_{0i}} = \frac{2n\cos i}{n\cos i + \frac{\mu}{\mu'}n'\cos\theta}$$

Note that: $n'\cos\theta = \sqrt{n'^2 - n^2\sin^2 i}$

For p-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'}n'\cos i - n\cos\theta}{\frac{\mu}{\mu'}n'\cos i + n\cos\theta} \qquad \frac{E'_0}{E_{0i}} = \frac{2n\cos i}{\frac{\mu}{\mu'}n'\cos i + n\cos\theta}$$

Note that: $n'\cos\theta = \sqrt{n'^2 - n^2\sin^2 i}$

Special case: normal incidence (i=0, θ=0)

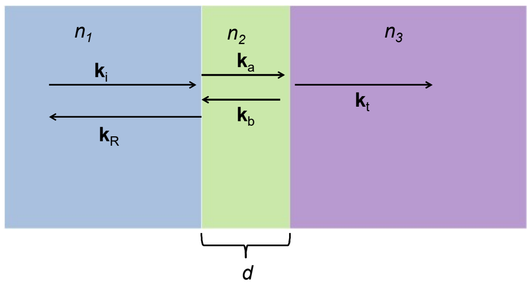
$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'}n' - n}{\frac{\mu}{\mu'}n' + n} \qquad \frac{E'_0}{E_{0i}} = \frac{2n}{\frac{\mu}{\mu'}n' + n}$$

Reflectance, transmittance:

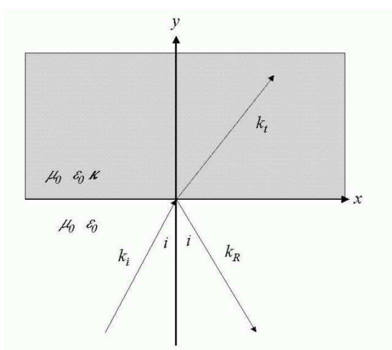
$$\mathrm{R} = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n' - n}{\frac{\mu}{\mu'}n' + n}\right|^2$$

$$\mathrm{T} = \left|\frac{E'_0}{E_{0i}}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'} = \left|\frac{2n}{\frac{\mu}{\mu'}n' + n}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'}$$

Multilayer dielectrics (Problem #7.2)



Extension of analysis to anisotropic media --



Consider the problem of determining the reflectance from an anisotropic medium with isotropic permeability μ_0 and anisotropic permittivity $\varepsilon_0 \boldsymbol{\kappa}$ where:

$$\boldsymbol{\kappa} \equiv \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}$$

By assumption, the wave vector in the medium is confined to the *x-y* plane and will be denoted by

$$\mathbf{k}_t \equiv \frac{\omega}{c}(n_x\hat{\mathbf{x}} + n_y\hat{\mathbf{y}}), \text{where } n_x \text{ and } n_y \text{ are to be determined.}$$

The electric field inside the medium is given by:

$$\mathbf{E} = (E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}} + E_z\hat{\mathbf{z}})e^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}.$$

Inside the anisotropic medium, Maxwell's equations are:

$$\nabla\cdot\mathbf{H}=0 \qquad \nabla\cdot\boldsymbol{\kappa}\cdot\mathbf{E}=0$$

$$\nabla\times\mathbf{E}-i\omega\mu_0\mathbf{H}=0 \qquad \nabla\times\mathbf{H}+i\omega\epsilon_0\boldsymbol{\kappa}\cdot\mathbf{E}=0$$

After some algebra, the equation for **E** is:

$$\begin{pmatrix} \kappa_{xx}-n_y^2 & n_xn_y & 0 \\ n_xn_y & \kappa_{yy}-n_x^2 & 0 \\ 0 & 0 & \kappa_{zz}-(n_x^2+n_y^2) \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}=0.$$

From **E, H** can be determined from

$$\mathbf{H}=\frac{1}{\mu_0 c}\left\{E_z(n_y\hat{\mathbf{x}}-n_x\hat{\mathbf{y}})+(E_yn_x-E_xn_y)\hat{\mathbf{z}}\right\}\mathrm{e}^{i\frac{\omega}{c}(n_xx+n_yy)-i\omega t}.$$

The fields for the incident and reflected waves are the same as for the isotropic case.

$$\mathbf{k}_i=\frac{\omega}{c}(\sin i\hat{\mathbf{x}}+\cos i\hat{\mathbf{y}}),$$

$$\mathbf{k}_R=\frac{\omega}{c}(\sin i\hat{\mathbf{x}}-\cos i\hat{\mathbf{y}}).$$

Note that, consistent with Snell's law: $n_x=\sin i$
Continuity conditions at the *y=0* plane must be applied for the following fields:

$$\mathbf{H}(x,0,z,t),\ E_x(x,0,z,t),\ E_z(x,0,z,t),\ \text{and}\ D_y(x,0,z,t).$$

There will be two different solutions, depending of the polarization of the incident field.

Solution for s-polarization

$$E_x=E_y=0 \quad \Rightarrow n_y^2=\kappa_{zz}-n_x^2$$

$$\mathbf{E}=E_z\hat{\mathbf{z}}\mathrm{e}^{i\frac{\omega}{c}(n_xx+n_yy)-i\omega t} \qquad \mathbf{H}=\frac{1}{\mu_0 c}\left\{E_z(n_y\hat{\mathbf{x}}-n_x\hat{\mathbf{y}})\right\}\mathrm{e}^{i\frac{\omega}{c}(n_xx+n_yy)-i\omega t}$$

E_z must be determined from the continuity conditions:

$$E_0+E_0''=E_z \qquad (E_0-E_0'')\cos i=E_zn_y \qquad (E_0+E_0'')\sin i=E_zn_x$$

$$\frac{E_0''}{E_0}=\frac{\cos i-n_y}{\cos i+n_y}.$$

Solution for p-polarization

$$E_z = 0 \quad \Rightarrow n_y^2 = \frac{\kappa_{xx}}{\kappa_{yy}}(\kappa_{yy} - n_x^2).$$

$$\mathbf{E} = E_x\left(\hat{\mathbf{x}} - \frac{\kappa_{xx}n_x}{\kappa_{yy}n_y}\hat{\mathbf{y}}\right)\mathrm{e}^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}.$$

$$\mathbf{H} = -\frac{E_x}{\mu_0 c}\frac{\kappa_{xx}}{n_y}\hat{\mathbf{z}}\mathrm{e}^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}.$$

E_x must be determined from the continuity conditions:

$$(E_0 - E_0'')\cos i = E_x \quad (E_0 + E_0'') = \frac{\kappa_{xx}}{n_y}E_x \quad (E_0 + E_0'')\sin i = \frac{\kappa_{xx}n_x}{n_y}E_x.$$

$$\frac{E_0''}{E_0} = \frac{\kappa_{xx}\cos i - n_y}{\kappa_{xx}\cos i + n_y}.$$

Extension of analysis to complex dielectric functions

For simplicity assume that $\mu = \mu_0$

Suppose the dielectric function is complex :

$$\varepsilon = \varepsilon_R + i\varepsilon_I \qquad \frac{\varepsilon}{\varepsilon_0} = (n_R + in_I)^2 \equiv \alpha + i\beta$$

$$n_R = \left(\frac{\sqrt{\alpha^2 + \beta^2} + \alpha}{2}\right)^{1/2} \qquad n_I = \left(\frac{\sqrt{\alpha^2 + \beta^2} - \alpha}{2}\right)^{1/2}$$

$$\mathbf{E}(\mathbf{r}, t) = \Re\left(\mathbf{E}_0 e^{i\frac{\omega}{c}(n\hat{\mathbf{k}}\cdot\mathbf{r} - ct)}\right) = \Re\left(\mathbf{E}_0 e^{i\frac{\omega}{c}(n_R\hat{\mathbf{k}}\cdot\mathbf{r} - ct)}\right)e^{-\frac{\omega}{c}n_I\hat{\mathbf{k}}\cdot\mathbf{r}}$$