

**PHY 712 Electrodynamics
9-9:50 AM MWF Olin 105**

Plan for Lecture 26:

Finish Chap. 11 and begin Chap. 14

- A. Electromagnetic field transformations & corresponding analysis of Liénard-Wiechert potentials for constant velocity sources
 - B. Radiation by moving charged particles

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Fri: 03/15/2019	No class	Spring Break	
21 Mon: 03/18/2019	Chap. 9	Radiation from localized oscillating sources	#15 3/22/2019
22 Wed: 03/20/2019	Chap. 9	Radiation from oscillating sources	#16 3/25/2019
23 Fri: 03/22/2019	Chap. 9 and 10	Radiation from oscillating sources	#17 3/27/2019
24 Mon: 03/25/2019	Chap. 11	Special Theory of Relativity	Pick topic 3/29/2019
25 Wed: 03/27/2019	Chap. 11	Special Theory of Relativity	#18 4/01/2019
26 Fri: 03/29/2019	Chap. 11	Special Theory of Relativity	#19 4/03/2019
27 Mon: 04/01/2019			
28 Wed: 04/03/2019			
29 Fri: 04/05/2019			
30 Mon: 04/08/2019			
31 Wed: 04/10/2019			
32 Fri: 04/12/2019			
33 Mon: 04/15/2019			
34 Wed: 04/17/2019			
Fri: 04/19/2019	No class	Good Friday	
35 Mon: 04/22/2019			
36 Wed: 04/24/2019			
Fri: 04/26/2019		Presentations I	
Mon: 04/29/2019		Presentations II	
Wed: 05/01/2019		Presentations III	

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$$F^{\alpha\beta} = \mathcal{L}_v^{-1\alpha\gamma} F^{\gamma\delta} \mathcal{L}_v^{-1\beta\delta}$$

$$\mathcal{L}_v^{-1} = \begin{pmatrix} \gamma_v & -\gamma_v \beta_v & 0 & 0 \\ -\gamma_v \beta_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -\gamma_v(E_y - \beta_v B_z) & -\gamma_v(E_z + \beta_v B_y) \\ E_x & 0 & -\gamma_v(B_z - \beta_v E_y) & \gamma_v(B_y + \beta_v E_z) \\ \gamma_v(E_y - \beta_v B_z) & \gamma_v(B_z - \beta_v E_y) & 0 & -B_x \\ \gamma_v(E_z + \beta_v B_y) & -\gamma_v(B_y + \beta_v E_z) & B_x & 0 \end{pmatrix}$$

Summary of results:

$$\begin{aligned} E_x' &= E_x & B_x' &= B_x \\ E_y' &= \gamma_v (E_y - \beta_v B_z) & B_y' &= \gamma_v (B_y + \beta_v E_z) \\ E_z' &= \gamma_v (E_z + \beta_v B_y) & B_z' &= \gamma_v (B_z - \beta_v E_y) \end{aligned}$$

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Example:

Fields in moving frame :

$$\mathbf{E}' = \frac{q}{r'^3} (x' \hat{\mathbf{x}} + y' \hat{\mathbf{y}}) = \frac{q(-vt' \hat{\mathbf{x}} + b \hat{\mathbf{y}})}{((-vt')^2 + b^2)^{3/2}}$$

$$\mathbf{B}' = 0$$

Fields in stationary frame :

$$E_x = E'_x$$

$$E_y = \gamma_v (E'_y + \beta_v B'_z)$$

$$E_z = \gamma_v (E'_z - \beta_v B'_y)$$

$$B_x = B'_x$$

$$B_y = \gamma_v (B'_y - \beta_v E'_z)$$

$$B_z = \gamma_v (B'_z + \beta_v E'_y)$$

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Example:

Fields in moving frame :

$$\mathbf{E}' = \frac{q}{r'^3} (x' \hat{\mathbf{x}} + y' \hat{\mathbf{y}}) = \frac{q(-vt' \hat{\mathbf{x}} + b \hat{\mathbf{y}})}{((-vt')^2 + b^2)^{3/2}}$$

$$\mathbf{B}' = 0$$

Fields in stationary frame :

$$E_x = E'_x = \frac{q(-vt')}{((-vt')^2 + b^2)^{3/2}}$$

$$E_y = \gamma_v (E'_y) = \frac{q(\gamma_v b)}{((-vt')^2 + b^2)^{3/2}}$$

$$B_z = \gamma_v (\beta_v E'_y) = \frac{q(\gamma_v \beta_v b)}{((-vt')^2 + b^2)^{3/2}}$$

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Example:

Fields in moving frame :

$$\mathbf{E}' = \frac{q}{r'^3} (x' \hat{\mathbf{x}} + y' \hat{\mathbf{y}}) = \frac{q(-vt' \hat{\mathbf{x}} + b \hat{\mathbf{y}})}{((-vt')^2 + b^2)^{3/2}}$$

$$\mathbf{B}' = 0$$

Fields in stationary frame :

$$E_x = E'_x = \frac{q(-v\gamma_v t)}{((-v\gamma_v t)^2 + b^2)^{3/2}}$$

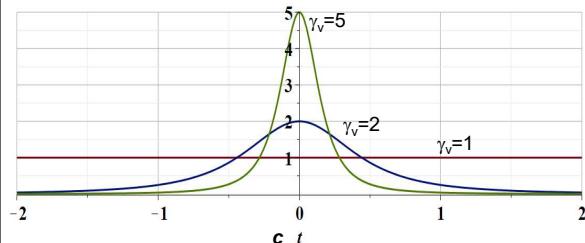
$$E_y = \gamma_v (E'_y) = \frac{q(\gamma_v b)}{((-v\gamma_v t)^2 + b^2)^{3/2}}$$

$$B_z = \gamma_v (\beta_v E'_y) = \frac{q(\gamma_v \beta_v b)}{((-v\gamma_v t)^2 + b^2)^{3/2}}$$

Expression in terms of consistent coordinates

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$$E_y = \frac{q(\gamma_v b)}{\left((-v\gamma_v t)^2 + b^2\right)^{3/2}} = \frac{q(\gamma_v b)}{\left((\gamma_v^2 - 1)c^2 t^2 + b^2\right)^{3/2}}$$



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Examination of this system from the viewpoint of the
the Liénard-Wiechert potentials –(Gaussian units)

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c} \right)^3} \left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c} \right) \left(1 - \frac{\mathbf{v}^2}{c^2} \right) + \left(\mathbf{R} \times \left(\left(\mathbf{R} - \frac{\mathbf{v}R}{c} \right) \times \frac{\dot{\mathbf{v}}}{c^2} \right) \right) \right]$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{q}{c} \left[\frac{-\mathbf{R} \times \mathbf{v}}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c} \right)^3} \left(1 - \frac{v^2}{c^2} + \frac{\dot{\mathbf{v}} \cdot \mathbf{R}}{c^2} \right) - \frac{\mathbf{R} \times \dot{\mathbf{v}} / c}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c} \right)^2} \right]$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{\mathbf{R} \times \mathbf{E}(\mathbf{r}, t)}{R}.$$

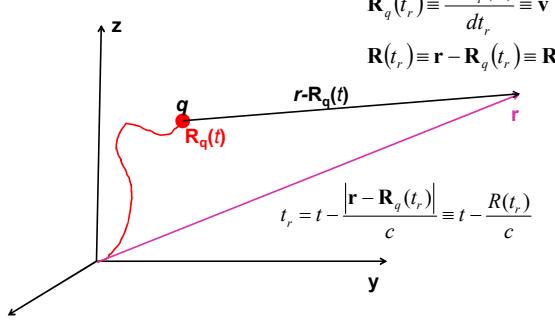
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Radiation from a moving charged particle

Variables (notations)



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Examination of this system from the viewpoint of the the Liénard-Wiechart potentials -(Gaussian units)

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c} \right) \left(1 - \frac{v^2}{c^2} \right) \right]$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{q}{c} \left[\frac{-\mathbf{R} \times \mathbf{v}}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left(1 - \frac{v^2}{c^2} \right) \right]$$

For our example:
 $\mathbf{R}_q(t_r) = vt_r \hat{\mathbf{x}}$
 $\mathbf{r} = b \hat{\mathbf{y}}$
 $\mathbf{R} = b \hat{\mathbf{y}} - vt_r \hat{\mathbf{x}}$
 $R = \sqrt{v^2 t_r^2 + b^2}$
 $\mathbf{v} = v \hat{\mathbf{x}}$
 $t_r = t - \frac{R}{c}$

This should be equivalent to the result given in Jackson (11.152):

$$\mathbf{E}(x, y, z, t) = \mathbf{E}(0, b, 0, t) = q \frac{-v\gamma \hat{\mathbf{x}} + \gamma b \hat{\mathbf{y}}}{(b^2 + (v\gamma t)^2)^{3/2}}$$

$$\mathbf{B}(x, y, z, t) = \mathbf{B}(0, b, 0, t) = q \frac{\gamma \beta b \hat{\mathbf{z}}}{(b^2 + (v\gamma t)^2)^{3/2}}$$

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Some details

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c} \right) \left(1 - \frac{v^2}{c^2} \right) \right]$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{q}{c} \left[\frac{-\mathbf{R} \times \mathbf{v}}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left(1 - \frac{v^2}{c^2} \right) \right]$$

For our example:
 $\mathbf{R}_q(t_r) = vt_r \hat{\mathbf{x}}$
 $\mathbf{r} = b \hat{\mathbf{y}}$
 $\mathbf{R} = b \hat{\mathbf{y}} - vt_r \hat{\mathbf{x}}$
 $R = \sqrt{v^2 t_r^2 + b^2}$
 $\mathbf{v} = v \hat{\mathbf{x}}$
 $t_r = t - \frac{R}{c}$

t_r must be a solution to a quadratic equation:

$$t_r - t = -\frac{R}{c} \Rightarrow t_r^2 - 2\gamma^2 t_r t + \gamma^2 t^2 - \gamma^2 b^2 / c^2 = 0$$

with the physical solution:

$$t_r = \gamma \left(\gamma t - \frac{\sqrt{(v\gamma t)^2 + b^2}}{c} \right)$$

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Some details continued:

Now we can express R as:

$$R = \gamma \left(-\beta v \gamma t + \sqrt{(v\gamma t)^2 + b^2} \right)$$

and the related quantities:

$$\mathbf{R} - \mathbf{v}R / c = -vt \hat{\mathbf{x}} + b \hat{\mathbf{y}}$$

$$R - \mathbf{v} \cdot \mathbf{R} / c = \frac{\sqrt{(v\gamma t)^2 + b^2}}{\gamma}$$

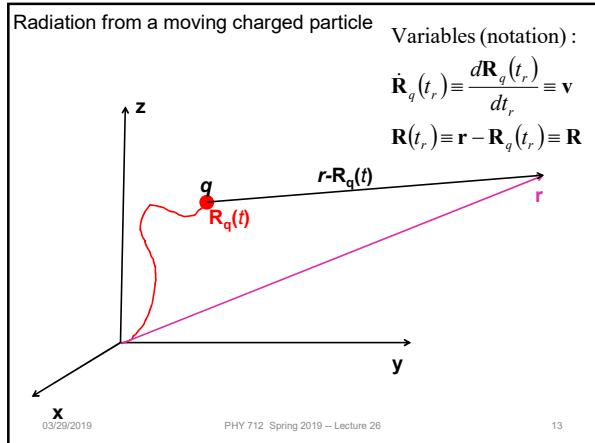
$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c} \right) \left(1 - \frac{v^2}{c^2} \right) \right] = q \frac{-v\gamma \hat{\mathbf{x}} + \gamma b \hat{\mathbf{y}}}{(b^2 + (v\gamma t)^2)^{3/2}}$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{q}{c} \left[\frac{-\mathbf{R} \times \mathbf{v}}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left(1 - \frac{v^2}{c^2} \right) \right] = q \frac{\gamma \beta b \hat{\mathbf{z}}}{(b^2 + (v\gamma t)^2)^{3/2}}$$

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Liénard-Wiechert fields (cgs Gaussian units):

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left[\left(\mathbf{R} - \frac{\mathbf{v} R}{c} \right) \left(1 - \frac{v^2}{c^2} \right) + \left(\mathbf{R} \times \left(\left(\mathbf{R} - \frac{\mathbf{v} R}{c} \right) \times \frac{\dot{\mathbf{v}}}{c^2} \right) \right) \right]. \quad (19)$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{q}{c} \left[\frac{-\mathbf{R} \times \mathbf{v}}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c} \right)^3} \left(1 - \frac{v^2}{c^2} + \frac{\dot{\mathbf{v}} \cdot \mathbf{R}}{c^2} \right) - \frac{\mathbf{R} \times \dot{\mathbf{v}}/c}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c} \right)^2} \right]. \quad (20)$$

In this case, the electric and magnetic fields are related according to

$$\mathbf{B}(\mathbf{r}, t) = \frac{\mathbf{R} \times \mathbf{E}(\mathbf{r}, t)}{R}. \quad (21)$$

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v} \quad \mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R} \quad \dot{\mathbf{v}} \equiv \frac{d^2\mathbf{R}_q(t_r)}{dt_r^2}$$

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Electric field far from source:

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left\{ \mathbf{R} \times \left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c} \right) \times \frac{\dot{\mathbf{v}}}{c^2} \right] \right\}$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{\mathbf{R} \times \mathbf{E}(\mathbf{r}, t)}{R}$$

$$\text{Let } \hat{\mathbf{R}} \equiv \frac{\mathbf{R}}{R} \quad \beta \equiv \frac{\mathbf{v}}{c} \quad \dot{\beta} \equiv \frac{\dot{\mathbf{v}}}{c}$$

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{cR(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^3} \left\{ \hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}] \right\}$$

$$\mathbf{B}(\mathbf{r},t) = \hat{\mathbf{R}} \times \mathbf{E}(\mathbf{r},t)$$

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Poynting vector:

$$\mathbf{S}(\mathbf{r}, t) = \frac{c}{4\pi} (\mathbf{E} \times \mathbf{B})$$

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{cR(1 - \beta \cdot \hat{\mathbf{R}})^3} \left\{ \hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \beta) \times \dot{\beta}] \right\}$$

$$\mathbf{B}(\mathbf{r}, t) = \hat{\mathbf{R}} \times \mathbf{E}(\mathbf{r}, t)$$

$$\mathbf{S}(\mathbf{r}, t) = \frac{c}{4\pi} \hat{\mathbf{R}} |\mathbf{E}(\mathbf{r}, t)|^2 = \frac{q^2}{4\pi RC^2} \hat{\mathbf{R}} \frac{|\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \hat{\beta}) \times \hat{\beta}]|^2}{(1 - \beta \cdot \hat{\mathbf{R}})^6}$$

Note: We have used the fact that

$$\hat{\mathbf{R}} \cdot \mathbf{E}(\mathbf{r}, t) = 0$$

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Power radiated

$$\mathbf{S}(\mathbf{r}, t) = \frac{c}{4\pi} \hat{\mathbf{R}} |\mathbf{E}(\mathbf{r}, t)|^2 = \frac{q^2}{4\pi RC^2} \hat{\mathbf{R}} \frac{|\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \beta) \times \dot{\hat{\beta}}]|^2}{(1 - \beta \cdot \hat{\mathbf{R}})^6}$$

$$\frac{dP}{d\Omega} = \mathbf{S} \cdot \hat{\mathbf{R}} R^2 = \frac{q^2}{4\pi c} \frac{\left| \hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}] \right|^2}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^6}$$

In the non-relativistic limit: $\beta \ll 1$

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c} |\hat{\mathbf{R}} \times [\hat{\mathbf{R}} \times \dot{\hat{\beta}}]|^2 = \frac{q^2}{4\pi c^3} |\dot{\hat{\mathbf{v}}}|^2 \sin^2 \Theta$$

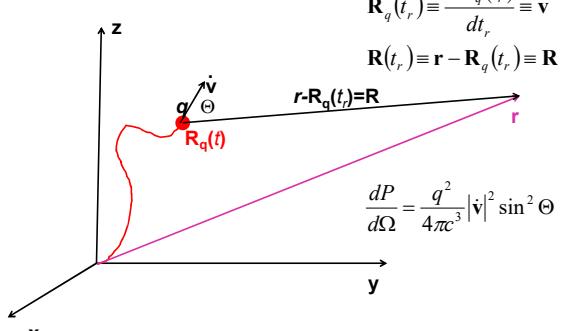
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Radiation from a moving charged particle

Variable (cont'd.)



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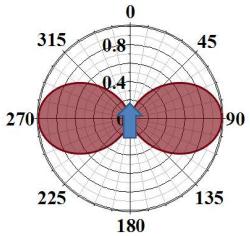
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Radiation power in non-relativistic case -- continued

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} |\dot{\mathbf{v}}|^2 \sin^2 \Theta$$

$$P = \int d\Omega \frac{dP}{d\Omega} = \frac{2}{3} \frac{q^2}{c^3} |\dot{\mathbf{v}}|^2$$



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