PHY 712 Electrodynamics
9-9:50 AM MWF Olin 105

Plan for Lecture 31:

Special Topics in Electrodynamics:

Cherenkov radiation

References: Jackson Chapter 13.4
Zangwill Chapter 23.7
Smith Chapter 6.4

23	Fri: 03/22/2019	Chap. 9 and 10	Radiation from oscillating sources	#17	3/27/2019
24	Mon: 03/25/2019	Chap. 11	Special Theory of Relativity	Pick topic	3/29/2019
25	Wed: 03/27/2019	Chap. 11	Special Theory of Relativity	#18	4/01/2019
26	Fri: 03/29/2019	Chap. 11	Special Theory of Relativity	#19	4/03/2019
27	Mon: 04/01/2019	Chap. 14	Radiation from accelerating charged particles	#20	4/05/2019
28	Wed: 04/03/2019	Chap. 14	Synchrotron radiation		
29	Fri: 04/05/2019	Chap. 14	Synchrotron radiation	#21	4/10/2019
30	Mon: 04/08/2019	Chap. 15	Radiation from collisions of charged particles	#22	4/12/2019
31	Wed: 04/10/2019	Chap. 13	Cherenkov radiation		
32	Fri: 04/12/2019		Special topic: E & M aspects of superconductivity		
33	Mon: 04/15/2019		Special topic: Aspects of optical properties of materials		
34	Wed: 04/17/2019	Chap. 1-15	Review		
	Fri: 04/19/2019	No class	*Good Friday*		
35	Mon: 04/22/2019	Chap. 1-15	Review		
36	Wed: 04/24/2019	Chap. 1-15	Review		
	Fri: 04/26/2019		Presentations I		
	Mon: 04/29/2019		Presentations II		
	Wed: 05/01/2019		Presentations III		

Colloquium: "Thermodynamics of Black Holes with positive Cosmological Constant and the Schottky Anomaly" – Wednesday, April 10, 2019, at 4:00 PM

Jennie Traschen, PhD
Department of Physics
University of Massachusetts Amherst
George P. Williams, Jr. Lecture Hall, (Olin 101)
Wednesday, April 10, 2019, at 4:00 PM

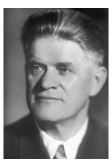
There will be a reception with refreshments at 3:30 PM in the lounge. All interested persons are cordially invited to attend.

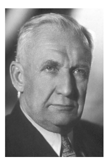
Cherenkov radiation

Cherenkov radiation emitted by the core of the Reed Research Reactor located at Reed College in Portland, Oregon, U.S. *Cherenkov radiation*. Photograph. *Encyclopædia Britannica Online*. Web. 12 Apr. 2013. http://www.britannica.com/EBchecked/media/174732

The Nobel Prize in Physics 1958

Pavel A. Cherenkov
Il´ja M. Frank
Igor Y. Tamm





Affiliation at the time of the award: P.N. Lebedev Physical Institute, Moscow, USSR

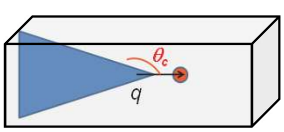
Prize motivation: "for the discovery and the interpretation of the Cherenkov effect."

https://www.nobelprize.org/prizes/physics/1958/ceremony-speech/

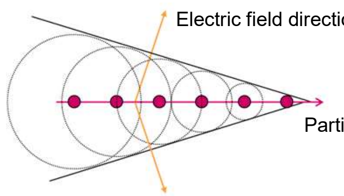
References for notes: Glenn S. Smith, *An Introduction to Electromagnetic Radiation* (Cambridge UP, 1997), Andrew Zangwill, Modern Electrodynamics (Cambridge UP, 2013)

Cherenkov radiation

Discovered ~1930; bluish light emitted by energetic charged particles traveling within dielectric materials



From: http://large.stanford.edu/courses/2014/ph241/alaeian2/



Maxwell's potential equations within a material having permittivity and permeability (Lorentz gauge; cgs Gaussian units)

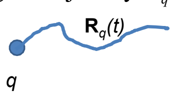
$$\nabla^2\Phi - \mu\varepsilon \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{4\pi}{\varepsilon}\rho$$

$$\nabla^2\mathbf{A} - \mu\varepsilon \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi\mu}{c}\mathbf{J}$$

Source: charged particle moving on trajectory $\mathbf{R}_q(t)$:

$$\rho(\mathbf{r},t) = q\delta\left(\mathbf{r}-\mathbf{R}_q(t)\right)$$

$$\mathbf{J}(\mathbf{r},t) = q\dot{\mathbf{R}}_q(t)\delta\left(\mathbf{r}-\mathbf{R}_q(t)\right)$$



Liénard-Wiechert potential solutions:

$$\Phi(\mathbf{r},t) = \frac{q}{\varepsilon}\frac{1}{\left|R(t_r) - \boldsymbol{\beta}_n\cdot\mathbf{R}(t_r)\right|}$$

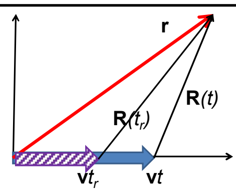
$$\mathbf{A}(\mathbf{r},t) = q\mu\frac{\boldsymbol{\beta}_n}{\left|R(t_r) - \boldsymbol{\beta}_n\cdot\mathbf{R}(t_r)\right|}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r)$$

$$\boldsymbol{\beta}_n(t_r) \equiv \frac{\dot{\mathbf{R}}_q(t_r)}{c_n} \qquad c_n \equiv \frac{c}{\sqrt{\mu\varepsilon}} \equiv \frac{c}{n}$$

$$t_r = t - \frac{R(t_r)}{c_n}$$

Consider a particle moving at constant velocity $\mathbf{v}$; $\quad v > c_n$



Some algebra

$$\mathbf{R}(t) = \mathbf{r} - \mathbf{v}t$$

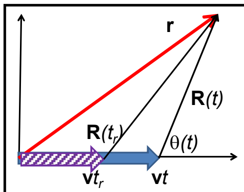
$$\mathbf{R}(t_r) = \mathbf{r} - \mathbf{v}t_r = \mathbf{R}(t) + \mathbf{v}(t - t_r)$$

$$(t - t_r)c_n = R(t_r) = \left|\mathbf{R}(t) + \mathbf{v}(t - t_r)\right|$$

Quadratic equation for $(t - t_r)c_n$:

$$((t - t_r)c_n)^2 = R^2(t) + 2\mathbf{R}(t)\cdot\boldsymbol{\beta}_n(t - t_r)c_n + \beta_n^{\,2}((t - t_r)c_n)^2$$

$$(t - t_r)c_n = \frac{-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n \pm\sqrt{(\mathbf{R}(t)\cdot\boldsymbol{\beta}_n)^2 - (\beta_n^{\,2} - 1)R^2(t)}}{\beta_n^{\,2} - 1}$$



$$\mathbf{R}(t_r) = \mathbf{r} - \mathbf{v}t_r = \mathbf{R}(t) + \mathbf{v}(t - t_r)$$

$$(t - t_r)c_n = R(t_r)$$

$$R(t_r) - \mathbf{R}(t_r)\cdot\boldsymbol{\beta}_n =$$

$$(t - t_r)c_n(1 - \beta_n^2) - \mathbf{R}(t)\cdot\boldsymbol{\beta}_n$$

$$= R(t_r)(1 - \beta_n^2) - \mathbf{R}(t)\cdot\boldsymbol{\beta}_n$$

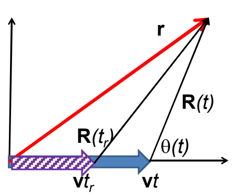
$$R(t_r) = \frac{-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n \pm\sqrt{(\mathbf{R}(t)\cdot\boldsymbol{\beta}_n)^2 - (\beta_n^{\,2} - 1)R^2(t)}}{\beta_n^{\,2} - 1}$$

$$R(t_r) = \frac{R(t)}{\beta_n^{\,2} - 1}\left(-\beta_n\cos\theta \pm\sqrt{1 - \beta_n^{\,2}\sin^2\theta}\right) = (t - t_r)c_n$$

$$R(t_r) - \mathbf{R}(t_r)\cdot\boldsymbol{\beta}_n = \mp R(t)\sqrt{1 - \beta_n^{\,2}\sin^2\theta}$$

Liénard-Wiechert potentials for two different retarded times:

$$\Phi(\mathbf{r},t) = \frac{q}{\varepsilon}\left.\frac{1}{R(t)\sqrt{1 - \beta_n^{\,2}\sin^2\theta}}\right|_{t_r}$$



$$\mathbf{A}(\mathbf{r},t) = q\mu\left.\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1 - \beta_n^{\,2}\sin^2\theta}}\right|_{t_r}$$

For $\beta_n > 1$, the range of θ is limited:

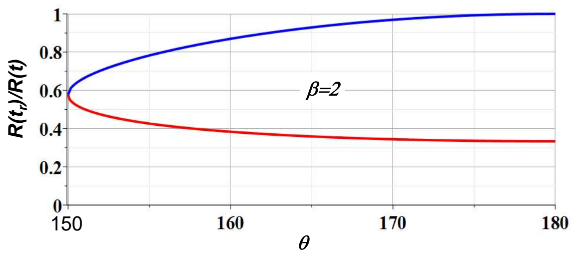
$$R(t_r) = \frac{R(t)}{\beta_n^{\,2} - 1}\left(-\beta_n\cos\theta \pm\sqrt{1 - \beta_n^{\,2}\sin^2\theta}\right) \geq 0$$

$$\Rightarrow |\sin\theta| \leq \frac{1}{\beta_n} \equiv |\sin\theta_c| \quad \text{and} \quad \pi \geq \theta_c \geq \pi/2$$

➔Diagram is not correct!

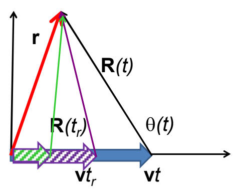
In this range, $\theta \geq \theta_c$

$$R(t_r) = \frac{R(t)}{\beta_n^2 - 1}\left(-\beta_n \cos\theta \pm \sqrt{1-\beta_n^2 \sin^2\theta}\right)$$



$\theta_c = 150^o$ for this case

Physical fields for $\beta_n > 1$ -- two retarded solutions contribute



$$\theta \le \sin^{-1}\left(\frac{1}{\beta_n}\right)$$

Define $\cos\theta_C \equiv -\sqrt{1-\frac{1}{\beta_n^2}}$

$\Rightarrow \cos\theta \le \cos\theta_C$

Adding two solutions; in terms of Heaviside $\Theta(x)$:

$$\Phi(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{1}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{A}(\mathbf{r},t) = 2q\mu\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

Physical fields for $\beta > 1$

$$\Phi(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{1}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{A}(\mathbf{r},t) = 2q\mu\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

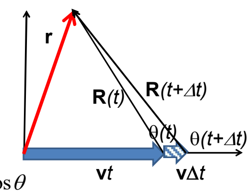
$$\mathbf{E}(\mathbf{r},t) = -\nabla\Phi - \frac{1}{c_n}\frac{\partial\mathbf{A}}{\partial t} \qquad \mathbf{B}(\mathbf{r},t) = \nabla\times\mathbf{A}$$

$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}}{(R(t))^2\sqrt{1-\beta_n^2\sin^2\theta}} \times$$

$$\left(\frac{\beta_n^2-1}{1-\beta_n^2\sin^2\theta}\Theta(\cos\theta_C - \cos\theta(t)) + \sqrt{\beta_n^2-1}\;\delta(\cos\theta_C - \cos\theta(t))\right)$$

$$\mathbf{B}(\mathbf{r},t) = -\beta_n \sin\theta\left(\hat{\theta}\times\mathbf{E}(\mathbf{r},t)\right)$$

Intermediate steps:



$$\frac{d\theta}{dt} = \frac{v\sin\theta}{R} \qquad \frac{dR}{dt} = -v\cos\theta$$

Using instantaneous polar coordinates: $\nabla \equiv \hat{\mathbf{R}}\frac{\partial}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial}{\partial\theta}$

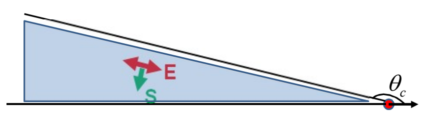
$$\nabla\Theta\left(\cos\theta_C - \cos\theta(t)\right) = \delta\left(\cos\theta_C - \cos\theta(t)\right)\frac{\sin\theta(t)}{R(t)}\hat{\boldsymbol{\theta}}$$

$$\frac{\partial\Theta\left(\cos\theta_C - \cos\theta(t)\right)}{\partial t} = \delta\left(\cos\theta_C - \cos\theta(t)\right)\frac{v\sin^2\theta(t)}{R(t)}$$

Cherenkov radiation observed near the angle θ_c -- continued

$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}}{(R(t))^2\sqrt{1-\beta_n^2\sin^2\theta}} \times$$
$$\left(\frac{\beta_n^2-1}{1-\beta_n^2\sin^2\theta}\Theta(\cos\theta_C - \cos\theta(t)) + \sqrt{\beta_n^2-1}\;\delta(\cos\theta_c - \cos\theta(t))\right)$$

$$\mathbf{B}(\mathbf{r},t) = -\beta_n\sin\theta\left(\hat{\theta}\times\mathbf{E}(\mathbf{r},t)\right)$$



Frequency dependence of intensity:

$$\frac{dI}{d\omega} \approx \frac{q^2}{c^2}\omega\left(1 - \frac{1}{\beta^2\epsilon(\omega)}\right)$$

A few details --

$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}}{(R(t))^2\sqrt{1-\beta_n^2\sin^2\theta}} \times$$
$$\left(\frac{\beta_n^2-1}{1-\beta_n^2\sin^2\theta}\Theta(\cos\theta_C - \cos\theta(t)) + \sqrt{\beta_n^2-1}\;\delta(\cos\theta_c - \cos\theta(t))\right)$$

$$\mathbf{B}(\mathbf{r},t) = -\beta_n\sin\theta\left(\hat{\theta}\times\mathbf{E}(\mathbf{r},t)\right)$$

$$\tilde{\mathbf{E}}(\mathbf{r},\omega) = \int_{-\infty}^{\infty} dt\; e^{-i\omega t}\mathbf{E}(\mathbf{r},t) \qquad \tilde{\mathbf{B}}(\mathbf{r},\omega) = \int_{-\infty}^{\infty} dt\; e^{-i\omega t}\mathbf{B}(\mathbf{r},t)$$

$$\langle\mathbf{S}(\mathbf{r},\omega)\rangle = \frac{c}{8\pi\mu}\;\tilde{\mathbf{E}}(\mathbf{r},\omega)\times\tilde{\mathbf{B}}^*(\mathbf{r},\omega)$$