PHY 712 Electrodynamics
9-9:50 AM MWF Olin 105

Plan for Lecture 34:

Review of electrodynamics

1. Review of Chap. 1-15
2. Evaluation forms

23	Fri: 03/22/2019	Chap. 9 and 10	Radiation from oscillating sources	#17	3/27/2019
24	Mon: 03/25/2019	Chap. 11	Special Theory of Relativity	Pick topic	3/29/2019
25	Wed: 03/27/2019	Chap. 11	Special Theory of Relativity	#18	4/01/2019
26	Fri: 03/29/2019	Chap. 11	Special Theory of Relativity	#19	4/03/2019
27	Mon: 04/01/2019	Chap. 14	Radiation from accelerating charged particles	#20	4/05/2019
28	Wed: 04/03/2019	Chap. 14	Synchrotron radiation		
29	Fri: 04/05/2019	Chap. 14	Synchrotron radiation	#21	4/10/2019
30	Mon: 04/08/2019	Chap. 15	Radiation from collisions of charged particles	#22	4/12/2019
31	Wed: 04/10/2019	Chap. 13	Cherenkov radiation		
32	Fri: 04/12/2019		Special topic: E & M aspects of superconductivity		
33	Mon: 04/15/2019		Special topic: Aspects of optical properties of materials		
34	Wed: 04/17/2019	Chap. 1-15	Review		
	Fri: 04/19/2019	No class	*Good Friday*		
35	Mon: 04/22/2019	Chap. 1-15	Review		
36	Wed: 04/24/2019	Chap. 1-15	Review		
	Fri: 04/26/2019		Presentations I		
	Mon: 04/29/2019		Presentations II		
	Wed: 05/01/2019		Presentations III		

Schedule for PHY 712 presentations

Friday, April 26, 2019

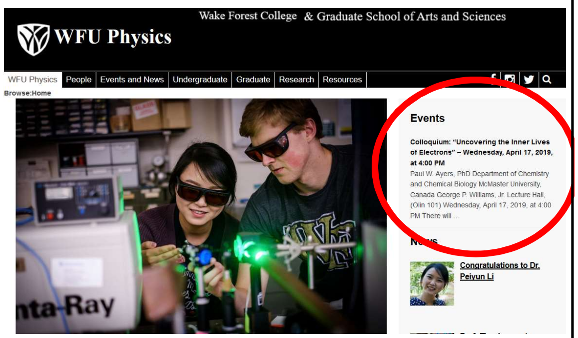
	Name	Topic
9:00-9:16	Ian Newsome	Current density from particle production in classical background E-field coupled to quantum scalar field. :)
9:17-9:31	Dizhou Wu	Ewald
9:32-9:48	Leda Gao	Hyperfine Hamiltonian

Monday, April 29, 2019

	Name	Topic
9:00-9:16	Shohreh Gholizadeh	3D FEM
9:17-9:31	Lindsey Gray	
9:32-9:48		

Wednesday, May 1, 2019

	Name	Topic
9:00-9:16	Eric	TBD
9:17-9:31	Ryan	Planetary Magnetism?
9:32-9:48		



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Units - SI vs Gaussian

Below is a table comparing SI and Gaussian unit systems. The fundamental units for each system are so labeled and are used to define the derived units.

Variable	SI		Gaussian		SI/Gaussian
	Unit	Relation	Unit	Relation	
length	m	fundamental	cm	fundamental	100
mass	kg	fundamental	gm	fundamental	1000
time	s	fundamental	s	fundamental	1
force	N	$kg \cdot m^2/s$	$dyne$	$gm \cdot cm^2/s$	10^5
current	A	fundamental	$statampere$	$statcoulomb/s$	$\frac{1}{10c}$
charge	C	$A \cdot s$	$statcoulomb$	$\sqrt{dyne \cdot cm^2}$	$\frac{1}{10c}$

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Basic equations of electrodynamics

$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P} = \epsilon\mathbf{E}$

$\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M} = \frac{1}{\mu}\mathbf{B}$

CGS (Gaussian)	SI
$\nabla \cdot \mathbf{D} = 4\pi\rho$	$\nabla \cdot \mathbf{D} = \rho$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$
$\mathbf{F} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})$	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
$u = \frac{1}{8\pi}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$	$u = \frac{1}{2}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$
$\mathbf{S} = \frac{c}{4\pi}(\mathbf{E} \times \mathbf{H})$	$\mathbf{S} = (\mathbf{E} \times \mathbf{H})$

$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon\mathbf{E}$

$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} = \frac{1}{\mu}\mathbf{B}$

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More relationships

CGS (Gaussian)

$$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P} = \epsilon\mathbf{E}$$

$$\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M} = \frac{1}{\mu}\mathbf{B}$$

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla\times\mathbf{A}$$

MKS (SI)

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon\mathbf{E}$$

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} = \frac{1}{\mu}\mathbf{B}$$

$$\mathbf{E} = -\nabla\Phi - \frac{\partial\mathbf{A}}{\partial t}$$

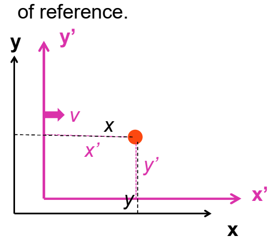
$$\mathbf{B} = \nabla\times\mathbf{A}$$

$$\epsilon \Leftrightarrow \epsilon/\epsilon_0$$

$$\mu \Leftrightarrow \mu/\mu_0$$

Notions of special relativity

- The basic laws of physics are the same in all frames of reference (at rest or moving at constant velocity).
- The speed of light in vacuum *c* is the same in all frames of reference.

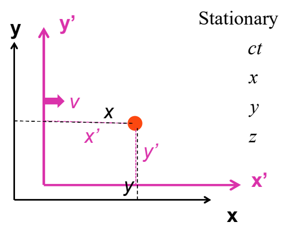


Lorentz transformations

Convenient notation :

$$\beta \equiv \frac{v}{c}$$

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$



Stationary frame		Moving frame
ct	$=$	$\gamma(ct'+\beta x')$
x	$=$	$\gamma(x'+\beta ct')$
y	$=$	y'
z	$=$	z'

Lorentz transformations -- continued

$\beta \equiv \frac{v}{c} \quad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$

For the moving frame with $\mathbf{v} = v\hat{\mathbf{x}}$:

$$\mathcal{L} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathcal{L}^{-1} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \mathcal{L} \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} \qquad \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \mathcal{L}^{-1} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

Notice:

$c^2t^2 - x^2 - y^2 - z^2 = c^2t'^2 - x'^2 - y'^2 - z'^2 = c^2\tau^2 \qquad \tau =$ proper time

Examples of other 4-vectors applicable to the Lorentz transformation:

$\beta \equiv \frac{v}{c} \quad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$

For the moving frame with $\mathbf{v} = v\hat{\mathbf{x}}$:

$$\mathcal{L} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathcal{L}^{-1} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \omega / c \\ k_x \\ k_y \\ k_z \end{pmatrix} = \mathcal{L} \begin{pmatrix} \omega' / c \\ k'_x \\ k'_y \\ k'_z \end{pmatrix} \qquad \begin{pmatrix} \omega' / c \\ k'_x \\ k'_y \\ k'_z \end{pmatrix} = \mathcal{L}^{-1} \begin{pmatrix} \omega / c \\ k_x \\ k_y \\ k_z \end{pmatrix}$$

Four-vectors:

Time and position: $\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \Rightarrow x^\alpha$ Frequency and wavevector: $\begin{pmatrix} \omega / c \\ k_x \\ k_y \\ k_z \end{pmatrix} \Rightarrow k^\alpha$

Charge and current: $\begin{pmatrix} c\rho \\ J_x \\ J_y \\ J_z \end{pmatrix} \Rightarrow J^\alpha$

Vector and scalar potentials: $\begin{pmatrix} \Phi \\ A_x \\ A_y \\ A_z \end{pmatrix} \Rightarrow A^\alpha$

4-vector relationships

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \Leftrightarrow \begin{pmatrix} A^0 \\ A^1 \\ A^2 \\ A^3 \end{pmatrix} \Leftrightarrow \left(A^0, \mathbf{A}\right): \text{ upper index 4-vector } A^\alpha \text{ for } (\alpha = 0,1,2,3)$$

Keeping track of signs -- lower index 4-vector $A_\alpha = \left(A^0, -\mathbf{A}\right)$

Derivative operators (defined with different sign convention):

$$\partial^\alpha = \left(\frac{\partial}{c\partial t}, -\nabla\right) \qquad \partial_\alpha = \left(\frac{\partial}{c\partial t}, \nabla\right)$$

Algebra rules for Lorentz transformations

For the moving frame with $\mathbf{v} = v\hat{\mathbf{x}}$:

$$\mathcal{L} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathcal{L}^{-1} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \omega/c \\ k_x \\ k_y \\ k_z \end{pmatrix} = \mathcal{L} \begin{pmatrix} \omega'/c \\ k'_x \\ k'_y \\ k'_z \end{pmatrix} \qquad \begin{pmatrix} \omega'/c \\ k'_x \\ k'_y \\ k'_z \end{pmatrix} = \mathcal{L}^{-1} \begin{pmatrix} \omega/c \\ k_x \\ k_y \\ k_z \end{pmatrix}$$

Using indices:

$\mathcal{L}^{\mu}_{\nu}$ $\mu \rightarrow$ row, $\nu \rightarrow$ column

$$k^\mu = \mathcal{L}^\mu_\nu k'^\nu$$

Example

Consider a wave in the moving frame:

$$k'^\mu = \begin{pmatrix} \omega'/c \\ \omega'/c \\ 0 \\ 0 \end{pmatrix} \quad \text{in the stationary frame:} \quad k^\mu = \mathcal{L}^\mu_\nu k'^\nu = \begin{pmatrix} \omega/c \\ \omega/c \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \omega = \omega' \sqrt{\frac{1+\beta}{1-\beta}}$$

Special theory of relativity and Maxwell's equations

Continuity equation: $\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$

Lorentz gauge condition: $\frac{1}{c}\frac{\partial \Phi}{\partial t} + \nabla \cdot \mathbf{A} = 0$

Potential equations: $\frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} - \nabla^2 \Phi = 4\pi\rho$

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2 \mathbf{A} = \frac{4\pi}{c}\mathbf{J}$$

Field relations: $\mathbf{E} = -\nabla \Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Lorentz transformations

$$\mathcal{L}(v) = \begin{pmatrix} \gamma_v & \gamma_v\beta_v & 0 & 0 \\ \gamma_v\beta_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Time and space: $x^\alpha = \mathcal{L}(v)x'^\alpha \equiv \mathcal{L}(v)^\alpha_\beta x'^\beta$

Charge and current: $J^\alpha = \mathcal{L}(v)J'^\alpha \equiv \mathcal{L}(v)^\alpha_\beta J'^\beta$

Vector and scalar potential: $A^\alpha = \mathcal{L}(v)A'^\alpha \equiv \mathcal{L}(v)^\alpha_\beta A'^\beta$

4-vector relationships

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \Leftrightarrow \begin{pmatrix} A^0 \\ A^1 \\ A^2 \\ A^3 \end{pmatrix} \Leftrightarrow \left(A^0, \mathbf{A}\right):\ \text{upper index 4-vector}\quad A^\alpha \text{ for } (\alpha = 0,1,2,3)$$

Keeping track of signs -- lower index 4-vector $A_\alpha = \left(A^0, -\mathbf{A}\right)$

Derivative operators (defined with different sign convention):

$$\partial^\alpha = \left(\frac{\partial}{c\partial t}, -\nabla\right) \qquad \partial_\alpha = \left(\frac{\partial}{c\partial t}, \nabla\right)$$

Special theory of relativity and Maxwell's equations

Continuity equation : $\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$ ➔ $\partial_\alpha J^\alpha = 0$

Lorentz gauge condition : $\frac{1}{c}\frac{\partial \Phi}{\partial t} + \nabla \cdot \mathbf{A} = 0$ ➔ $\partial_\alpha A^\alpha = 0$

Potential equations :

$$\left.\begin{array}{l} \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} - \nabla^2 \Phi = 4\pi\rho \\ \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2 \mathbf{A} = \frac{4\pi}{c}\mathbf{J} \end{array}\right\} \partial_\alpha \partial^\alpha A^\beta = \frac{4\pi}{c} J^\beta$$

Field relations :

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$

➔ ??

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Electric and Magnetic field relationships

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \qquad E_x = -\frac{\partial \Phi}{\partial x} - \frac{\partial A_x}{c\partial t} = -\left(\partial^0 A^1 - \partial^1 A^0\right)$$

$$E_y = -\frac{\partial \Phi}{\partial y} - \frac{\partial A_y}{c\partial t} = -\left(\partial^0 A^2 - \partial^2 A^0\right)$$

$$E_z = -\frac{\partial \Phi}{\partial z} - \frac{\partial A_z}{c\partial t} = -\left(\partial^0 A^3 - \partial^3 A^0\right)$$

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = -\left(\partial^2 A^3 - \partial^3 A^2\right)$$

$$B_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = -\left(\partial^3 A^1 - \partial^1 A^3\right)$$

$$B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = -\left(\partial^1 A^2 - \partial^2 A^1\right)$$

Field strength tensor $F^{\alpha\beta} \equiv \left(\partial^\alpha A^\beta - \partial^\beta A^\alpha\right)$

$$F^{\alpha\beta} \equiv \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

Transformation of field strength tensor

$$F^{\alpha\beta} = \mathcal{L}(v)^\alpha_\gamma F'^{\gamma\delta} \mathcal{L}(v)^\beta_\delta \qquad \mathcal{L}(v) = \begin{pmatrix} \gamma_v & \gamma_v\beta_v & 0 & 0 \\ \gamma_v\beta_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E'_x & -\gamma_v\left(E'_y + \beta_v B'_z\right) & -\gamma_v\left(E'_z - \beta_v B'_y\right) \\ E'_x & 0 & -\gamma_v\left(B'_z + \beta_v E'_y\right) & \gamma_v\left(B'_y - \beta_v E'_z\right) \\ \gamma_v\left(E'_y + \beta_v B'_z\right) & \gamma_v\left(B'_z + \beta_v E'_y\right) & 0 & -B'_x \\ \gamma_v\left(E'_z - \beta_v B'_y\right) & -\gamma_v\left(B'_y - \beta_v E'_z\right) & B'_x & 0 \end{pmatrix}$$

Inverse transformation of field strength tensor

$$F'^{\alpha\beta} = \mathcal{L}(v)^{-1\alpha}{}_{\gamma} F^{\gamma\delta} \mathcal{L}(v)^{-1\beta}{}_{\delta} \qquad \mathcal{L}(v)^{-1} = \begin{pmatrix} \gamma_v & -\gamma_v\beta_v & 0 & 0 \\ -\gamma_v\beta_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$F'^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -\gamma_v\left(E_y - \beta_v B_z\right) & -\gamma_v\left(E_z + \beta_v B_y\right) \\ E_x & 0 & -\gamma_v\left(B_z - \beta_v E_y\right) & \gamma_v\left(B_y + \beta_v E_z\right) \\ \gamma_v\left(E_y - \beta_v B_z\right) & \gamma_v\left(B_z - \beta_v E_y\right) & 0 & -B_x \\ \gamma_v\left(E_z + \beta_v B_y\right) & -\gamma_v\left(B_y + \beta_v E_z\right) & B_x & 0 \end{pmatrix}$$

Summary of results:

$$E'_x = E_x \qquad B'_x = B_x$$

$$E'_y = \gamma_v\left(E_y - \beta_v B_z\right) \qquad B'_y = \gamma_v\left(B_y + \beta_v E_z\right)$$

$$E'_z = \gamma_v\left(E_z + \beta_v B_y\right) \qquad B'_z = \gamma_v\left(B_z - \beta_v E_y\right)$$

Comparison of the two transformations

$$F^{\alpha\beta} = \mathcal{L}(v)^{\alpha}_{\gamma} F'^{\gamma\delta} \mathcal{L}(v)^{\beta}_{\delta} \qquad \mathcal{L}(v) = \begin{pmatrix} \gamma_v & \gamma_v\beta_v & 0 & 0 \\ \gamma_v\beta_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E'_x & -\gamma_v\left(E'_y + \beta_v B'_z\right) & -\gamma_v\left(E'_z - \beta_v B'_y\right) \\ E'_x & 0 & -\gamma_v\left(B'_z + \beta_v E'_y\right) & \gamma_v\left(B'_y - \beta_v E'_z\right) \\ \gamma_v\left(E'_y + \beta_v B'_z\right) & \gamma_v\left(B'_z + \beta_v E'_y\right) & 0 & -B'_x \\ \gamma_v\left(E'_z - \beta_v B'_y\right) & -\gamma_v\left(B'_y - \beta_v E'_z\right) & B'_x & 0 \end{pmatrix}$$

$$F'^{\alpha\beta} = \mathcal{L}(v)^{-1\alpha}{}_{\gamma} F'^{\gamma\delta} \mathcal{L}(v)^{-1\beta}{}_{\delta} \qquad \mathcal{L}(v)^{-1} = \begin{pmatrix} \gamma_v & -\gamma_v\beta_v & 0 & 0 \\ -\gamma_v\beta_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$F'^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -\gamma_v\left(E_y - \beta_v B_z\right) & -\gamma_v\left(E_z + \beta_v B_y\right) \\ E_x & 0 & -\gamma_v\left(B_z - \beta_v E_y\right) & \gamma_v\left(B_y + \beta_v E_z\right) \\ \gamma_v\left(E_y - \beta_v B_z\right) & \gamma_v\left(B_z - \beta_v E_y\right) & 0 & -B_x \\ \gamma_v\left(E_z + \beta_v B_y\right) & -\gamma_v\left(B_y + \beta_v E_z\right) & B_x & 0 \end{pmatrix}$$