

```

> restart; assume(a,'positive'); with(LinearAlgebra);
[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm,
BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column,
ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix,
ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation,
CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal,
DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers,
Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm,
GaussianElimination, GenerateEquations, GenerateMatrix, Generic,
GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt,
HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix,
HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal,
IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main,
LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd,
MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm,
MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor,
Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix,
Permanent, Pivot, PopovForm, QRDecomposition, RandomMatrix, RandomVector, Rank,
RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension,
RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm,
SingularValues, SmithForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis,
SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm,
UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply,
VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip ]

```

Define lattice translation

```
> T1 := Vector(3, [a, 0, 0]); T2 := Vector(3, [0, a, 0]); T3 := Vector(3, [0, 0, a]);
```

$$\begin{aligned}
T1 &:= \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} \\
T2 &:= \begin{bmatrix} 0 \\ a \\ 0 \end{bmatrix} \\
T3 &:= \begin{bmatrix} 0 \\ 0 \\ a \end{bmatrix}
\end{aligned} \tag{2}$$

Define reciprocal lattice translation

```
> G1 := Vector(3, [2·Pi/a, 0, 0]); G2 := Vector(3, [0, 2·Pi/a, 0]); G3 := Vector(3, [0, 0,
2·Pi/a]);
```

$$\begin{aligned}
 G1 &:= \begin{bmatrix} \frac{2\pi}{a\sim} \\ 0 \\ 0 \end{bmatrix} \\
 G2 &:= \begin{bmatrix} 0 \\ \frac{2\pi}{a\sim} \\ 0 \end{bmatrix} \\
 G3 &:= \begin{bmatrix} 0 \\ 0 \\ \frac{2\pi}{a\sim} \end{bmatrix}
 \end{aligned} \tag{3}$$

> tau := Vector(3, [0.5·a, 0.5·a, 0.5·a]);

$$\tau := \begin{bmatrix} 0.5 a\sim \\ 0.5 a\sim \\ 0.5 a\sim \end{bmatrix} \tag{4}$$

>

>  $\eta := \frac{4}{a^2}; \Omega := a^3; con1 := \frac{4 \cdot \text{Pi}}{\text{Omega}}; con2 := \sqrt{\frac{\eta}{\text{Pi}}};$

$$\eta := \frac{4}{a\sim^2}$$

$$\Omega := a\sim^3$$

$$con1 := \frac{4\pi}{a\sim^3}$$

$$con2 := \frac{2}{a\sim \sqrt{\pi}}$$

(5)

*Initial terms -- Cl-Cl and Cs-Cs*

>

> tot := -evalf(con2·2);

$$tot := -\frac{2.256758334}{a\sim} \tag{6}$$

> **for** n **from** -8 **by** 1 **while** n < 8 **do**  
**for** l **from** -8 **by** 1 **while** l < 8 **do**

**for** m **from** -8 **by** 1 **while** m < 8 **do**  
**if** (n ≠ 0 or m ≠ 0 or l ≠ 0)

then g := (n·G1 + m·G2 + l·G3) ;

$$tot := tot + evalf\left(2 \cdot con1 \cdot (1 - \exp(-I))\right)$$

$$\begin{aligned}
 & \cdot \text{DotProduct}(g, \tau)) \cdot \frac{\exp\left(-\frac{\text{DotProduct}(g, g)}{\text{eta}}\right)}{\text{DotProduct}(g, g)} \Bigg) \quad \text{end if end do end do end do;} \\
 & \text{evalf}(tot); \\
 & - \frac{2.256758334}{a\sim} + \frac{0.0003951360355 + 1.906745200 \cdot 10^{-24} I}{a\sim}
 \end{aligned} \tag{7}$$

$$\begin{aligned}
 > \text{for } n \text{ from } -8 \text{ by } 1 \text{ while } n < 8 \text{ do} & \text{for } m \text{ from } -8 \text{ by } 1 \text{ while } m < 8 \text{ do} \\
 & \text{for } l \text{ from } -8 \text{ by } 1 \text{ while } l < 8 \text{ do} \quad t := (n \cdot T1 + m \cdot T2 + l \cdot T3) ; \\
 & \quad tot := tot - \text{evalf}\left(\frac{2 \cdot \left(\text{erfc}\left(\frac{\text{sqrt}(\text{eta})}{2} \cdot \text{VectorNorm}(\tau + t, 2)\right)\right)}{\text{VectorNorm}(\tau + t, 2)}\right) ; \\
 & \text{if } (n \neq 0 \text{ or } m \neq 0 \text{ or } l \neq 0) \text{ then} \quad tot := tot \\
 & \quad + \text{evalf}\left(\frac{2 \cdot \text{erfc}\left(\frac{\text{sqrt}(\text{eta})}{2} \cdot \text{VectorNorm}(t, 2)\right)}{\text{VectorNorm}(t, 2)}\right) \text{ end if end do end do end do;} \\
 & \text{evalf}(tot); \\
 & - \frac{4.071118106}{a\sim} + \frac{0.0003951360355 + 1.906745200 \cdot 10^{-24} I}{a\sim}
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 > \text{Re}(\%); \\
 & - \frac{4.070722970}{a\sim}
 \end{aligned} \tag{9}$$