**PHY 712 Electrodynamics
9-9:50 AM MWF Olin 103**

Plan for Lecture 1:

Reading: Appendix 1 and Chapters I&1

1. **Course structure and expectations**
2. **Units – SI vs Gaussian**
3. **Electrostatics – Poisson equation**

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http://users.wfu.edu/natalie/s20phy712/

PHY 712 Electrodynamics

MWF 9-9:50 AM | OPL 103 | http://www.wfu.edu/~natalie/s20phy712/

Instructor: Natalie Holzwarth **Phone:**758-5510 **Office:**300 OPL **e-mail:**natalie@wfu.edu

- General information
- Syllabus and homework assignments
- Lecture notes
- Some presentation ideas

Last modfied: Monday, 06-Jan-2020 11:09:02 EST

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**Spring 2020 Schedule
for N. A. W. Holzwarth**

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-10:00	Electrodynamics PHY712	Physics Research	Electrodynamics PHY712	Physics Research	Electrodynamics PHY712
10:00-12:30	Office Hours		Office Hours		Office Hours
12:30-1:00				Condensed Matter Theory Journal Club	
1:00-2:00	Quantum Mechanics II PHY742		Quantum Mechanics II PHY742		Quantum Mechanics II PHY742
2:00-3:00	Physics Research		Physics Research	Physics Research	Physics Research
3:00-5:00			Physics Colloquium		

Please email natalie@wfu.edu if you would like to meet with me at any time.

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Physics Colloquium series – Wednesdays 3-4 PM
followed by refreshments

WFU Physics Colloquium Schedule — Spring 2020

Previous and future colloquia

All seminars will be held at 3 PM in Room 101 of Olin Physical Laboratory (unless noted otherwise); there will be refreshments at approximately 4:00 PM in the lobby following each seminar.

Wed. Jan. 15, 2020 — Dr. Subhasish Mandal, Rutgers University, New Jersey –"First-Principles Investigation on Quantum Materials Using Beyond-DFT Methods" (host: D. Kim-Shapiro)

Wed. Jan. 22, 2020 — Dr. Ongun Ozcelik, University of California, San Diego (host: D. Kim-Shapiro)

Wed. Jan. 29, 2020 — Dr. Steve Winter, Institute for Theoretical Physics, Frankfurt Germany (host: D. Kim-Shapiro)

Wed. Feb. 5, 2020 — Dr. Sharmila Shirodkar, Rice University, Houston, TX (host: D. Kim-Shapiro)

Wed. Feb. 12, 2020 — Samuel D. Flynn, WFU alum and NCSU graduate student (host: P. Anderson)

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Physics Colloquium series – Wednesdays 3-4 PM
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WFU Physics Colloquium Schedule — Spring 2020

Previous and future colloquia

All seminars will be held at 3 PM in Room 101 of Olin Physical Laboratory (unless noted otherwise); there will be refreshments at approximately 4:00 PM in the lobby following each seminar.

Wed. Mar. 18, 2020 — Professor April M. Kloxin,University of Delaware (host: O. Jurchescu)

Wed. Mar. 25 2020 — Dr. Samindranath Mitra, Editor, Physical Review Letters (host: N. Holzwarth)

Wed. Apr. 1, 2020 — Dr. Gail H. Marcus, Independent Consultant on Nuclear Power Technology and Policy based in Washington DC — "From Fukushima to the Future: Lessons Learned and New Developments" (Part of Special Event: "Remembering the Fukushima Nuclear Disaster" in coordination with the Department of East Asian Languages and Cultures; physics host: D. Kim-Shapiro)

Thurs. Apr. 2, 2020 — Dr. Michael J. Marcus, Marcus Spectrum Solutions LLC — "Can we have BOTH 5G mobile serive and good hurrican forecasts? — Anatomy of a difficult technical public policy dilemma" (host: D. Kim-Shapiro)
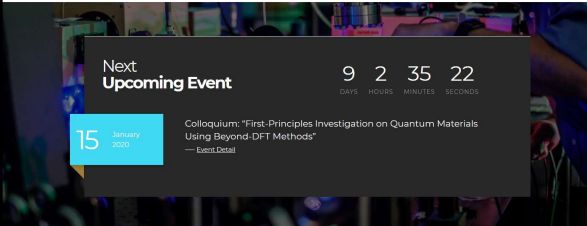
Wed. Apr. 8 2020 — Professor Sandra Liss, Radford University (host: P. Anderson)

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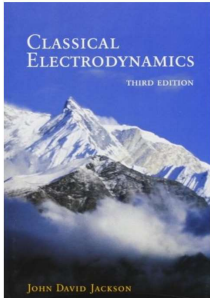New notification of events on Physics web page –
https://www.physics.wfu.edu/
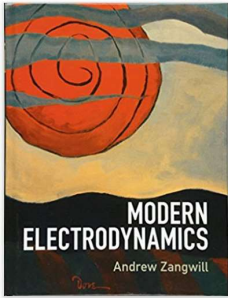
WFU PHYSICS

Next
Upcoming Event

9 2 35 22
DAYS HOURS MINUTES SECONDS

15 January 2020

Colloquium: "First-Principles Investigation on Quantum Materials Using Beyond-DFT Methods"
— Event Detail

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Textbook

Optional supplement

CLASSICAL ELECTRODYNAMICS
THIRD EDITION

JOHN DAVID JACKSON

MODERN ELECTRODYNAMICS
Andrew Zangwill

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PHY 712 Electrodynamics

MWF 9-9:50 AM | OPL 103 | http://www.wfu.edu/~natalie/s20phy712/

Instructor: Natalie Holzwarth | Phone: 758-5510 | Office: 300 OPL | e-mail: natalie@wfu.edu

General Information

This course is a one semester survey of Electrodynamics at the graduate level, using the textbook: **Classical Electrodynamics**, 3rd edition, by John David Jackson (John Wiley & Sons, Inc., 1999) -- "**JDJ**". link to errata The more recent textbook: **Modern Electrodynamics**, by Andrew Zangwill (Cambridge University Press, 2013) will be used as a supplement.

It is likely that your grade for the course will depend upon the following factors:

Problem sets*	45%
Presentation	10%
Exams	45%

*The schedule notes the "due" date for each assignment. Homeworks may be turned in 1 lecture past their due date without grade penalty. After that, the homework grade will be reduced by 10% for each succeeding late date. According to the honor system, all work submitted for grading purposes should represent the student's own best efforts. This means that students who work together on homework assignments should all contribute roughly equally and independently verify all derivations and results.

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Course schedule for Spring 2020

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	HW	Due date
1	Mon: 01/13/2020	Chap. 1 & Appen.	Introduction, units and Poisson equation	#1	01/17/2020
2	Wed: 01/15/2020	Chap. 1	Electrostatic energy calculations		
3	Fri: 01/17/2020	Chap. 1	Electrostatic potentials and fields		
	Mon: 01/20/2020	No class	Martin Luther King Holiday		
4	Wed: 01/22/2020	Chap. 1 - 3	Poisson's equation in 2 and 3 dimensions		
5	Fri: 01/24/2020	Chap. 1 - 3	Brief introduction to numerical methods		
6	Mon: 01/27/2020	Chap. 2 & 3	Image charge constructions		
7	Wed: 01/29/2020	Chap. 2 & 3	Cylindrical and spherical geometries		
8	Fri: 01/31/2020	Chap. 3 & 4	Spherical geometry and multipole moments		
9	Mon: 02/03/2020	Chap. 4	Dipoles and Dielectrics		

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Comment about HW #1: (Jackson problem 1.5)

The time-averaged potential of a neutral hydrogen atom is given by:

$$\Phi(r) = \frac{q}{4\pi\epsilon_0} \frac{e^{-2r/a_0}}{r}\left(1 + \frac{r}{a_0}\right)$$

where q denotes the magnitude of the elementary charge of an electron or proton and where a_0 denotes the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your results physically.

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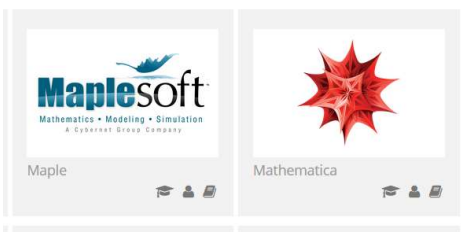
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Additional information –
  Mid-term exam March 2-6, 2020 (same as QM II)
  Presentations for both E&M and QM II – will be
    further discussed in QM II

Question – handouts or materials only on web?

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Remember to check your algebraic manipulation software --

https://software.wfu.edu/audience/students/

Maple   Mathematica

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Some Ideas for Computational Project

The purpose of the "Computational Project" is to provide an opportunity for you to study a topic of your choice in greater depth. The general guideline for your choice of project is that it should have something to do with electrodynamics, and there should be some degree of computation or analysis with the project. The completed project will include a short write-up and a ~20min presentation to the class. You may design your own project or use one of the following list (which will be updated throughout the term).

- Evaluate the Ewald sum of various ionic crystals using Maple or a programing language. (Template available in Fortran code.)
- Work out the details of the finite difference or finite element methods.
- Work out the details of the hyperfine Hamiltonian as discussed in Chapter 5 of Jackson.
- Work out the details of Jackson problem 7.2 and related problems.
- Work out the details of reflection and refraction from birefringent materials.
- Analyze the Kramers-Kronig transform of some optical data or calculations.
- Determine the classical electrodynamics associated with an infrared or optical laser.
- Analyze the radiation intensity and spectrum from an interesting source such as an atomic or molecular transition, a free electron laser, etc.
- Work out the details of Jackson problem 14.15.

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Material discussed in Appendix of textbook --

Units - SI vs Gaussian

Coulomb's Law

$$F = K_C \frac{q_1 q_2}{r_{12}^2}.\qquad(1)$$

Ampere's Law

$$F = K_A \frac{i_1 i_2}{r_{12}^2} \, d\mathbf{s_1} \times d\mathbf{s_2} \times \hat{\mathbf{r}}_{12},\qquad(2)$$

In the equations above, the current and charge are related by $i_1 = dq_1/dt$ for all unit systems. The two constants K_C and K_A are related so that their ratio K_C/K_A has the units of $(m/s)^2$ and it is *experimentally* known that the ratio has the value $K_C/K_A = c^2$, where c is the speed of light.

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Units - SI vs Gaussian – continued

The choices for these constants in the SI and Gaussian units are given below:

	CGS (Gaussian)	SI
K_C	1	$\frac{1}{4\pi\epsilon_0}$
K_A	$\frac{1}{c^2}$	$\frac{\mu_0}{4\pi}$

Here, $\frac{\mu_0}{4\pi} \equiv 10^{-7} N/A^2$ and $\frac{1}{4\pi\epsilon_0} = c^2 \cdot 10^{-7} N/A^2 = 8.98755 \times 10^9 N \cdot m^2/C^2.$

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Units - SI vs Gaussian – continued

Below is a table comparing SI and Gaussian unit systems. The fundamental units for each system are so labeled and are used to define the derived units.

Variable	SI		Gaussian		SI/Gaussian
	Unit	Relation	Unit	Relation	
length	m	fundamental	cm	fundamental	100
mass	kg	fundamental	gm	fundamental	1000
time	s	fundamental	s	fundamental	1
force	N	$kg \cdot m^2/s$	$dyne$	$gm \cdot cm^2/s$	10^5
current	A	fundamental	$statampere$	$statcoulomb/s$	$\frac{1}{10c}$
charge	C	$A \cdot s$	$statcoulomb$	$\sqrt{dyne \cdot cm^2}$	$\frac{1}{10c}$

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Units - SI vs Gaussian – continued

One advantage of the Gaussian system is that the field vectors: $\mathbf{E}, \mathbf{D}, \mathbf{B}, \mathbf{H}, \mathbf{P}, \mathbf{M}$ all have the same physical dimensions., In vacuum, the following equalities hold: $\mathbf{B} = \mathbf{H}$ and $\mathbf{E} = \mathbf{D}$. Also, in the Gaussian system, the dielectric and permittivity constants ϵ and μ are dimensionless.

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Basic equations of electrodynamics

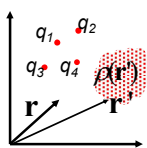| CGS (Gaussian)	SI
$\nabla \cdot \mathbf{D} = 4\pi\rho$	$\nabla \cdot \mathbf{D} = \rho$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$
$\mathbf{F} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})$	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
$u = \frac{1}{8\pi}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$	$u = \frac{1}{2}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$
$\mathbf{S} = \frac{c}{4\pi}(\mathbf{E} \times \mathbf{H})$	$\mathbf{S} = (\mathbf{E} \times \mathbf{H})$

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Units choice for this course:
SI units for Jackson in Chapters 1-10
Gaussian units for Jackson in Chapters 11-16

Electrostatics

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_i q_i \frac{\mathbf{r}-\mathbf{r}_i}{|\mathbf{r}-\mathbf{r}_i|^3}$$

$$= \frac{1}{4\pi\epsilon_0}\int d^3 r' \rho(\mathbf{r}') \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}$$

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Electrostatics

Discrete versus continuous charge distributions

In terms of Dirac delta function:

$$\rho(\mathbf{r}) = \sum_i q_i \ \delta(\mathbf{r}-\mathbf{r}_i)$$

Digression: Note that in cartesian coordinates --

$$\delta(\mathbf{r}-\mathbf{r}_i) = \delta(x-x_i)\delta(y-y_i)\delta(z-z_i)$$

in spherical polar coordinates --

$$\delta(\mathbf{r}-\mathbf{r}_i) = \frac{1}{r^2}\delta(r-r_i)\delta(\cos\theta - \cos\theta_i)\delta(\phi - \phi_i)$$

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Differential equations --

Electrostatics

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$$

$$\nabla \times \mathbf{E} = 0$$

Electrostatic potential

$$\mathbf{E} = -\nabla\Phi(r).$$

$$\nabla^2\Phi(r) = -\rho(r)/\epsilon_0.$$

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Relationship between integral and differential forms of electrostatics --

Differential form

$\nabla^2 \Phi(\mathbf{r}) = -\rho(\mathbf{r}) / \epsilon_0$

Integral form

$\Phi(\mathbf{r}) =$

$\frac{1}{4\pi\epsilon_0} \int d^3 r' \frac{\rho(\mathbf{r'})}{|\mathbf{r} - \mathbf{r'}|}$

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Relationship between integral and differential forms of electrostatics --

Need to show: $\nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r'}|} \right) = -4\pi \delta^3 (\mathbf{r} - \mathbf{r'}).$

Noting that

$\int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r'}}} d^3 r \, \delta^3(\mathbf{r} - \mathbf{r'}) f(\mathbf{r}) = f(\mathbf{r'}),$

we see that we must show that

$\int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r'}}} d^3 r \, \nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r'}|} \right) f(\mathbf{r}) = -4\pi f(\mathbf{r'}).$

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We introduce a small radius a such that:

$$\frac{1}{|\mathbf{r} - \mathbf{r'}|} = \lim_{a \to 0} \frac{1}{\sqrt{|\mathbf{r} - \mathbf{r'}|^2 + a^2}}.$$

For a fixed value of a,

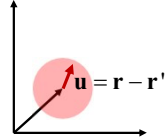$$\nabla^2 \frac{1}{\sqrt{|\mathbf{r} - \mathbf{r'}|^2 + a^2}} = \frac{-3a^2}{(|\mathbf{r} - \mathbf{r'}|^2 + a^2)^{5/2}}.$$

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If the function $f(\mathbf{r})$ is continuous, we can make a Taylor expansion of it about the point $\mathbf{r} = \mathbf{r}'$, keeping only the first term. The integral over the small sphere about $\mathbf{r}'$ can be carried out analytically, by changing to a coordinate system centered at $\mathbf{r}'$;

so that

$$\int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \ \nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) f(\mathbf{r}) \approx f(\mathbf{r}') \lim_{a \to 0} \int_{u < R} d^3u \frac{-3a^2}{(u^2 + a^2)^{5/2}}.$$

$$\mathbf{u} = \mathbf{r} - \mathbf{r}'$$

$$\int_{u < R} d^3u \frac{-3a^2}{(u^2 + a^2)^{5/2}} = 4\pi \int_0^R du \frac{-3a^2 u^2}{(u^2 + a^2)^{5/2}} = 4\pi \frac{-R^3}{(R^2 + a^2)^{3/2}}.$$

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$$\int_{u < R} d^3u \frac{-3a^2}{(u^2 + a^2)^{5/2}} = 4\pi \int_0^R du \frac{-3a^2 u^2}{(u^2 + a^2)^{5/2}} = 4\pi \frac{-R^3}{(R^2 + a^2)^{3/2}}.$$

$$\text{For } a \ll R, \quad 4\pi \frac{-R^3}{\left(R^2 + a^2\right)^{3/2}} \approx -4\pi$$

➔ $$\int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r \ \nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) f(\mathbf{r}) \approx f(\mathbf{r}')(-4\pi),$$

➔ $$\nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) = -4\pi \delta^3(\mathbf{r} - \mathbf{r}')$$

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Example in HW1

The electrostatic potential of a neutral H atom is given by:

$$\Phi(r) = \frac{q}{4\pi\epsilon_0} \frac{e^{-\alpha r}}{r} \left(1 + \frac{\alpha r}{2}\right).$$

Find the charge density (both continuous and discrete) for this potential.

Hint #1: For continuous contribution you can use

the identity: $$\nabla^2 \Phi(r) = \frac{1}{r} \frac{\partial^2 (r\Phi(r))}{\partial r^2}$$

Hint #2: Don't forget to consider possible discrete contributions.

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