PHY 712 Electrodynamics

12-12:50 AM MWF via video link:
https://wakeforest-university.zoom.us/my/natalie.holzwarth

Plan for Lecture 29:

Finish reading Chap. 14 –

Radiation from charged particles

1. **Review of synchrotron radiation**
2. **Free electron laser**

In this lecture, we will summarize what we learned about synchrotron radiation and also discuss some other radiation results

21	Mon: 03/23/2020	Chap. 9	Radiation from localized oscillating sources	#17	03/25/2020
22	Wed: 03/25/2020	Chap. 9	Radiation from oscillating sources	#18	03/27/2020
23	Fri: 03/27/2020	Chap. 9 and 10	Radiation from oscillating sources	#19	03/30/2020
24	Mon: 03/30/2020	Chap. 11	Special Theory of Relativity	#20	04/03/2020
25	Wed: 04/01/2020	Chap. 11	Special Theory of Relativity		
26	Fri: 04/03/2020	Chap. 11	Special Theory of Relativity	#21	04/06/2020
27	Mon: 04/06/2020	Chap. 14	Radiation from accelerating charged particles	#22	04/08/2020
28	Wed: 04/08/2020	Chap. 14	Synchrotron radiation		
	Fri: 04/10/2020	No class	*Good Friday*		
29	Mon: 04/13/2020	Chap. 14	Synchrotron radiation	#23	04/15/2020
30	Wed: 04/15/2020	Chap. 15	Radiation from collisions of charged particles		
31	Fri: 04/17/2020	Chap. 13	Cherenkov radiation		
32	Mon: 04/20/2020		Special topic: E & M aspects of superconductivity		
33	Wed: 04/22/2020		Special topic: Aspects of optical properties of materials		
34	Fri: 04/24/2020				
35	Mon: 04/27/2020				
36	Wed: 04/29/2020		Review		

One hopefully simple homework for this lecture.

Comment on HW 22

PHY 712 -- Assignment #22

April 6, 2020

Continue reading Chap. 14 in **Jackson** .

1. Consider an electron moving at constant speed $\beta c \approx c$ in a circular trajectory of radius ρ. Its total energy is $E = \gamma m c^2$. Determine the ratio of the energy lost during one full cycle to its total energy. Evaluate the expression for an electron with total energy 400 GeV in a synchroton of radius $\rho = 10^3$ m.

According to Jackson Eq. 14.32, the radiated power in one cycle is

$$\delta E = \frac{4\pi}{3}\frac{e^2}{\rho}\beta^3\gamma^4 \quad \text{while the particle energy is } E = \gamma mc^2$$

$$\frac{\delta E}{E} = \frac{4\pi}{3}\frac{e^2}{mc^2\rho}\beta^3\gamma^3$$

What is β? What is γ? What is mc^2?

Some hints about “back of the envelope” calculation methods.

Advice about remaining topics to cover

- Cherenkov radiation
- Radiation from collisions of charged particles
- Electrodynamics of superconductivity
- Optical properties of materials

Please vote on these or suggest other topics that you prefer to cover, time permitting.

Spectral composition of electromagnetic radiation -- continued

The spectral intensity per unit solid angle Ω and frequency ω for a particle of charge q, with trajectory $\mathbf{R}_q(t)$ and velocity $\frac{d\mathbf{R}_q(t)}{dt} \equiv \boldsymbol{\beta}(t)c$, depends on the following integral:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt\; e^{i\omega\left(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right)}\;\left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}(t)\right)\right]\right|^2$$

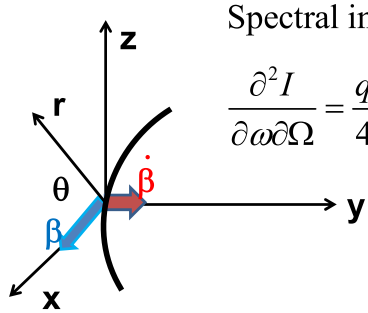
Recall that the spectral intensity is related to the time integrated power:

$$\int_{-\infty}^{\infty} dt \frac{dP(t)}{d\Omega} = \int_{-\infty}^{\infty} d\omega \frac{\partial^2 I}{\partial\omega\partial\Omega}$$

Review of key points.

Spectral intensity relationship:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt\; e^{i\omega\left(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right)}\;\left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}(t)\right)\right]\right|^2$$



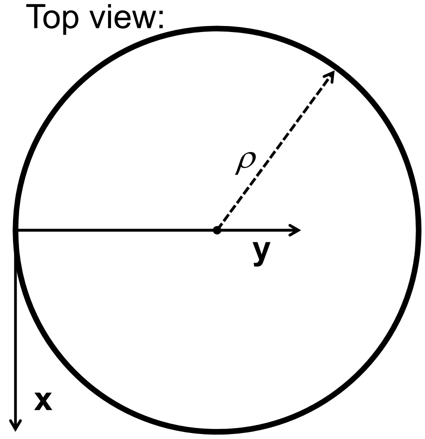
$$\mathbf{R}_q(t) = \rho\hat{\mathbf{x}}\sin(vt/\rho) + \rho\hat{\mathbf{y}}\left(1-\cos(vt/\rho)\right)$$

$$\boldsymbol{\beta}(t) = \beta\left(\hat{\mathbf{x}}\cos(vt/\rho) + \hat{\mathbf{y}}\sin(vt/\rho)\right)$$

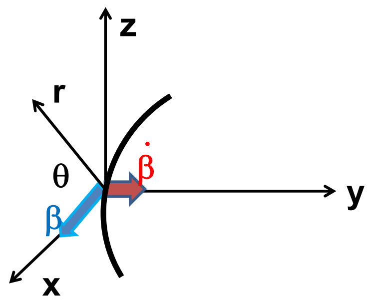
For convenience, choose:

$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$

Top view:



Geometry of particle moving in a circular path.



$$\mathbf{R}_q(t) = \rho\hat{\mathbf{x}}\sin(vt/\rho) + \rho\hat{\mathbf{y}}\left(1-\cos(vt/\rho)\right)$$

$$\boldsymbol{\beta}(t) = \beta\left(\hat{\mathbf{x}}\cos(vt/\rho) + \hat{\mathbf{y}}\sin(vt/\rho)\right)$$

For convenience, choose:

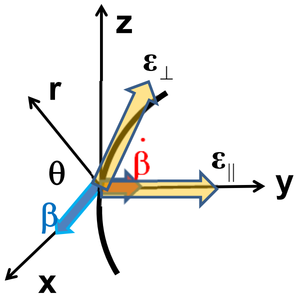
$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$

Note that we have previous shown that in the radiation zone, the Poynting vector is in the $\hat{\mathbf{r}}$ direction; we can then choose to analyze two orthogonal polarization directions:

$$\boldsymbol{\varepsilon}_{\parallel} = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_{\perp} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$$

$$\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \boldsymbol{\beta}\right) = \beta\left(-\boldsymbol{\varepsilon}_{\parallel}\sin(vt/\rho) + \boldsymbol{\varepsilon}_{\perp}\sin\theta\cos(vt/\rho)\right)$$

Note that there are two perpendicular polarizations conveniently defined as shown.



$$\boldsymbol{\varepsilon}_{\parallel} = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_{\perp} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$$

$$\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \boldsymbol{\beta}\right) =$$

$$\beta\left(-\boldsymbol{\varepsilon}_{\parallel}\sin\left(vt/\rho\right) + \boldsymbol{\varepsilon}_{\perp}\sin\theta\cos\left(vt/\rho\right)\right)$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} \hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\beta}) \mathrm{e}^{i\omega(t - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)} dt\right|^2$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2\omega^2\beta^2}{4\pi^2 c}\left\{ |C_{\parallel}(\omega)|^2 + |C_{\perp}(\omega)|^2 \right\} \qquad \text{where} \quad \beta \equiv \frac{v}{c}$$

$$C_{\parallel}(\omega) = \int_{-\infty}^{\infty} dt \sin(vt/\rho) \mathrm{e}^{i\omega(t - \frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$C_{\perp}(\omega) = \int_{-\infty}^{\infty} dt \sin\theta\cos(vt/\rho) \mathrm{e}^{i\omega(t - \frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

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The analytic expressions for the radiation amplitudes for the two polarizations are given here.

PHYSICAL REVIEW VOLUME 75, NUMBER 12 JUNE 15, 1949

On the Classical Radiation of Accelerated Electrons

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts

(Received March 8, 1949)

This paper is concerned with the properties of the radiation from a high energy accelerated electron, as recently observed in the General Electric synchrotron. An elementary derivation of the total rate of radiation is first presented, based on Larmor's formula for a slowly moving electron, and arguments of relativistic invariance. We then construct an expression for the instantaneous power radiated by an electron moving along an arbitrary, prescribed path. By casting this result into various forms, one obtains the angular distribution, the spectral distribution, or the combined angular and spectral distributions of the radiation. The method is based on an examination of the rate at which the electron irreversibly transfers energy to the electromagnetic field, as determined by half the difference of retarded and advanced electric field intensities. Formulas are obtained for an arbitrary charge-current distribution and then specialized to a point charge. The total radiated power and its angular distribution are obtained for an arbitrary trajectory. It is found that the direction of motion is a strongly preferred direction of emission at high energies. The spectral distribution of the radiation depends upon the detailed motion over a time interval large compared to the period of the radiation. However, the narrow cone of radiation generated by an energetic electron indicates that only a small part of the trajectory is effective in producing radiation observed in a given direction, which also implies that very high frequencies are emitted. Accordingly, we evaluate the spectral and angular distributions of the high frequency radiation by an energetic electron, in their dependence upon the parameters characterizing the instantaneous orbit. The average spectral distribution, as observed in the synchrotron measurements, is obtained by averaging the electron energy over an acceleration cycle. The entire spectrum emitted by an electron moving with constant speed in a circular path is also discussed. Finally, it is observed that quantum effects will modify the classical results here obtained only at extraordinarily large energies.

EARLY in 1945, much attention was focused on the design of accelerators for the production of very high energy electrons and other charged particles.[1] In connection with this activity, the author investigated in some detail the limitations to the

is instantaneously at rest is

$$P=\frac{2}{3}\frac{e^2}{c^3}\left(\frac{d\mathbf{v}}{dt}\right)^2=\frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{d\mathbf{p}}{dt}\right)^2. \qquad \text{(I.1)}$$

These results were obtained by Julian Schwinger in 1949.

Evaluation of integrals representing distant charged particles moving in a circular trajectory such that the spectrum represents a superposition of light generated over many complete circles. In this case, there is an interference effect which results in the spectrum consisting of discrete multiples of v/ρ.

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2\omega^2\beta^2}{4\pi^2 c}\left\{|C_{\parallel}(\omega)|^2 + |C_{\perp}(\omega)|^2\right\}$$

$$C_{\parallel}(\omega) = \int_{-\infty}^{\infty} dt \sin(vt/\rho) \mathrm{e}^{i\omega(t - \frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$C_{\perp}(\omega) = \int_{-\infty}^{\infty} dt \sin\theta\cos(vt/\rho) \mathrm{e}^{i\omega(t - \frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

Schwinger directly evaluated the amplitude equations using some cleaver identities.

Useful identity involving Bessel functions

$$\mathrm{e}^{-iA\sin\alpha} = \sum_{m=-\infty}^{\infty} J_m(A)\mathrm{e}^{-im\alpha} \qquad \text{Here } J_m(A) \text{ is a}$$

Bessel function of integer order m.

In our case, $A = \frac{\omega\rho}{c}\cos\theta$ and $\alpha = \frac{vt}{\rho}$.

$$C_{\parallel}(\omega) = \int_{-\infty}^{\infty} dt \sin(vt/\rho)\mathrm{e}^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$= \frac{c}{-i\omega\rho}\frac{\partial}{\partial\cos\theta}\int_{-\infty}^{\infty} dt\mathrm{e}^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$= \frac{c}{-i\omega\rho}\frac{\partial}{\partial\cos\theta}\sum_{m=-\infty}^{\infty} J_m\left(\frac{\omega\rho}{c}\cos\theta\right)2\pi\delta(\omega - m\frac{v}{\rho}).$$

Some of the identities repeated from last time.

Astronomical synchrotron radiation -- continued:

Note that:

$$\int_{-\infty}^{\infty} dt e^{i(\omega - m\frac{v}{\rho})t} = 2\pi\delta(\omega - m\frac{v}{\rho}).$$

$$\Rightarrow C_{\parallel}(\omega) = 2\pi i \sum_{m=-\infty}^{\infty} J_m^{'}\left(\frac{\omega\rho}{c}\cos\theta\right)\delta(\omega - m\frac{v}{\rho}),$$

where $J_m^{'}(A) \equiv \frac{dJ_m(A)}{dA}$

Similarly:

$$C_{\perp}(\omega) = \int_{-\infty}^{\infty} dt \sin\theta \cos(vt/\rho) e^{i\omega(t - \frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$= 2\pi \frac{\tan\theta}{v/c} \sum_{m=-\infty}^{\infty} J_m\left(\frac{\omega\rho}{c}\cos\theta\right)\delta(\omega - m\frac{v}{\rho}).$$

More details.

Astronomical synchrotron radiation -- continued:

In both of the expressions, the sum over *m* includes both negative and positive values. However, only the positive values of ω and therefore positive values of *m* are of interest. Using the identity: $J_{-m}(A) = (-1)^m J_m(A)$, the result becomes:

$$\frac{d^2I}{d\omega d\Omega} =$$

$$\frac{q^2\omega^2\beta^2}{c}\sum_{m=0}^{\infty}\delta(\omega - m\frac{v}{\rho})\left\{\left[J_m'\left(\frac{\omega\rho}{c}\cos\theta\right)\right]^2 + \frac{\tan^2\theta}{v^2/c^2}\left[J_m\left(\frac{\omega\rho}{c}\cos\theta\right)\right]^2\right\}.$$

These results were derived by Julian Schwinger (Phys. Rev. **75**, 1912-1925 (1949)). The discrete case is similar to the result quoted in Problem 14.15 in Jackson's text.

Comment on the relevance of the results.

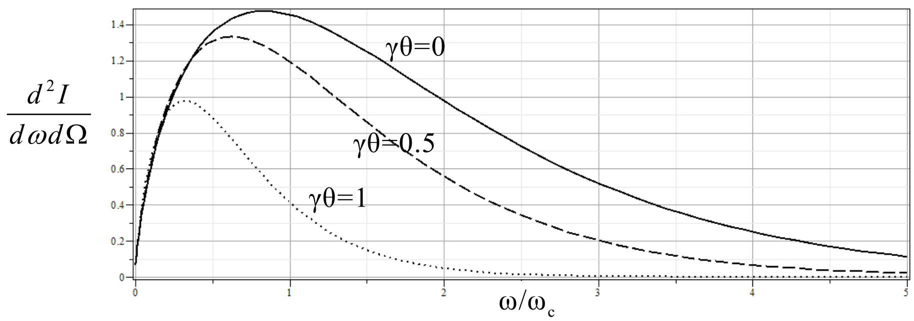
Recall the previous result for man-made synchrotrons used as light sources. In this case, the light is produced by short bursts of electrons moving close to the speed of light ($v \approx c(1-1/(2\gamma^2))$) passing a beam line port. In addition, because of the design of the radiation ports, $\theta \approx 0$, and the relevant integration times t are close to $t \approx 0$. This results in the form shown in Eq. 14.79 of your text. It is convenient to rewrite this form in terms of a critical frequency $\omega_c \equiv \dfrac{3c\gamma^3}{2\rho}$. In that range, the differential intensity takes the form:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{3q^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2 (1+\gamma^2\theta^2)^2 \left\{\left[K_{2/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2 + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}\left[K_{1/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2\right\}$$

At the other extreme, for man made synchrotrons, the intensity pattern is concentrated tangentially to electron path as represented by the Bessel functions of the third kind. It is convenient to define the critical frequency omega$_c$.

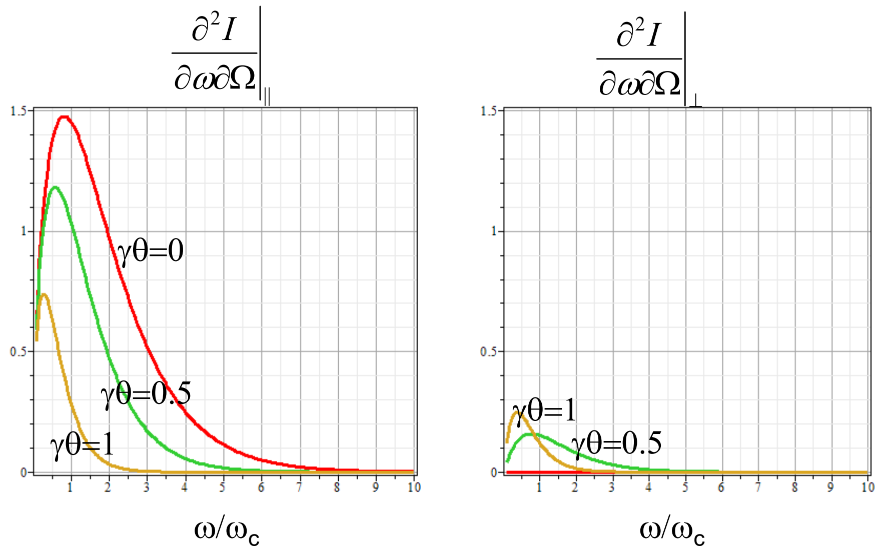
$$\frac{d^2I}{d\omega d\Omega} = \frac{3q^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2 (1+\gamma^2\theta^2)^2 \left\{\left[K_{2/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2 + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}\left[K_{1/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2\right\}$$

By plotting the intensity as a function of ω, we see that the intensity is largest near $\omega \approx \omega_c$. The plot below shows the intensity as a function of ω/ω_c for $\gamma\theta=0$, 0.5 and 1:



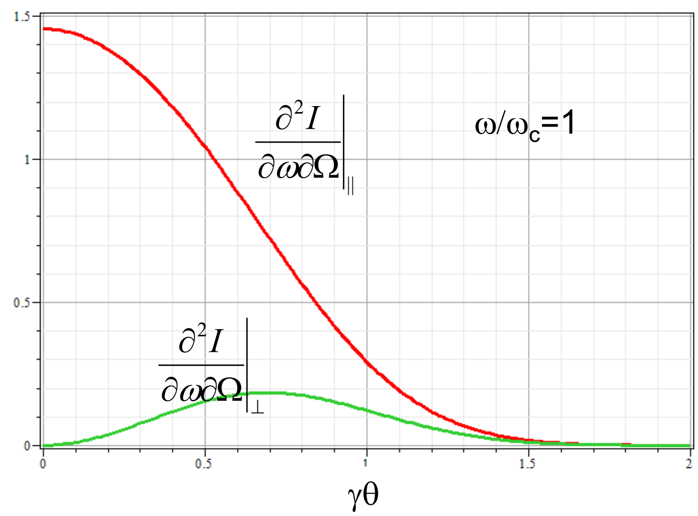
In this case the spectral frequency is scaled by the critical frequency omega$_c$.

Plots of the intensity function for synchrotron radiation



The intensity is highly peaked at angle 0 and is much more intense for the "parallel" polarization.

Plots of the intensity function for synchrotron radiation



Plots of the intensity at the critical frequency as a function of angle

Synchrotron facilities worldwide
https://lightsources.org/

There are more than 50 light sources in the world (operational, or under construction). This page lists all the members of the lightsources.org collaboration.

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If you are interested in man made light sources, this website, collects information from many installations throughout the world.

Free electron laser: Classical Theory of Free-Electron Lasers

A text for students and researchers

Eric B Szarmes

Department of Physics and Astronomy, University of Hawai'i at Mānoa,

ISBN 978-1-6270-5573-4 (ebook)
ISBN 978-1-6270-5572-7 (print)
ISBN 978-1-6270-5680-9 (mobi)

DOI 10.1088/978-1-6270-5573-4

https://iopscience.iop.org/book/978-1-6270-5573-4

Another technology that generates intense light, is the so-called Free Electron Laser. This reference is a good source for some of the design features and background physics.

1.1 The free-electron laser

A free-electron laser (FEL) is a laser source that produces spatially and temporally coherent optical radiation by stimulated emission, where in place of an atomic or molecular medium to provide amplification the gain medium is comprised of a beam of relativistic electrons traveling in a vacuum through a periodic magnetic field. The basic components common to all FELs are a relativistic electron beam, a periodic magnetic structure (an undulator or wiggler magnet of spatial period λ_w), and an optical resonator providing feedback and amplification. (X-ray FELs such as the Linac Coherent Light Source at Stanford omit the optical resonator by necessity and achieve the required gain on a single pass.) The features that make FELs particularly useful as research devices are the unique combination of continuous and broadband tunability, high peak and average power, and spatial and temporal coherence.

The design features include accelerating charges, but also use wave guides.

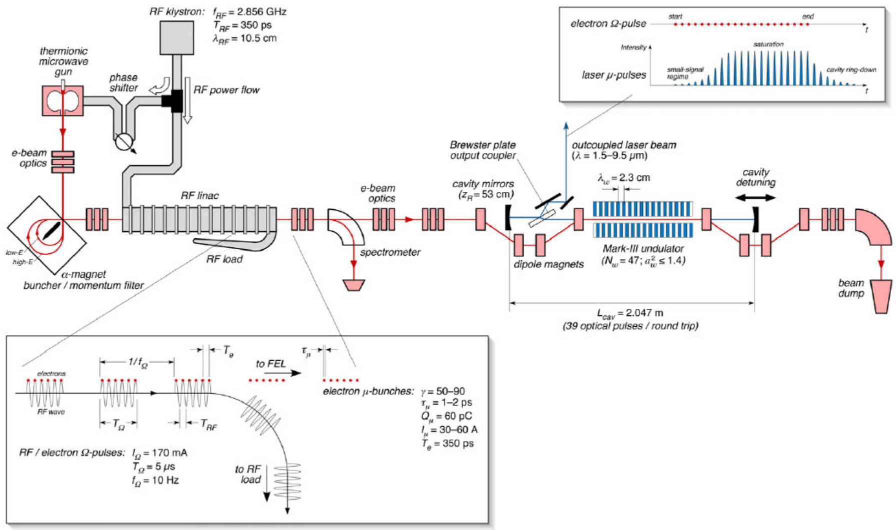
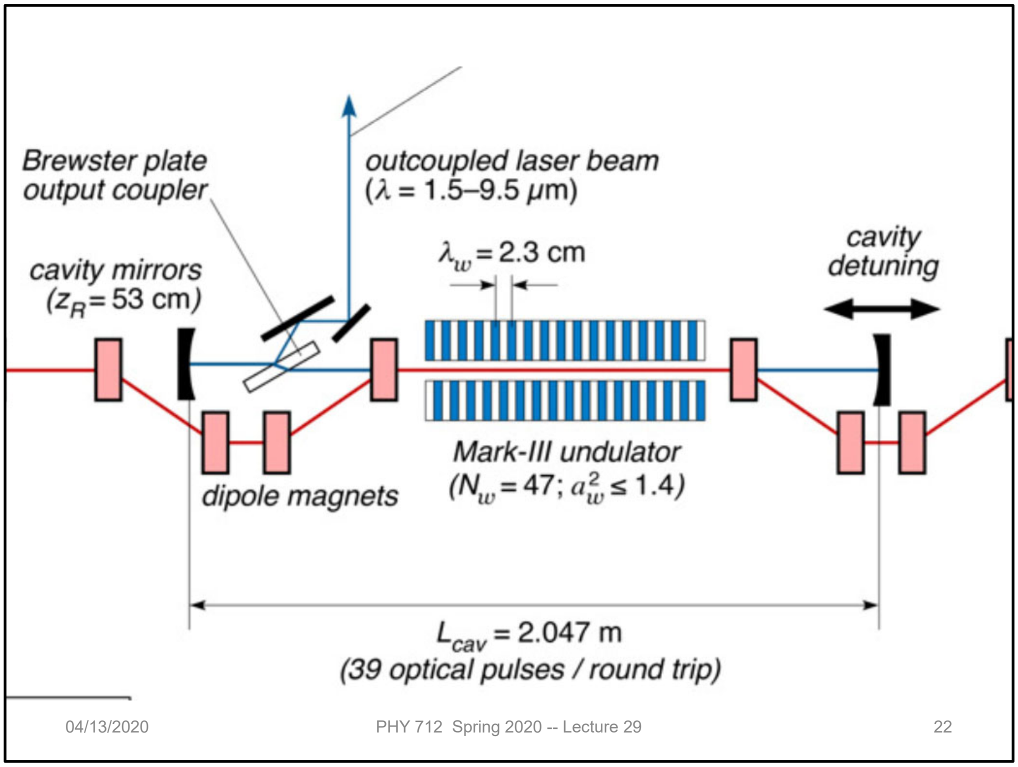


Figure 1.1. The MkIII RF-linac FEL.

Diagram from the text.



Portion of the device where the light forms a standing wave between two mirrors and the electron is manipulated with periodic magnetics.

Electron emission in periodic magnet

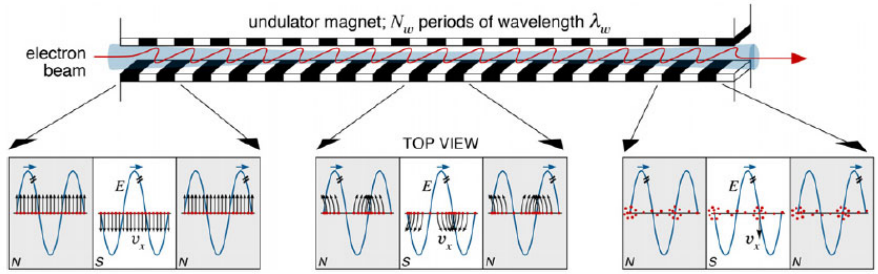


Figure 1.3. Conceptual illustration of the bunching mechanism in an FEL.

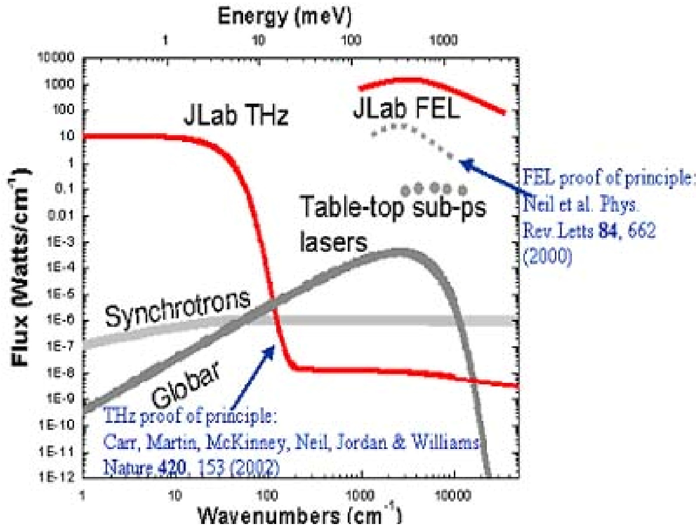
Because of Doppler shift and Thompson scattering, effective

wavelength is: $$\lambda = \frac{\lambda_w}{2\gamma_z^2}$$

frequency is: $$\omega = \frac{4\pi}{\lambda_w}\gamma_z^2 c$$

Lots of physical processes are engineered into this device.

Spectral output for FEL at Jefferson Lab (in Virginia)



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Data from Free Electron Laser facility in Virginia showing the range of intensities of various light sources

REVIEW: LASER TECHNOLOGY

Free-Electron Lasers: Status and Applications

Patrick G. O'Shea[1] and Henry P. Freund[2]

SCIENCE VOL 292 8 JUNE 2001

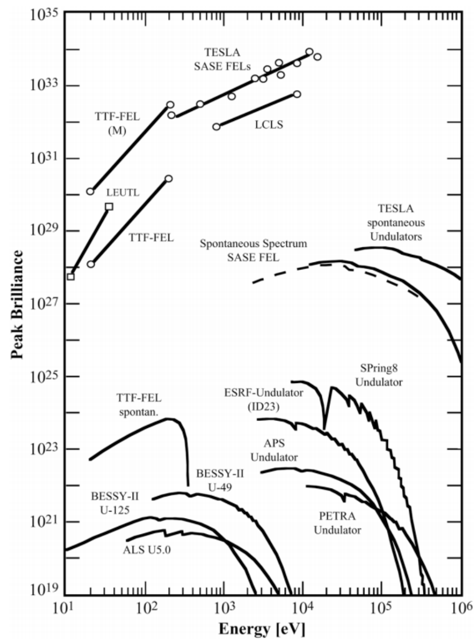


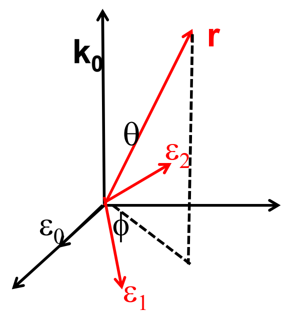
Fig. 3. Peak brilliance of x-ray FELs and undulators for spontaneous radiation at the TESLA Test Facility, in comparison with synchrotron radiation sources. Brilliance is expressed as photons s^{-1} $mrad^{-2}$ mm^{-2} per 0.1% bandwidth. For comparison, the spontaneous spectrum of x-ray FEL undulators is also shown. The label TTF-FEL indicates design values for the FEL at the TESLA Test Facility, with (M) for the planned seeded version (*28*).

04/13/2020

Data from a review article on Free Electron Lasers

Back to fundamental processes – Thompson and Compton scattering

Some details of scattering of electromagnetic waves incident on a particle of charge q and mass m_q



$$\mathbf{E}(\mathbf{r},t) = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k_0}\cdot\mathbf{r}-i\omega t}\right)$$

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For next time.