PHY 712 Electrodynamics

12-12:50 AM MWF via video link:
https://wakeforest-university.zoom.us/my/natalie.holzwarth

Plan for Lecture 33:

Special Topics in Electrodynamics:

Electromagnetic aspects of superconductivity

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In this lecture we will discuss some of the aspects of superconductivity that involve electromagnetism, without getting into the quantum mechanical mechanisms.

21	Mon: 03/23/2020	Chap. 9	Radiation from localized oscillating sources	#17	03/25/2020
22	Wed: 03/25/2020	Chap. 9	Radiation from oscillating sources	#18	03/27/2020
23	Fri: 03/27/2020	Chap. 9 and 10	Radiation from oscillating sources	#19	03/30/2020
24	Mon: 03/30/2020	Chap. 11	Special Theory of Relativity	#20	04/03/2020
25	Wed: 04/01/2020	Chap. 11	Special Theory of Relativity		
26	Fri: 04/03/2020	Chap. 11	Special Theory of Relativity	#21	04/06/2020
27	Mon: 04/06/2020	Chap. 14	Radiation from accelerating charged particles	#22	04/08/2020
28	Wed: 04/08/2020	Chap. 14	Synchrotron radiation		
	Fri: 04/10/2020	No class	*Good Friday*		
29	Mon: 04/13/2020	Chap. 14	Synchrotron radiation	#23	04/15/2020
30	Wed: 04/15/2020	Chap. 15	Radiation from collisions of charged particles	#24	04/17/2020
31	Fri: 04/17/2020	Chap. 15	Radiation from collisions of charged particles		
32	Mon: 04/20/2020	Chap. 13	Cherenkov radiation		
33	Wed: 04/22/2020		Special topic: E & M aspects of superconductivity		
34	Fri: 04/24/2020		Special topic: Aspects of optical properties of materials		
35	Mon: 04/27/2020		Review		
36	Wed: 04/29/2020		Review		

Important dates: Final exams available May1; due May 11
Projects and outstanding HW due May 11

Please note the important dates.

Advice about projects

Each project should be roughly ~ 5 pages (using word, latex, annotated Mathematica or Maple, etc.)

It should contain the following

1. Introduction and motivation
2. Some detailed derivation and/or numerical work
3. Conclusions and summary of what you learned
4. Bibliography (If you have chosen to review a literature paper, please include its pdf file if possible.)

Hopefully this is manageable…

Special topic: Electromagnetic properties of superconductors

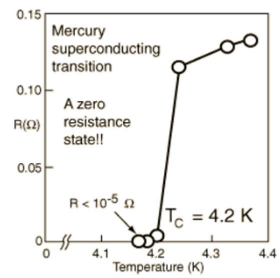
Ref:D. Teplitz, editor, Electromagnetism – paths to research, Plenum Press (1982); Chapter 1 written by Brian Schwartz and Sonia Frota-Pessoa

History:

1908 H. Kamerlingh Onnes successfully liquified He

1911 H. Kamerlingh Onnes discovered that Hg at 4.2 K has vanishing resistance

1957 Theory of superconductivity by Bardeen, Cooper, and Schrieffer



These notes are partly based on the Teplitz textbook and other sources. Interestingly this is an example of a physical phenomenon stumping the theorists for nearly 50 years. The theorists are still arguing.

Fritz London 1900-1954

Fritz London, 1947, photo: Lotte Meitner-Graf

Fritz London, one of the most distinguished scientists on the Duke University faculty, was an internationally recognized theorist in Chemistry, Physics and the Philosophy of Science. He was born in Breslau, Germany (now Wroclaw, Poland) in 1900. In 1933 he was

He immigrated to the United States in 1939, and came to Duke University, first as a Professor of Chemistry. In 1949 he received a joint appointment in Physics and Chemistry and became a James B. Duke Professor. In 1953 he became the 5th recipient of the Lorentz medal, awarded by the Royal Netherlands Academy of Sciences, and was the first American citizen to receive this honor. He died in Durham in 1954.

https://phy.duke.edu/about/history/historical-faculty/fritz-london

The ideas we will discuss are largely due to Fritz London who developed a phenomenological theory before the microscopic materials mechanisms were developed by Bardeen, Cooper, and Schrieffer a few years after he died.

Some phenomenological theories < 1957 thanks to F. London

Drude model of conductivity in "normal" materials

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E} - m\frac{\mathbf{v}}{\tau}$$

$$\mathbf{v}(t) = \mathbf{v}_0 e^{-t/\tau} - \frac{e\mathbf{E}\tau}{m}$$

$$\mathbf{J} = -ne\mathbf{v}; \qquad \text{for } t >> \tau \qquad \Rightarrow \qquad \mathbf{J} = \frac{ne^2\tau}{m}\mathbf{E} \equiv \sigma\mathbf{E}$$

Note: Equations are in cgs Gaussian units.

London model of conductivity in superconducting materials; $\tau \to \infty$

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E}$$

$$\frac{d\mathbf{v}}{dt} = -\frac{e\mathbf{E}}{m} \qquad \frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

From Maxwell's equations:

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

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These equations represent models of idealized electrons in metals, starting with the Drude model which we previously discussed. The symbol tau represents a "relaxation" time; n represents the number density.

Some phenomenological theories < 1957

London model of conductivity in superconducting materials

$$\frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

From Maxwell's equations:

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} \qquad \nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\left(\nabla\times\mathbf{B}\right) = -\nabla^2\mathbf{B} = \frac{4\pi}{c}\nabla\times\mathbf{J} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2}$$

$$-\nabla^2\frac{\partial\mathbf{B}}{\partial t} = \frac{4\pi}{c}\nabla\times\frac{\partial\mathbf{J}}{\partial t} - \frac{1}{c^2}\frac{\partial^3\mathbf{B}}{\partial t^3}$$

$$-\nabla^2\frac{\partial\mathbf{B}}{\partial t} = \frac{4\pi ne^2}{mc}\nabla\times\mathbf{E} - \frac{1}{c^2}\frac{\partial^3\mathbf{B}}{\partial t^3}$$

$$-\nabla^2\frac{\partial\mathbf{B}}{\partial t} = -\frac{4\pi ne^2}{mc^2}\frac{\partial\mathbf{B}}{\partial t} - \frac{1}{c^2}\frac{\partial^3\mathbf{B}}{\partial t^3}$$

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B} = 0 \qquad \text{with } \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

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Following the logic of London's equations. Here lambda which comes out of the analysis is a parameter with units of length.

London model – continued

London model of conductivity in superconducting materials

$$\frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B} = 0 \qquad \text{with } \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

For slowly varying solution:

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2}\right)\mathbf{B} = 0 \qquad \text{for } \frac{\partial \mathbf{B}}{\partial t} = \hat{\mathbf{z}}\frac{\partial B_z(x,t)}{\partial t}:$$

$$\Rightarrow \frac{\partial B_z(x,t)}{\partial t} = \frac{\partial B_z(0,t)}{\partial t}e^{-x/\lambda_L}$$

London leap: $B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

Consistent results for current density:

$$\frac{4\pi}{c}\nabla\times\mathbf{J} = -\nabla^2\mathbf{B} = -\frac{1}{\lambda_L^2}\mathbf{B} \qquad \mathbf{J} = \hat{\mathbf{y}}J_y(x) \quad \Rightarrow J_y(x) = \lambda_L\frac{ne^2}{mc}\mathrm{B}_z(0)\,\mathrm{e}^{-x/\lambda_L}$$

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Fancy thinking with the time dependence. The result shows that the B field decays within the material within a distance lambda. Similarly, the current density also decays within the material.

London model – continued

Penetration length for superconductor: $\lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$ Typically, $\lambda_L \approx 10^{-7} m$

$B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

Vector potential for $\mathbf{B} = \nabla \times \mathbf{A}$ and $\nabla \cdot \mathbf{A} = 0$:

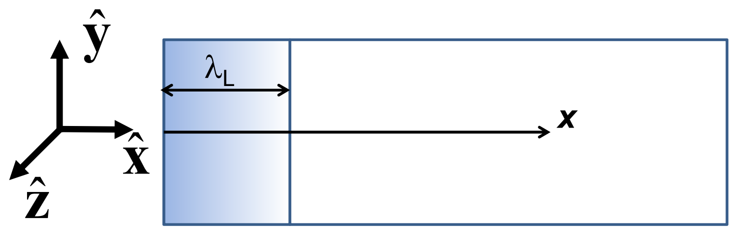
$\mathbf{A} = \hat{\mathbf{y}} A_y(x)$ $\quad A_y(x) = -\lambda_L B_z(0) e^{-x/\lambda_L}$

Note that: $\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}$

$-\nabla^2 \mathbf{A} = \frac{4\pi}{c}\mathbf{J} \Rightarrow \nabla^2 \mathbf{A} + \frac{4\pi}{c}\mathbf{J} = 0$

Recall form for current density: $J_y(x) = \lambda_L \frac{ne^2}{mc} \mathrm{B}_z(0)\, \mathrm{e}^{-x/\lambda_L}$

$\Rightarrow \mathbf{J} + \frac{ne^2}{mc}\mathbf{A} = 0$ or $\frac{ne}{m}\left(m\mathbf{v} + \frac{e}{c}\mathbf{A} \right) = 0$



The conclusion is that the current and magnetic field are excluded from the bulk of the superconductor; they are confined within a length lambda at the surface.

Behavior of superconducting material – exclusion of magnetic field according to the London model

Penetration length for superconductor: $\lambda_L^2 \equiv \dfrac{mc^2}{4\pi n e^2}$

$$B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$$

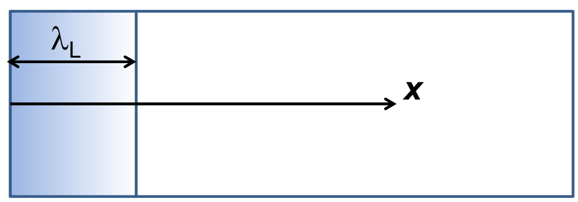
Vector potential for $\nabla \cdot \mathbf{A} = 0$:

$$\mathbf{A} = \hat{\mathbf{y}} A_y(x) \qquad A_y(x) = -\lambda_L B_z(0) e^{-x/\lambda_L}$$

Current density: $J_y(x) = \lambda_L \dfrac{ne^2}{mc} \mathrm{B}_z(0)\,\mathrm{e}^{-x/\lambda_L}$

$$\Rightarrow \mathbf{J} + \frac{ne^2}{mc}\mathbf{A} = 0 \quad \text{or} \quad \frac{ne}{m}\left(m\mathbf{v} + \frac{e}{c}\mathbf{A}\right) = 0$$

Typically, $\lambda_L \approx 10^{-7} m$



Lambda is also called the London penetration length.

Behavior of magnetic field lines near superconductor

normal state:

superconducting state:

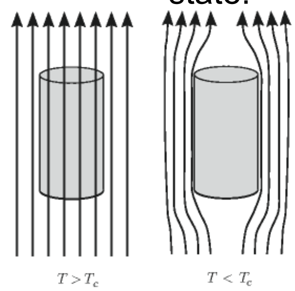
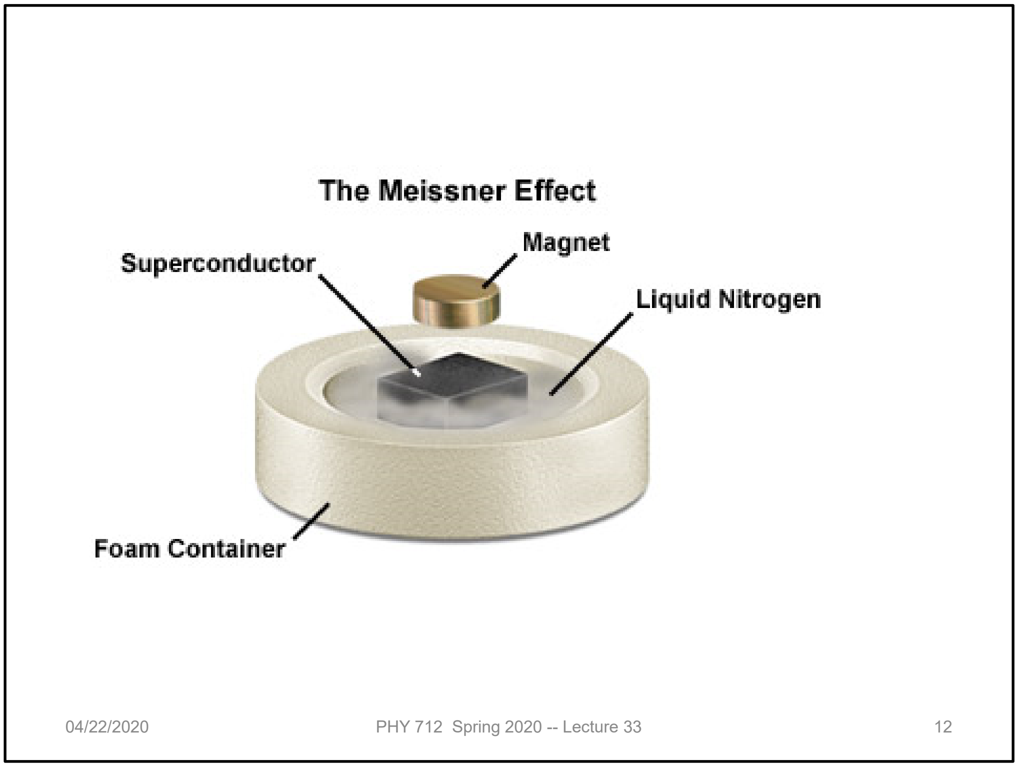


Figure 18.2 Exclusion of a weak external magnetic field from the interior of a superconductor.

An illustration of the phenomenon in three dimensions.



Demonstration of the magnetic field effects when a small permanent magnetic is put above a superconducting magnetic. In this case the liquid N2 is needed to produce the superconducting phase of the material.

Need to consider phase equilibria between "normal" and superconducting state as a function of temperature and applied magnetic fields.

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$$

Within the superconductor, if $\mathbf{B} = 0$

$$\text{then} \quad \mathbf{H} + 4\pi\mathbf{M} = 0 \quad \text{or} \quad \mathbf{M} = -\frac{\mathbf{H}}{4\pi}$$

Interesting properties of the magnetization field of a superconductor.

Magnetization field

Treating London current in terms of corresponding magnetization field **M**:

$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$

$\Rightarrow$ For $x >> \lambda_L$, $\quad \mathbf{H} = -4\pi\mathbf{M}, \qquad \mathbf{M}(\mathbf{H}) = -\frac{\mathbf{H}}{4\pi}$

Gibbs free energy associated with magnetization for superconductor:

$$G_S(H_a) = G_S(H=0) - \int_0^{H_a} dH M(H) = G_S(0) - \int_0^{H_a} dH\left(\frac{-H}{4\pi}\right) = G_S(0) + \frac{1}{8\pi}H_a^2$$

This relation is true for an applied field $H_a \leq H_C$ when the superconducting and normal Gibbs free energies are equal:

$G_S(H_C) = G_N(H_C) \approx G_N(H=0)$

Condition at phase boundary between normal and superconducting states:

$$G_N(H_C) \approx G_N(0) = G_S(H_C) = G_S(0) + \frac{1}{8\pi}H_C^2 \qquad \text{At } T=0K$$

$$\Rightarrow G_S(0) - G_N(0) = -\frac{1}{8\pi}H_C^2$$

$$G_S(H_a) - G_N(H_a) = \begin{cases} -\frac{1}{8\pi}\left(H_C^2 - H_a^2\right) & \text{for } H_a < H_C \\ 0 & \text{for } H_a > H_C \end{cases}$$

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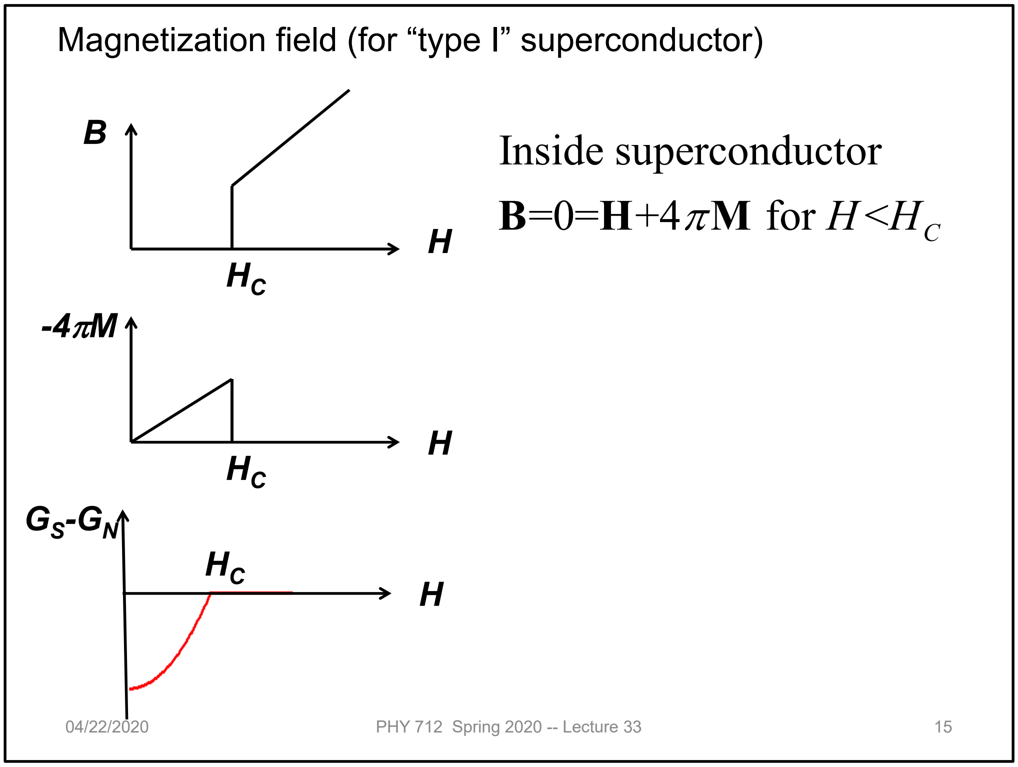
Here we need to consider thermodynamics of phase change. The Gibbs free energy of the superconducting state can be estimated. An applied magnetic field can raise the Gibbs free energy so that the superconducting phase is less favorable than the normal phase.



Plets of fields and Gibbs energy as a function of the applied field H.

PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. BARDEEN, L. N. COOPER,† AND J. R. SCHRIEFFER‡
Department of Physics, University of Illinois, Urbana, Illinois
(Received July 8, 1957)

$$G_S(0) - G_N(0) = -\frac{H_C^2}{8\pi} \approx -2N(E_F)(\hbar\omega)^2 e^{-2/(N(E_F)V)}$$

characteristic phonon energy

density of electron states at E_F

attraction potential between electron pairs

Briefly, BCS theory estimated the energy of a superconductor relative to a normal metal at room temperature

Temperature dependence of critical field

$$H_c(T) \approx H_c(0)\left(1-\left(\frac{T}{T_c}\right)^2\right)$$

From PR **108**, 1175 (1957) Bardeen, Cooper, and Schrieffer, “Theory of Superconductivity”

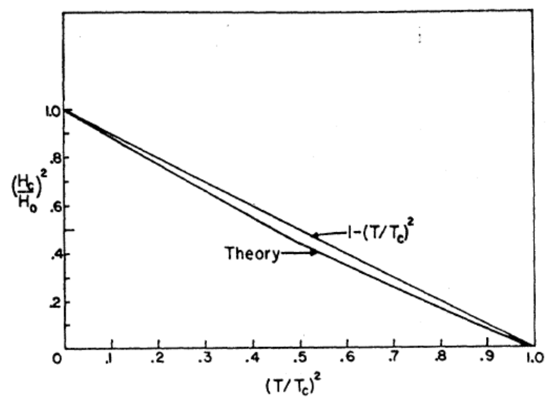


FIG. 2. Ratio of the critical field to its value at $T=0°K$ *vs* $(T/T_c)^2$. The upper curve is the $1-(T/T_c)^2$ law of the Gorter-Casimir theory and the lower curve is the law predicted by the theory in the weak-coupling limit. Experimental values generally lie between the two curves.

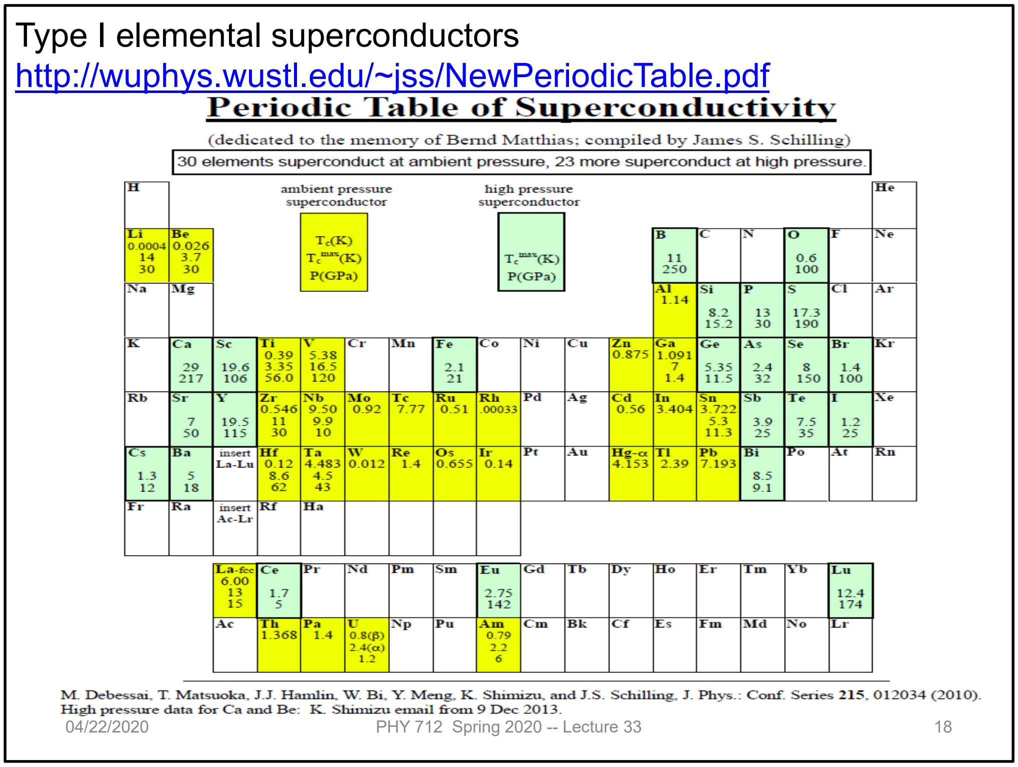
$$T_c \approx \frac{\hbar\omega}{k} e^{-2/(N(E_F)V)}$$

characteristic phonon energy

density of electron states at E_F

attraction potential between electron pairs

The energy and critical field depends on temperature in a characteristic way predicted by the theory.



Some elemental super conductors.

Type I superconductors:

$$H_c(T) = H_c(0)\left(1 - \frac{T^2}{T_c^2}\right)$$

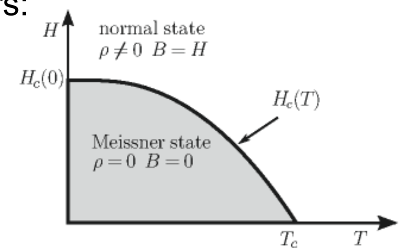


Figure 18.3 Schematic phase diagram illustrating normal and superconducting regions of a type-I superconductor.

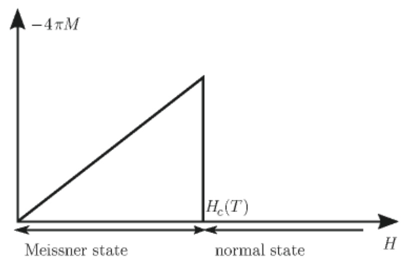


Figure 18.4 Magnetization versus applied field for type-I superconductors.

This discussion is relevant to “type I” superconductors.

Type II superconductors

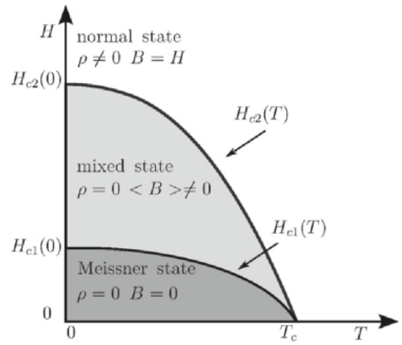


Figure 18.5 Schematic phase diagram illustrating normal, mixed and Meissner regions of a type-II superconductor (the vanishingly small resistivity of the mixed state occurs if flux lines are "pinned" by appropriate material defects); in the mixed state, $\langle B \rangle$ denotes the average magnetic field in the superconductor.

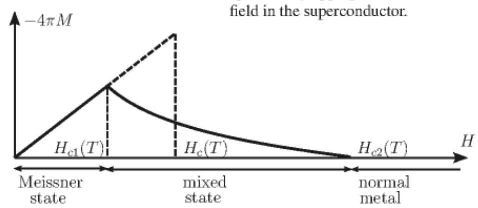


Figure 18.6 Magnetization versus applied field H for a type-II superconductor. The equivalent area construction of the thermodynamic field $H_c(T)$ is also illustrated.

Type II superconductors are more complicated. This model is more consistent with the so called high temperature superconductors.

Quantization of current flux associated with the superconducting state (Ref: Ashcroft and Mermin, ***Solid State Physics***)

From the London equations for the interior of the superconductor:

$$\left(m\mathbf{v}+\frac{e}{c}\mathbf{A}\right) = 0$$

Now suppose that the current carrier is a pair of electrons characterized by a wavefunction of the form $\psi = |\psi| e^{i\phi}$

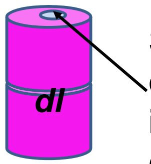
The quantum mechanical current associated with the electron pair is

$$\mathbf{j} = -\frac{e\hbar}{2mi}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) - \frac{2e^2}{mc}\mathbf{A}|\psi|^2$$

$$= -\left(\frac{e\hbar}{m}\nabla\phi + \frac{2e^2}{mc}\mathbf{A}\right)|\psi|^2$$

Part of the story is that there can be (quantized) fields (vortices) within type II superconductors. This slide discusses some aspects of the currents.

Quantization of current flux associated with the superconducting state -- continued



Suppose a superconducting material has a cylindrical void. Evaluate the integral of the current in a closed path within the superconductor containing the void.

$$\oint \mathbf{j} \cdot d\mathbf{l} = 0 = -\oint \left(\frac{e\hbar}{m} \nabla \phi + \frac{2e^2}{mc} \mathbf{A} \right) |\psi|^2 \cdot d\mathbf{l}$$

$$\oint \mathbf{A} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \int \mathbf{B} \cdot d\mathbf{a} = \Phi \quad \text{magnetic flux}$$

$$\oint \nabla \phi \cdot d\mathbf{l} = 2\pi n \qquad \text{for some integer } n$$

$$\Rightarrow \text{Quantization of flux in the void: } |\Phi| = n \frac{hc}{2e} \equiv n\Phi_0$$

Such "vortex" fields can exist within type II superconductors.

The analysis follows from the notion that the wavefunction of the superconducting "particle" has a non-trivial phase factor.

Table 18.1 Critical temperature of some selected superconductors, and zero-temperature critical field. For elemental materials, the thermodynamic critical field $H_c(0)$ is given in gauss. For the compounds, which are type-II superconductors, the upper critical field $H_{c2}(0)$ is given in Tesla ($1\ \mathrm{T} = 10^4$ G). The data for metallic elements and binary compounds of V and Nb are taken from G. Burns (1992). The data for MgB_2 and iron pnictide are taken from the references cited in the text, and refer to the two principal crystallographic axes. The data for the other compounds are taken from D. R. Harshman and A. P. Mills, Phys. Rev. B *45*, 10684 (1992)]. A more extensive list of data can be found in the mentioned references.

Metallic elements	$T_c(K)$	$H_c(0)$ (gauss)
Al	1.17	105
Sn	3.72	305
Pb	7.19	803
Hg	4.15	411
Nb	9.25	2060
V	5.40	1410
Binary compounds	$T_c(K)$	$H_{c2}(0)$ (Tesla)
V_3Ga	16.5	27
V_3Si	17.1	25
Nb_3Al	20.3	34
Nb_3Ge	23.3	38
MgB_2	40	$\approx 5; \approx 20$
Other compounds	$T_c(K)$	$H_{c2}(0)$ (Tesla)
UPt_3 (heavy fermion)	0.53	2.1
$PbMo_6S_8$ (Chevrel phase)	12	55
$\kappa-[BEDT-TTF]_2Cu[NCS]_2$ (organic phase)	10.5	≈ 10
Rb_2CsC_{60} (fullerene)	31.3	≈ 30
$NdFeAsO_{0.7}F_{0.3}$ (iron pnictide)	47	$\approx 30;\ \approx 50$
Cuprate oxides	$T_c(K)$	$H_{c2}(0)$ (Tesla)
$La_{2-x}Sr_xCuO_4$ ($x \approx 0.15$)	38	≈ 45
$YBa_2Cu_3O_7$	92	≈ 140
$Bi_2Sr_2CaCu_2O_8$	89	≈ 107
$Tl_2Ba_2Ca_2Cu_3O_{10}$	125	≈ 75

Some superconducting materials listed on the web.

Crystal structure of one of the high temperature superconductors

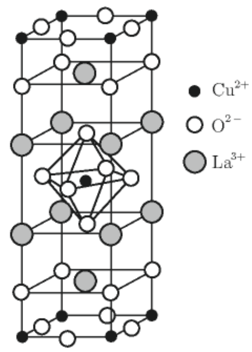


Figure 18.1 Crystal structure of the ceramic material La_2CuO_4. Appropriately doped, lanthanum-based cuprates opened the path to high-T_c superconductivity in 1986.

One of the high temperature superconducting materials.

Some details of single vortex in type II superconductor

London equation without vortices:

$$\frac{4\pi}{c}\nabla\times\mathbf{J} = -\nabla^2\mathbf{B} = -\frac{1}{\lambda_L^2}\mathbf{B} \qquad \text{where} \quad \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

Equation for field with single quantum of vortex along z - axis:

$$\nabla^2\mathbf{B} - \frac{1}{\lambda_L^2}\mathbf{B} = -\frac{\Phi_0}{\lambda_L^2}\hat{\mathbf{z}}\delta(\mathbf{r}) \qquad \Phi_0 = \frac{hc}{2e} \qquad \mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$$

Solution: $\mathbf{B}(\mathbf{r}) = \hat{\mathbf{z}}\frac{\Phi_0}{2\pi\lambda_L^2}K_0\left(\frac{r}{\lambda_L}\right)$

Check:

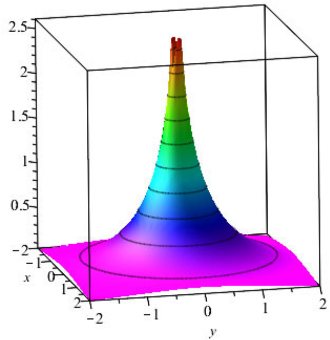
For $r > 0$ $\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{\lambda_L^2}\right)K_0\left(\frac{r}{\lambda_L}\right) = 0$

For $r \to 0$ $2\pi\int_0^r dr'\, r'\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{\lambda_L^2}\right)K_0\left(\frac{r}{\lambda_L}\right) = -2\pi$

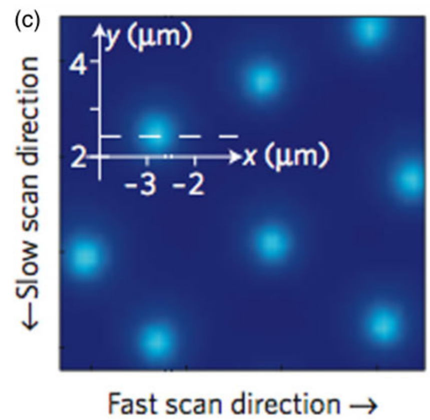
Since $\mathrm{K}_0(u) \underset{u\to 0}{\approx} -\ln u$

Equations demonstrating that vortex solutions are consistent with London's model.

$$\mathbf{B}(\mathbf{r}) = \hat{\mathbf{z}}\,\frac{\Phi_0}{2\pi\lambda_L^2}\,K_0\left(\frac{r}{\lambda_L}\right)$$



Scanning probe images of vortices in YBCO at 22 K



IOP PUBLISHING

Rep. Prog. Phys. 73 (2010) 126501 (36pp) doi:10.1088/0034-4885/73/12/126501

Fundamental studies of superconductors using scanning magnetic imaging

J R Kirtley

Center for Probing the Nanoscale, Stanford University, Stanford, CA, USA

Scanning probe techniques can be used to visualize the magnetic vortices.